19. (a) False. If $a = 3$, $b = 4$, then $(a+b)^2 = 7^2 = 49$, but $a^2 + b^2 = 3^2 + 4^2 = 9 + 16 = 25$. (b) False. If $a = -2$, $b = -3$, then $a > b$, but $a^2 = 4 < 9 = b^2$. (c) True.
21. (a) True. (b) False. If $f(x) = x + 1$ and $g(x) = x + 2$, Then $f(x) \cdot g(x) = x^2 + 3x + 2$ is not a linear function. (c) True.
23. (a) If a and b are prime numbers, then $a + b$ is prime. False: take $a = 3$ and $b = 5$.
    (b) If a and b are prime numbers, then $a + b$ is not prime. False: take $a = 2$ and $b = 3$.
    (c) If x is a prime number, then x is odd. False: take $x = 2$. (This is the only counterexample.)
    (d) If x is a prime number, then x is even. False: take $x = 3$ (or 5 or 7 or…)
25. (a) If x is a solution of $x + 5 = 9$, then x is odd. False: take $x = 4$.
    (b) If a 3-sided polygon has equal sides, then it is a triangle. True. (We also have non-equilateral triangles.)
    (c) If a person is a calculus student, then that person studies hard. False (unfortunately), but we won't mention names.
    (d) If x is a (real number) solution of $x^2 - 5x + 6 = 0$, then x is even. False: take $x = 3$.

Section 1.0

1. (a) $m = \frac{y-9}{x-3}$. If $x = 2.97$, $m = \frac{-0.1791}{-0.03} = 5.97$. If $x = 3.001$, $m = \frac{0.006001}{0.001} = 6.001$. If $x = 3 + h$,
$$m = \frac{(3+h)^2 - 9}{(3+h) - 3} = \frac{9 + 6h + h^2 - 9}{h} = 6 + h$$
(b) When h is close to 0, $6 + h$ is close to 6.
3. (a) $m = \frac{y-4}{x-2}$. If $x = 1.99$, $m = \frac{-0.0499}{-0.01} = 4.99$. If $x = 2.004$, $m = \frac{0.020016}{0.004} = 5.004$. If $x = 2 + h$:
$$m = \frac{\left[(2+h)^2 + (2+h) - 2\right] - 4}{(2+h) - 2} = 5 + h$$
(b) When h is very small, $5 + h$ is very close to 5.
5. All of these answers are **approximate**. Your answers should be close to these numbers.
   (a) average rate of temperature change $\approx$
$$\frac{80^\circ - 64^\circ}{1 \text{ p.m.} - 9 \text{ a.m.}} = \frac{16^\circ}{4 \text{ hours}} = 4 \frac{\circ}{\text{hour}}$$
   (b) At 10 a.m., temperature was rising about 5° per hour; at 7 p.m., its was rising about $-10^\circ/\text{hr}$ (**falling** about $10^\circ/\text{hr}$).
7. All of these answers are **approximate**.
   (a) average velocity $\approx \dfrac{300 \text{ ft} - 0 \text{ ft}}{20 \text{ sec} - 0 \text{ sec}} = 15 \dfrac{\text{ft}}{\text{sec}}$
   (b) average velocity $\approx \dfrac{100 \text{ ft} - 200 \text{ ft}}{30 \text{ sec} - 10 \text{ sec}} = -5 \dfrac{\text{ft}}{\text{sec}}$
   (c) At $t = 10$ seconds, velocity $\approx$ 30 feet per second (between 20 and 35 ft/sec); at $t = 20$ seconds, velocity ≈ -1 feet per second; at $t = 30$ seconds, velocity ≈ -40 feet per second.
9. (a) $A(0) = 0$, $A(1) = 3$, $A(2) = 6$, $A(2.5) = 7.5$, $A(3) = 9$ (b) The area of the rectangle bounded below by the t-axis, above by the line $y = 3$, on the left by the vertical line $t = 1$ and on the right by the vertical line $t = 4$. (c) Graph of $y = 3x$.

