3
Derivatives and Graphs

In this chapter, we explore what the first and second derivatives of a function tell us about the graph of that function and apply this graphical knowledge to locate the extreme values of a function.

3.1 Finding Maximums and Minimums

In theory and applications, we often want to maximize or minimize some quantity. An engineer may want to maximize the speed of a new computer or minimize the heat produced by an appliance. A manufacturer may want to maximize profits and market share or minimize waste. A student may want to maximize a grade in calculus or minimize the hours of study needed to earn a particular grade.

Many natural objects follow minimum or maximum principles, so if we want to model natural phenomena we may need to maximize or minimize. A light ray travels along a "minimum time" path. The shape and surface texture of some animals tend to minimize or maximize heat loss. Systems reach equilibrium when their potential energy is minimized. A basic tenet of evolution is that a genetic characteristic that maximizes the reproductive success of an individual will become more common in a species.

Calculus provides tools for analyzing functions and their behavior and for finding maximums and minimums.

Methods for Finding Maximums and Minimums

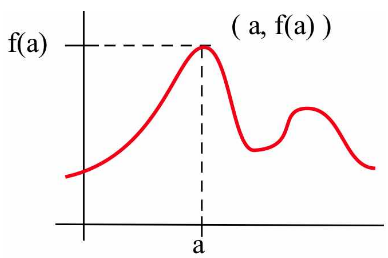
We can try to find where a function f is largest or smallest by evaluating f at lots of values of x, a method that is not very efficient and may not find the exact place where f achieves its extreme value. If we try hundreds or thousands of values for x, however, then we can often find a value of f that is close to the maximum or minimum. In general, this type of exhaustive search is only practical if you have a computer do the work.

The graph of a function provides a visual way of examining lots of values of f, and it is a good method, particularly if you have a computer to do the work for you. It is still inefficient, however, as you (or a computer) still need to evaluate the function at hundreds or thousands of inputs in order to create the graph—and we still may not find the exact location of the maximum or minimum.

Calculus provides ways to drastically narrow the number of points we need to examine to find the exact locations of maximums and minimums. Instead of examining f at thousands of values of x, calculus can often guarantee that the maximum or minimum must occur at one of three or four values of x, a substantial improvement in efficiency.

A Little Terminology

Before we examine how calculus can help us find maximums and minimums, we need to carefully define these concepts.



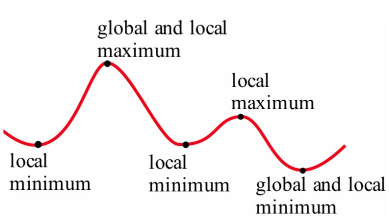
Definitions:

- f has a **maximum** or **global maximum** at $x = a$ if $f(a) \geq f(x)$ for all x in the domain of f.
- The **maximum value** of f is then $f(a)$ and this maximum value of f **occurs at** a.
- The **maximum point** on the graph of f is $(a, f(a))$.

The previous definition involves the overall biggest value a function attains on its entire domain. We are sometimes interested in how a function behaves locally rather than globally.

Definition: f has a **local** or **relative maximum** at $x = a$ if $f(a) \geq f(x)$ for all x "near" a, (that is, in some open interval that contains a).

Global and local **minimums** are defined similarly by replacing the $\geq$ symbol with $\leq$ in the previous definitions.



Definition:

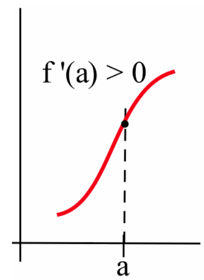
f has a **global extreme** at $x = a$
if $f(a)$ is a global maximum or minimum.

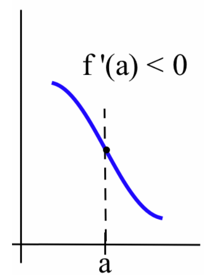
See the margin figure for graphical examples of local and global extremes of a function.

You should notice that every global extreme is also a local extreme, but there are local extremes that are not global extremes. If $h(x)$ is the height of the earth above sea level at location x, then the global maximum of h is $h(\text{summit of Mt. Everest}) = 29{,}028$ feet. The local maximum of h for the United States is $h(\text{summit of Mt. McKinley}) = 20{,}320$ feet. The local minimum of h for the United States is $h(\text{Death Valley}) =$ -282 feet.

Finding Maximums and Minimums of a Function

One way to narrow our search for a maximum value of a function f is to eliminate those values of x that, for some reason, cannot possibly make f maximum.





Theorem:

If $f'(a) > 0$ or $f'(a) < 0$
then $f(a)$ is not a local maximum or minimum.

Proof. Assume that $f'(a) > 0$. By definition:

$$f'(a) = \lim_{\Delta x \to 0} \frac{f(a + \Delta x) - f(a)}{\Delta x}$$

so $f'(a) = \lim_{\Delta x \to 0} \frac{f(a + \Delta x) - f(a)}{\Delta x} > 0$. This means that the right and left limits are both positive: $f'(a) = \lim_{\Delta x \to 0^+} \frac{f(a + \Delta x) - f(a)}{\Delta x} > 0$ and $f'(a) = \lim_{\Delta x \to 0^-} \frac{f(a + \Delta x) - f(a)}{\Delta x} > 0$.

Considering the right limit, we know that if we restrict $\Delta x > 0$ to be sufficiently small, we can guarantee that $\frac{f(a + \Delta x) - f(a)}{\Delta x} > 0$ so, multiplying each side of this last inequality by the positive number Δx, we have $f(a + \Delta x) - f(a) > 0 \Rightarrow f(a + \Delta x) > f(a)$ for all sufficiently small values of $\Delta x > 0$, so any open interval containing $x = a$ will also contain values of x with $f(x) > f(a)$. This tell us that $f(a)$ is not a maximum.

Considering the left limit, we know that if we restrict $\Delta x < 0$ to be sufficiently small, we can guarantee that $\frac{f(a + \Delta x) - f(a)}{\Delta x} > 0$ so, multiplying each side of this last inequality by the negative number Δx, we have $f(a + \Delta x) - f(a) < 0 \Rightarrow f(a + \Delta x) < f(a)$ for all sufficiently small values of $\Delta x < 0$, so any open interval containing $x = a$ will also contain values of x with $f(x) < f(a)$. This tell us that $f(a)$ is not a minimum.

The argument for the "$f'(a) < 0$" case is similar. □

When we evaluate the derivative of a function f at a point $x = a$, there are only four possible outcomes: $f'(a) > 0$, $f'(a) < 0$, $f'(a) = 0$ or $f'(a)$ is undefined. If we are looking for extreme values of f, then we can eliminate those points at which f' is positive or negative, and only two possibilities remain: $f'(a) = 0$ or $f'(a)$ is undefined.

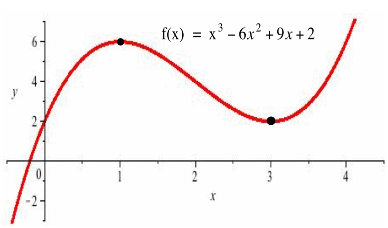
Theorem:

If	f is defined on an open interval
	and $f(a)$ is a local extreme of f
then	either $f'(a) = 0$ or f is not differentiable at a.

Example 1. Find the local extremes of $f(x) = x^3 - 6x^2 + 9x + 2$.

Solution. An extreme value of f can occur only where $f'(x) = 0$ or where f is not differentiable; $f(x)$ is a polynomial, so it is differentiable for all values of x, and we can restrict our attention to points where $f'(x) = 0$.

$$f'(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3) = 3(x-1)(x-3)$$

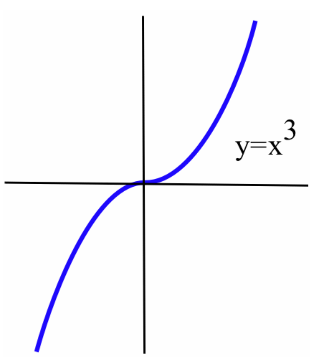


so $f'(x) = 0$ only at $x = 1$ and $x = 3$.

The only possible locations of local extremes of f are at $x = 1$ and $x = 3$. We don't know yet whether $f(1)$ or $f(3)$ is a local extreme of f, but we can be certain that no other point is a local extreme. The graph of f (see margin) shows that $(1, f(1)) = (1, 6)$ appears to be a local maximum and $(3, f(3)) = (3, 2)$ appears to be a local minimum. This function does not have a global maximum or minimum. ◀

Practice 1. Find the local extremes of $f(x) = x^2 + 4x - 5$ and of $g(x) = 2x^3 - 12x^2 + 7$.

It is important to recognize that the two conditions "$f'(a) = 0$" or "f not differentiable at a" do not guarantee that $f(a)$ is a local maximum or minimum. They only say that $f(a)$ **might** be a local extreme or that $f(a)$ is a **candidate** for being a local extreme.

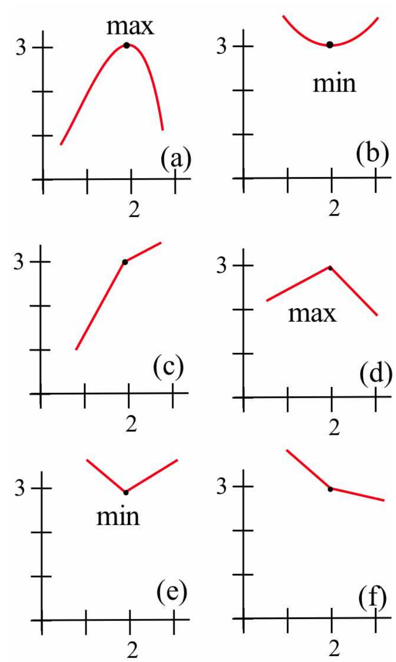


Example 2. Find all local extremes of $f(x) = x^3$.