Section 1.1

1. (a) 2 (b) 1 (c) DNE (does not exist) (d) 1
3. (a) 1 (b) -1 (c) -1 (d) 2
5. (a) -7 (b) $\frac{13}{0}$ (DNE)
7. (a) 0.54 (radian mode!) (b) -0.318 (c) -0.54
9. (a) 0 (b) 0 (c) 0
11. (a) 0 (b) -1 (c) DNE
13. The one- and two-sided limits agree at $x = 1$, $x = 4$ and $x = 5$, but not at $x = 2$:

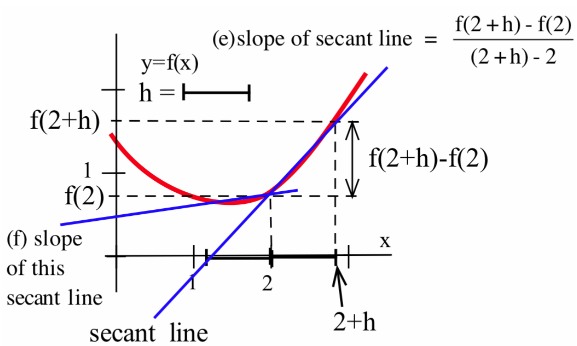
$\lim_{x\to1^-} g(x) = 1$	$\lim_{x\to1^+} g(x) = 1$	$\lim_{x\to1} g(x) = 1$
$\lim_{x\to2^-} g(x) = 1$	$\lim_{x\to2^+} g(x) = 4$	$\lim_{x\to2} g(x)$ DNE
$\lim_{x\to4^-} g(x) = 2$	$\lim_{x\to4^+} g(x) = 2$	$\lim_{x\to4} g(x) = 2$
$\lim_{x\to5^-} g(x) = 1$	$\lim_{x\to5^+} g(x) = 1$	$\lim_{x\to5} g(x) = 1$

15. (a) 1.0986 (b) 1
17. (a) 0.125 (b) 3.5
19. (a) $A(0) = 0$, $A(1) = 2.25$, $A(2) = 5$, $A(3) = 8.25$ (b) $A(x) = 2x + \frac{1}{4}x^2$ (c) The area of the trapezoid bounded below by the t-axis, above by the line $y = \frac{1}{2}t + 2$, on the left by the vertical line $t = 1$ and on the right by the vertical line $t = 3$.

Section 1.2

1. (a) 2 (b) 0 (c) DNE (d) 1.5

3. (a) 1 (b) 3 (c) 1 (d) ≈ 0.8

5. 

7. (a) 2 (b) -1 (c) DNE (d) 2 (e) 2 (f) 2 (g) 1 (h) 2 (i) DNE

9. (a) When $v = 0$, $L = A$. (b) 0

11. (a) 4 (b) 1 (c) 2 (d) 0 (e) 1 (f) 1

13. (a) Slope of the line tangent to the graph of $y = \cos(x)$ at the point $(0, 1)$. (b) slope $= 0$

15. (a) ≈ 1 (b) ≈ 3.43 (c) ≈ 4

17. At $x = -1$: "connected and smooth"; at $x = 0$: "connected with a corner"; at $x = 1$: "simple hole"; at $x = 2$: "vertical jump"; at $x = 3$: "simple hole"; at $x = 4$: "corner"; at $x = 5$: "smooth"

19. Many lists will work. Here is one example: Put $a_n = 2 + \frac{1}{n}$ so a_n approaches 2 and $\frac{|a_n - 2|}{a_n - 2} = 1$ for all n. Put $b_n = 2 - \frac{1}{n}$ so $b_n \to 2$ and $\frac{|b_n - 2|}{b_n - 2} = -1$ for all n.

21. $-x^2 \le x^2 \cos\left(\frac{1}{x^2}\right) \le x^2$ so limit is 0

23. $-x^2 \le x^2 \sin\left(\frac{1}{x}\right) \le x^2$ so limit is 3

25. $\frac{1}{x^2} - 1 < \left\lfloor \frac{1}{x^2} \right\rfloor \le \frac{1}{x^2}$ so limit is 1

Section 1.3

1. Discontinuous at 1, 3 and 4.

3. (a) Discontinuous at $x = 3$: fails condition (i) there. (b) At $x = 2$: fails (i). (c) Where $\cos(x) < 0$ (for example, at $x = \pi$): fails (i). (d) Where x^2 is an integer (for example, at $x = 1$ or $x = 2$): fails (ii). (e) Where $\sin(x) = 0$ (for example, at $x = 0$, $x = \pm\pi, \pm 2\pi \ldots$): fails (i). (f) At $x = 0$: fails (i). (g) At $x = 0$: fails (i). (h) At $x = 3$: fails (i). (i) At $x = \frac{\pi}{2}$: fails (i).