Solution. $f(x) = x^3$ is differentiable for all x, and $f'(x) = 3x^2$ equals 0 only at $x = 0$, so the only candidate is the point $(0, 0)$. But if $x > 0$ then $f(x) = x^3 > 0 = f(0)$, so $f(0)$ is not a local maximum. Similarly, if $x < 0$ then $f(x) = x^3 < 0 = f(0)$ so $f(0)$ is not a local minimum. The point $(0, 0)$ is the only candidate to be a local extreme of f, but this candidate did not turn out to be a local extreme of f. The function $f(x) = x^3$ does not have any local extremes. ◀

If	$f'(a) = 0$ or f is not differentiable at a
then	the point $(a, f(a))$ is a candidate to be a local extreme but may not actually be a local extreme.

Practice 2. Sketch the graph of a differentiable function f that satisfies the conditions: $f(1) = 5$, $f(3) = 1$, $f(4) = 3$ and $f(6) = 7$; $f'(1) = 0$, $f'(3) = 0$, $f'(4) = 0$ and $f'(6) = 0$; the only local maximums of f are at $(1, 5)$ and $(6, 7)$; and the only local minimum is at $(3, 1)$.



Is $f(a)$ a Maximum or Minimum or Neither?

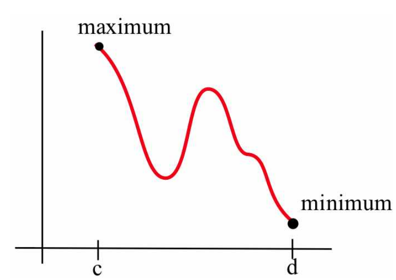
Once we have found the candidates $(a, f(a))$ for extreme points of f, we still have the problem of determining whether the point is a maximum, a minimum or neither.

One method involves graphing (or letting a calculator graph) the function near a, and then drawing a conclusion from the graph. All of the graphs in the margin have $f(2) = 3$, and on each of the graphs $f'(2)$ either equals 0 or is undefined. It is clear from the graphs that the point $(2, 3)$ is: a local maximum in (a) and (d); a local minimum in (b) and (e); and not a local extreme in (c) and (f).

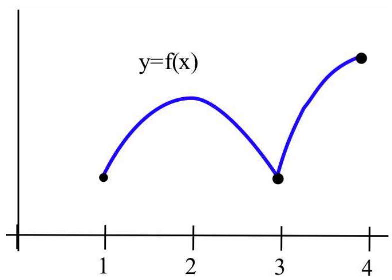
In Sections 3.3 and 3.4, we will investigate how information about the first and **second** derivatives of f can help determine whether the candidate $(a, f(a))$ is a maximum, a minimum or neither.

Endpoint Extremes

So far we have discussed finding extreme values of functions over the entire real number line or on an open interval, but in practice we may need to find the extreme of a function over some closed interval $[c, d]$. If an extreme value of f occurs at $x = a$ between c and d ($c < a < d$) then the previous reasoning and results still apply: either $f'(a) = 0$ or f is not differentiable at a. On a closed interval, however, there is one more possibility: an extreme can occur at an **endpoint** of the closed interval (see margin): at $x = c$ or $x = d$.



We can extend our definition of a local extreme at $x = a$ (which requires $f(a) \geq f(x)$ [or $f(a) \leq f(x)$] for all x in some *open* interval containing a) to include $x = a$ being the endpoint of a closed interval: $f(a) \geq f(x)$ [or $f(a) \leq f(x)$] for all x in an interval of the form $[a, a + h)$ (for left endpoints) or $(a - h, a]$ (for right endpoints), where $h > 0$ is a number small enough to guarantee the "half-open" interval is in the domain of $f(x)$. Using this extended definition, the function in the margin has a local maximum (which is also a global maximum) at $x = c$ and a local minimum (also a global minimum) at $x = d$.



Practice 3. List all of the extremes $(a, f(a))$ of the function in the margin figure on the interval $[1, 4]$ and state whether $f'(a) = 0$, f is not differentiable at a, or a is an endpoint.

Example 3. Find the extreme values of $f(x) = x^3 - 3x^2 - 9x + 5$ for $-2 \le x \le 6$.

Solution. We need to find investigate points where $f'(x) = 0$, points where f is not differentiable, and the endpoints:

- $f'(x) = 3x^2 - 6x - 9 = 3(x+1)(x-3)$, so $f'(x) = 0$ only at $x = -1$ and $x = 3$.
- f is a polynomial, so it is differentiable everywhere.
- The endpoints of the interval are $x = -2$ and $x = 6$.

Altogether we have four points in the interval to examine, and any extreme values of f can only occur when x is one of those four points: $f(-2) = 3$, $f(-1) = 10$, $f(3) = -22$ and $f(6) = 59$. The (global) minimum of f on $[-2, 6]$ is -22 when $x = 3$, and the (global) maximum of f on $[-2, 6]$ is 59 when $x = 6$. ◀

Sometimes the function we need to maximize or minimize is more complicated, but the same methods work.

Example 4. Find the extreme values of $f(x) = \frac{1}{3}\sqrt{64 + x^2} + \frac{1}{5}(10 - x)$ for $0 \le x \le 10$.

Solution. This function comes from an application we will examine in section 3.5. The only possible locations of extremes are where $f'(x) = 0$ or $f'(x)$ is undefined or where x is an endpoint of the interval $[0, 10]$.

$$\begin{aligned} f'(x) &= \mathbf{D}\left(\frac{1}{3}\left(64 + x^2\right)^{\frac{1}{2}} + \frac{1}{5}(10 - x)\right) \\ &= \frac{1}{3} \cdot \frac{1}{2}(64 + x^2)^{-\frac{1}{2}} \cdot 2x - \frac{1}{5} \\ &= \frac{x}{3\sqrt{64 + x^2}} - \frac{1}{5} \end{aligned}$$

To find where $f'(x) = 0$, set the derivative equal to 0 and solve for x:

$$\frac{x}{3\sqrt{64 + x^2}} - \frac{1}{5} = 0 \Rightarrow \frac{x}{3\sqrt{64 + x^2}} = \frac{1}{5} \Rightarrow \frac{x^2}{576 + 9x^2} = \frac{1}{25}$$
$$\Rightarrow 16x^2 = 576 \Rightarrow x = \pm 6$$

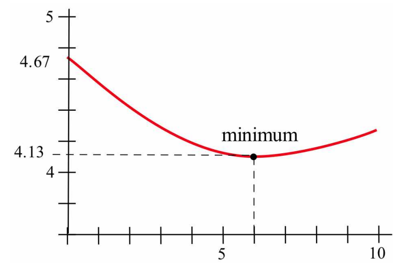
but only $x = 6$ is in the interval $[0, 10]$. Evaluating f at this point gives $f(6) \approx 4.13$.

We can evaluate the formula for $f'(x)$ for any value of x, so the derivative is always defined.

Finally, the interval $[0,10]$ has two endpoints, $x = 0$ and $x = 10$, and $f(0) \approx 4.67$ while $f(10) \approx 4.27$.

The maximum of f on $[0,10]$ must occur at one of the points $(0, 4.67)$, $(6, 4.13)$ and $(10, 4.27)$, and the minimum must occur at one of these three points as well.

The maximum value of f is 4.67 at $x = 0$, and the minimum value of f is 4.13 at $x = 6$. ◀



Practice 4. Rework the previous Example to find the extreme values of $f(x) = \frac{1}{3}\sqrt{64 + x^2} + \frac{1}{5}(10 - x)$ for $0 \le x \le 5$.

Critical Numbers

The points at which a function **might** have an extreme value are called **critical numbers**.

> **Definitions**: A **critical number** for a function f is a value $x = a$ in the domain of f so that:
>
> - $f'(a) = 0$ or
> - f is not differentiable at a or
> - a is an **endpoint** of a closed interval to which f is restricted.

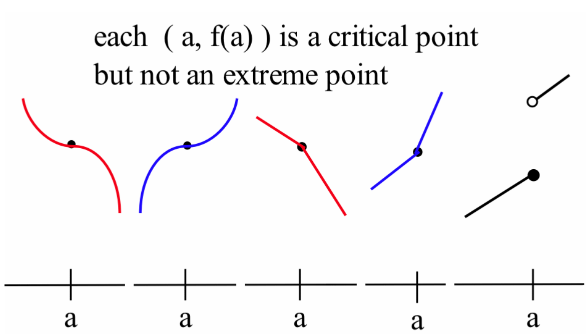
If we are trying to find the extreme values of f on an **open** interval $c < x < d$ or on the entire number line, then the set of inputs to which f is restricted will not include any endpoints, so we will not need to worry about any endpoint critical numbers.

We can now give a very succinct description of where to look for extreme values of a function.

> An extreme value of f can only occur at a critical number.

The critical numbers only give **possible** locations of extremes; some critical numbers are not locations of extremes. In other words, critical numbers are the **candidates** for the locations of maximums and minimums.

Section 3.5 is devoted entirely to translating and solving maximum and minimum problems.