5. (a) $f(x) = 0$ for at least 3 values of x in the interval $0 \le x \le 5$. (b) 1 (c) 3 (d) 2 (e) Yes. (It does not have to happen, but it is possible.)

7. (a) $f(0) = 0$, $f(3) = 9$ and $0 \le 2 \le 9$; $c = \sqrt{2} \approx 1.414$ (b) $f(-1) = 1$, $f(2) = 4$ and $1 \le 3 \le 4$; $c = \sqrt{3} \approx 1.732$ (c) $f(0) = 0$, $f(\frac{\pi}{2} = 1$ and $0 \le \frac{1}{2} \le 1$; $c = \arcsin(\frac{1}{2}) \approx 0.524$ (d) $f(0) = 0$, $f(1) = 1$ and $0 \le \frac{1}{3} \le 1$; $c = \frac{1}{3}$ (e) $f(2) = 2$, $f(5) = 20$ and $2 \le 4 \le 20$; $c = \frac{1+\sqrt{17}}{2} \approx 2.561$ (f) $f(1) = 0$, $f(10) \approx 2.30$ and $0 \le 2 \le 2.30$; $c = e^2 \approx 7.389$

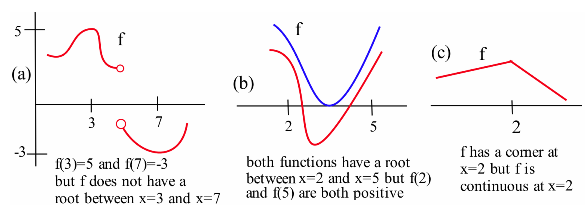
9. Neither student is correct. The bisection algorithm converges to the root labeled C.

11. (a) D (b) D (c) hits B

13. $[-0.9375, -0.875]$, root ≈ -0.879; $[1.3125, 1.375]$, root ≈ 1.347; $[2.5, 2.5625]$, root ≈ 2.532

15. $[2.3125, 2.375]$, root ≈ 2.32

17. $[-0.375, -0.3125]$, root ≈ -0.32

19. 

21. (a) $A(2.1) - A(2)$ is the area of the region bounded below by the t-axis, above by the graph of $y = f(t)$, on the left by the vertical line $t = 2$, and on the right by the vertical line $t = 2.1$. $\frac{A(2.1) - A(2)}{0.1} \approx f(2)$ or $f(2.1) \Rightarrow \frac{A(2.1) - A(2)}{0.1} \approx 1$
(b) $A(4.1) - A(4)$ is the area of the region bounded below by the t-axis, above by the graph of $y = f(t)$, on the left by the vertical line $t = 4$, and on the right by the vertical line $t = 4.1$. $\frac{A(4.1) - A(4)}{0.1} \approx f(4) \approx 2$

23. (a) Yes (you justify). (b) Yes. (c) Try it.

Section 1.4

1. (a) If x is within $\frac{1}{2}$ unit of 3 then $2x+1$ is within 1 unit of 7. (b) 0.3 (c) 0.02 (d) $\frac{\epsilon}{2}$

3. (a) If x is within $\frac{1}{4}$ unit of 2 then $4x-3$ is within 1 unit of 5. (b) 0.1 (c) 0.02 (d) $\frac{\epsilon}{4}$

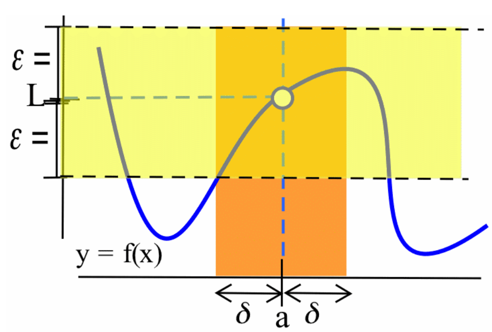
5. In 1, $m=2$, $\delta=\frac{\epsilon}{2}$; in 2, $m=3$, $\delta=\frac{\epsilon}{3}$; in 3, $m=4$, $\delta=\frac{\epsilon}{4}$; in 4, $m=5$, $\delta=\frac{\epsilon}{5}$. In general: $\delta=\frac{\epsilon}{|m|}$.