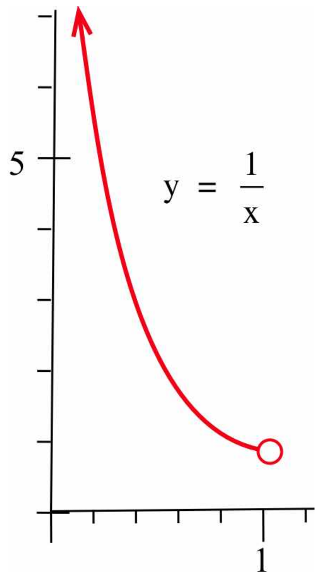


Which Functions Have Extremes?

Some functions don't have extreme values: Example 2 showed that $f(x) = x^3$ (defined on the entire number line) did not have a maximum or minimum.

Example 5. Find the extreme values of $f(x) = x$.

Solution. Because $f'(x) = 1 > 0$ for all x, the first theorem in this section guarantees that f has no extreme values. The function $f(x) = x$ does not have a maximum or minimum on the real number line. ◀



How would the situation change if we changed the interval in this example to $(0,1]$? To $[1,2]$?

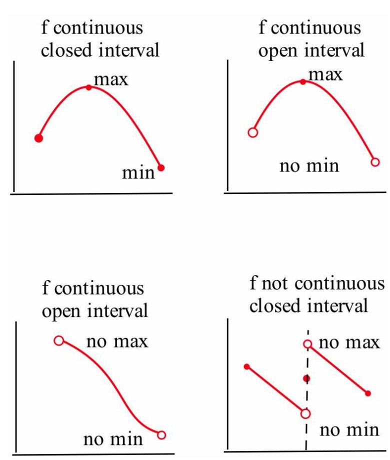
With the previous function, the domain was so large that we could always make the function output larger or smaller than any given value by choosing an appropriate input x. The next example shows that we can encounter the same difficulty even on a "small" interval.

Example 6. Show that $f(x) = \frac{1}{x}$ does not have a maximum or minimum on the interval $(0,1)$.

Solution. f is continuous for all $x \neq 0$ so f is continuous on the interval $(0,1)$. For $0 < x < 1$, $f(x) = \frac{1}{x} > 0$ and for any number a strictly between 0 and 1, we can show that $f(a)$ is neither a maximum nor a minimum of f on $(0,1)$, as follows.

Pick b to be any number between 0 and a: $0 < b < a$. Then $f(b) = \frac{1}{b} > \frac{1}{a} = f(a)$, so $f(a)$ is not a maximum. Similarly, pick c to be any number between a and 1: $a < c < 1$. Then $f(a) = \frac{1}{a} > \frac{1}{c} = f(c)$, so $f(a)$ is not a minimum. The interval $(0,1)$ is not "large," yet f does not attain an extreme value anywhere in $(0,1)$. ◀

The Extreme Value Theorem provides conditions that guarantee a function to have a maximum and a minimum.



Extreme Value Theorem:

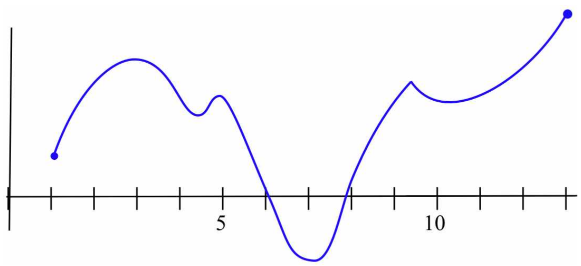
If f is continuous on a **closed** interval $[a,b]$
then f attains both a maximum and minimum on $[a,b]$.

The proof of this theorem is difficult, so we omit it. The margin figure illustrates some of the possibilities for continuous and discontinuous functions on open and closed intervals.

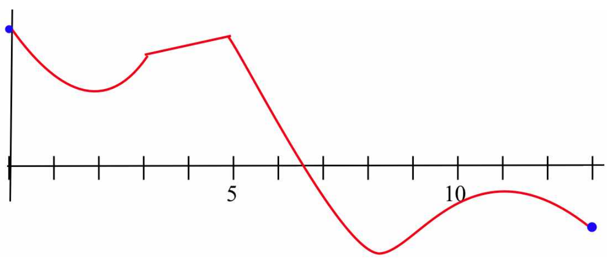
The Extreme Value Theorem guarantees that certain functions (continuous ones) on certain intervals (closed ones) must have maximums and minimums. Other functions on other intervals **may** or **may not** have maximums and minimums.

3.1 Problems

1. Label all of the local maximums and minimums of the function in the figure below. Also label all of the critical points.



2. Label the local extremes and critical points of the function graphed below.



In Problems 3–22, find all of the critical points and local maximums and minimums of each function.

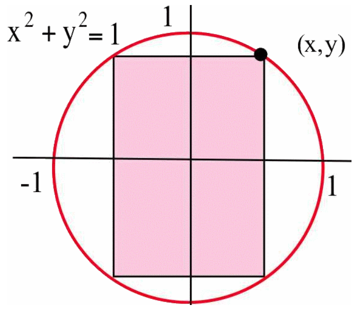
3. $f(x) = x^2 + 8x + 7$
4. $f(x) = 2x^2 - 12x + 7$
5. $f(x) = \sin(x)$
6. $f(x) = x^3 - 6x^2 + 5$
7. $f(x) = \sqrt[3]{x}$
8. $f(x) = 5x - 2$
9. $f(x) = xe^{5x}$
10. $f(x) = \sqrt[3]{1 + x^2}$
11. $f(x) = (x-1)^2(x-3)$
12. $f(x) = \ln(x^2 - 6x + 11)$
13. $f(x) = 2x^3 - 96x + 42$
14. $f(x) = 5x + \cos(2x + 1)$
15. $f(x) = e^{-(x-2)^2}$
16. $f(x) = |x + 5|$
17. $f(x) = \dfrac{x}{1 + x^2}$
18. $f(x) = \dfrac{x^3}{1 + x^4}$
19. $f(x) = (x-2)^{\frac{2}{3}}$
20. $f(x) = \left(x^2 - 1\right)^{\frac{2}{3}}$
21. $f(x) = \sqrt[3]{x^2 - 4}$
22. $f(x) = \sqrt[3]{x - 2}$

23. Sketch the graph of a continuous function f with:
    (a) $f(1) = 3$, $f'(1) = 0$ and the point $(1, 3)$ a relative maximum of f.
    (b) $f(2) = 1$, $f'(2) = 0$ and the point $(2, 1)$ a relative minimum of f.
    (c) $f(3) = 5$, f is not differentiable at $x = 3$, and the point $(3, 5)$ a relative maximum of f.
    (d) $f(4) = 7$, f is not differentiable at $x = 4$, and the point $(4, 7)$ a relative minimum of f.
    (e) $f(5) = 4$, $f'(5) = 0$ and the point $(5, 4)$ not a relative minimum or maximum of f.
    (f) $f(6) = 3$, f not differentiable at 6, and $(6, 3)$ not a relative minimum or maximum of f.

In Problems 24–37, find all critical points and local extremes of each function on the given intervals.

24. $f(x) = x^2 - 6x + 5$ on the entire real number line
25. $f(x) = x^2 - 6x + 5$ on $[-2, 5]$
26. $f(x) = 2 - x^3$ on the entire real number line
27. $f(x) = 2 - x^3$ on $[-2, 1]$
28. $f(x) = x^3 - 3x + 5$ on the entire real number line
29. $f(x) = x^3 - 3x + 5$ on $[-2, 1]$
30. $f(x) = x^5 - 5x^4 + 5x^3 + 7$ on $(-\infty, \infty)$
31. $f(x) = x^5 - 5x^4 + 5x^3 + 7$ on $[0, 2]$
32. $f(x) = \dfrac{1}{x^2 + 1}$ on $(-\infty, \infty)$
33. $f(x) = \dfrac{1}{x^2 + 1}$ on $[1, 3]$
34. $f(x) = 3\sqrt{x^2 + 4} - x$ on $(-\infty, \infty)$
35. $f(x) = 3\sqrt{x^2 + 4} - x$ on $[0, 2]$
36. $f(x) = xe^{-5x}$ on $(-\infty, \infty)$
37. $f(x) = x^3 - \ln(x)$ on $\left[\frac{1}{2}, 2\right]$
38. (a) Find two numbers whose sum is 22 and whose product is as large as possible. (Suggestion: call the numbers x and $22 - x$).
    (b) Find two numbers whose sum is $A > 0$ and whose product is as large as possible.

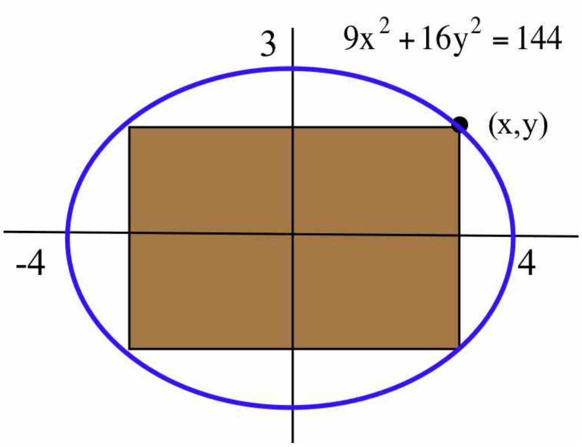
39. Find the coordinates of the point in the first quadrant on the circle $x^2 + y^2 = 1$ so that the rectangle in the figure below has the largest possible area.



40. Find the coordinates of the point in the first quadrant on the ellipse $9x^2 + 16y^2 = 144$ so that the rectangle in the figure below has:

(a) the largest possible area.

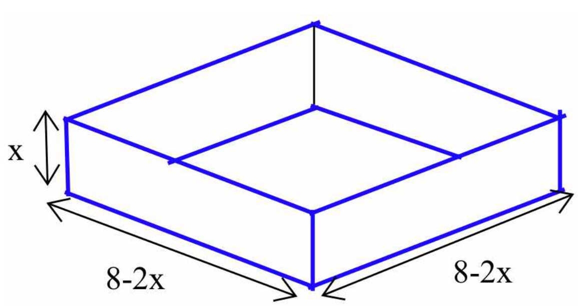
(b) The smallest possible area.



41. Find the value for x so the box shown below has:

(a) the largest possible volume.

(b) The smallest possible volume.



42. Find the radius and height of the cylinder that has the largest volume ($V = \pi r^2 h$) if the sum of the radius and height is 9.

43. Suppose you are working with a polynomial of degree 3 on a closed interval.

(a) What is the largest number of critical points the function can have on the interval?

(b) What is the smallest number of critical points it can have?

(c) What are the patterns for the most and fewest critical points a polynomial of degree n on a closed interval can have?

44. Suppose you have a polynomial of degree 3 divided by a polynomial of degree 2 on a closed interval.

(a) What is the largest number of critical points the function can have on the interval?

(b) What is the smallest number of critical points it can have?

45. Suppose $f(1) = 5$ and $f'(1) = 0$. What can we conclude about the point $(1, 5)$ if:

(a) $f'(x) < 0$ for $x < 1$ and $f'(x) > 0$ for $x > 1$?

(b) $f'(x) < 0$ for $x < 1$ and $f'(x) < 0$ for $x > 1$?

(c) $f'(x) > 0$ for $x < 1$ and $f'(x) < 0$ for $x > 1$?

(d) $f'(x) > 0$ for $x < 1$ and $f'(x) > 0$ for $x > 1$?

46. Suppose $f(2) = 3$ and f is continuous but not differentiable at $x = 2$. What can we conclude about the point $(2, 3)$ if:

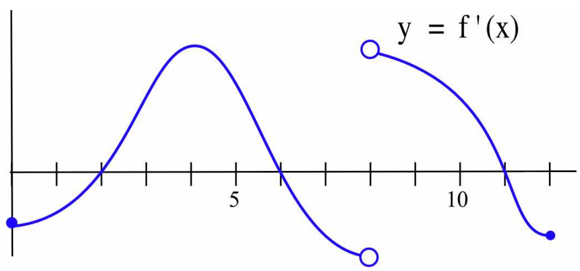
(a) $f'(x) < 0$ for $x < 2$ and $f'(x) > 0$ for $x > 2$?

(b) $f'(x) < 0$ for $x < 2$ and $f'(x) < 0$ for $x > 2$?

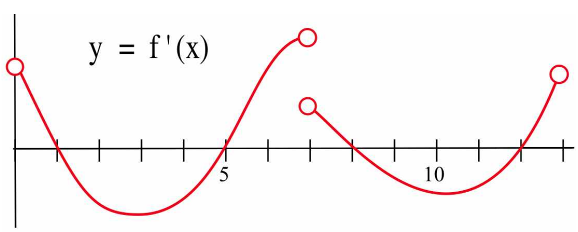
(c) $f'(x) > 0$ for $x < 2$ and $f'(x) < 0$ for $x > 2$?

(d) $f'(x) > 0$ for $x < 2$ and $f'(x) > 0$ for $x > 2$?

47. The figure below shows the graph of $f'(x)$, which is continuous on $(0, 12)$ except at $x = 8$.
   (a) Which values of x are critical points of $f(x)$?
   (b) At which values of x does f attain a local maximum?
   (c) At which values of x does f attain a local minimum?



48. The figure below shows the graph of $f'(x)$, which is continuous on $(0, 13)$ except at $x = 7$.
   (a) Which values of x are critical points?
   (b) At which values of x does f attain a local maximum?
   (c) At which values of x does f attain a local minimum?



49. State the contrapositive form of the Extreme Value Theorem.

50. Imagine the graph of $f(x) = 1 - x$. Does f have a **maximum** value for x in the given interval?

   (a) $[0, 2]$ (b) $[0, 2)$ (c) $(0, 2]$
   (d) $(0, 2)$ (e) $(1, \pi]$

51. Imagine the graph of $f(x) = 1 - x$. Does f have a **minimum** value for x in the given interval?

   (a) $[0, 2]$ (b) $[0, 2)$ (c) $(0, 2]$
   (d) $(0, 2)$ (e) $(1, \pi]$

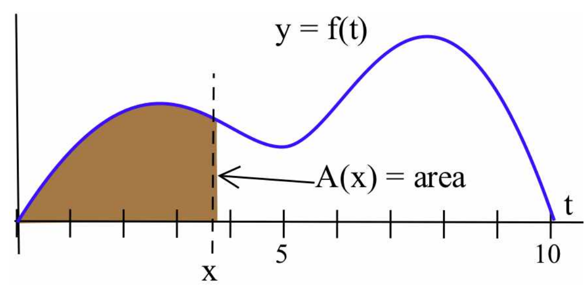
52. Imagine the graph of $f(x) = x^2$. Does f have a **maximum** value for x in the given interval?

   (a) $[-2, 3]$ (b) $[-2, 3)$ (c) $(-2, 3]$
   (d) $[-2, 1)$ (e) $(-2, 1]$

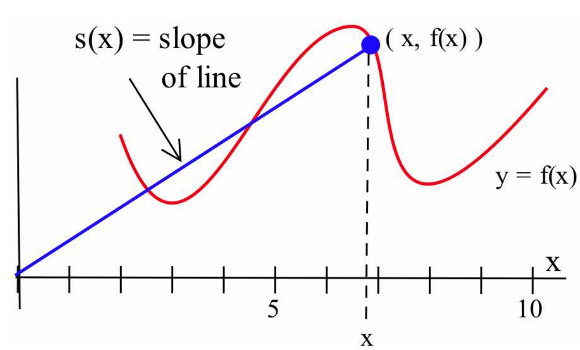
53. Imagine the graph of $f(x) = x^2$. Does f have a **minimum** value for x in the interval I?

   (a) $[-2, 3]$ (b) $[-2, 3)$ (c) $(-2, 3]$
   (d) $[-2, 1)$ (e) $(-2, 1]$

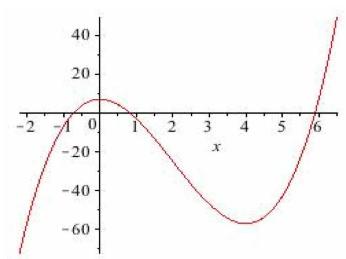
54. Define $A(x)$ to be the **area** bounded between the t-axis, the graph of $y = f(t)$ and a vertical line at $t = x$ (see figure below).
   (a) At what value of x is $A(x)$ minimum?
   (b) At what value of x is $A(x)$ maximum?



55. Define $S(x)$ to be the **slope** of the line through the points $(0, 0)$ and $(x, f(x))$ in the figure below.
   (a) At what value of x is $S(x)$ minimum?
   (b) At what value of x is $S(x)$ maximum?

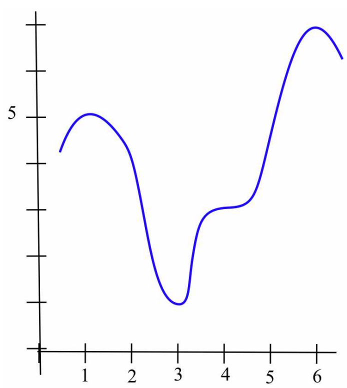


3.1 Practice Answers



1. $f(x) = x^2 + 4x - 5$ is a polynomial so f is differentiable for all x and $f'(x) = 2x + 4$; $f'(x) = 0$ when $x = -2$ so the only candidate for a local extreme is $x = -2$. Because the graph of f is a parabola opening up, the point $(-2, f(-2)) = (-2, -9)$ is a local minimum.

   $g(x) = 2x^3 - 12x^2 + 7$ is a polynomial so g is differentiable for all x and $g'(x) = 6x^2 - 24x = 6x(x-4)$ so $g'(x) = 0$ when $x = 0$ or 4, so the only candidates for a local extreme are $x = 0$ and $x = 4$. The graph of g (see margin) indicates that g has a local maximum at $(0, 7)$ and a local minimum at $(4, -57)$.



2. See the margin figure.

x	$f(x)$	$f'(x)$	max/min
1	5	0	local max
3	1	0	local min
4	3	0	neither
6	7	0	local max

3. $(1, f(1))$ is a global minimum; $x = 1$ is an endpoint
   $(2, f(2))$ is a local maximum; $f'(2) = 0$
   $(3, f(3))$ is a local/global minimum; f is not differentiable at $x = 3$
   $(4, f(4))$ is a global maximum; $x = 4$ is an endpoint

4. This is the same function used in Example 4, but now the interval is $[0, 5]$ instead of $[0, 10]$. See the Example for the calculations.

   Critical points:

   - endpoints: $x = 0$ and $x = 5$
   - f is differentiable for all $0 < x < 5$: none
   - $f'(x) = 0$: none in $[0, 5]$

   $f(0) \approx 4.67$ is the maximum of f on $[0, 5]$;
   $f(5) \approx 4.14$ is the minimum of f on $[0, 5]$.