7. 0.02 inches

9. (a) Any value of x between $\sqrt[3]{7.5}\approx 1.957$ and $\sqrt[3]{8.5}\approx 2.043$: If x is within 0.043 units of 2 then x^3 will be within 0.5 units of 8. (b) Any x between $\sqrt[3]{7.95}\approx 1.9958$ and $\sqrt[3]{8.05}\approx 2.0042$: If x is within 0.0042 units of 2 then x^3 will be within 0.05 units of 8.

11. (a) Any value of x between 0 and 8: If x is within 3 units of 3 then $\sqrt{1+x}$ will be within 1 unit of 2. (b) Any x between 2.99920004 and 3.00080004: If x is within 0.00079996 units of 3 then $\sqrt{1+x}$ will be within 0.0002 units of 2.

13. 0.0059964 inches

15. 

17. On your own.

19. Assume $\lim_{x\to 2} f(x)$ exists. Let $\epsilon=0.1$, so there must be a δ so that:

$$2-\delta<x<2+\delta \Rightarrow L-0.1<f(x)<L+0.1$$

Now, $f(x)=4$ for any x with $2-\delta<x<2$, so it must be true that:

$$\begin{aligned}2-\delta<x<2 &\Rightarrow L-0.1<4<L+0.1\\ &\Rightarrow L>3.9\end{aligned}$$

Similarly, $f(x)=3$ for any x with $2<x<2+\delta$, so it must be true that:

$$\begin{aligned}2<x<2+\delta &\Rightarrow L-0.1<3<L+0.1\\ &\Rightarrow L<3.1\end{aligned}$$

But no value of L can simultaneously satisfy $L>3.9$ and $L<3.1$, so we have reached a contradiction and our assumption must be false: $\lim_{x\to 2} f(x)$ does not exist.

21. Assume $\lim_{x\to 2} f(x)$ exists. Let $\epsilon=0.1$, so there must be a δ so that:

$$2-\delta<x<2+\delta \Rightarrow L-0.1<f(x)<L+0.1$$

We can assume that $\delta\leq 1$ (if not, we can replace our initial δ with $\delta=1$ because any smaller value will also work). Now, $f(x)=x$ for any x with $2-\delta<x<2$, so it must be true that:

$$\begin{aligned}2-\delta<x<2 &\Rightarrow L-0.1<x<L+0.1\\ &\Rightarrow L<0.1+x<2.1\end{aligned}$$

On the other hand, $f(x)=6-x$ for any x with $2<x<2+\delta\leq 3$, so it must be true that:

$$\begin{aligned}2<x<2+\delta &\Rightarrow L-0.1<6-x<L+0.1\\ &\Rightarrow L>5.9-x\geq 2.9\end{aligned}$$

(because $x\leq 3$). But no value of L can simultaneously satisfy $L<2.1$ and $L>2.9$, so we have reached a contradiction and our assumption must be false: $\lim_{x\to 2} f(x)$ does not exist.

23. Given any $\epsilon>0$, we know $\frac{\epsilon}{2}>0$, so there is a number $\delta_f>0$ such that $|x-a|<\delta_f \Rightarrow |f(x)-L|<\frac{\epsilon}{2}$. Likewise, there is a number $\delta_g>0$ such that $|x-a|<\delta_g \Rightarrow |g(x)-L|<\frac{\epsilon}{2}$. Let δ be the smaller of δ_f and δ_g. If $|x-a|<\delta$ then $|f(x)-L|<\frac{\epsilon}{2}$ and $|g(x)-M|<\frac{\epsilon}{2}$ so:

$$\begin{aligned}&|(f(x)-g(x))-(L-M))|\\ &\quad=|(f(x)-L)-(g(x)-M)|\\ &\quad\leq|f(x)-L|+|g(x)-M|\\ &\quad<\frac{\epsilon}{2}+\frac{\epsilon}{2}=\epsilon\end{aligned}$$

so $f(x)-g(x)$ is within ϵ of $L-M$ whenever x is within δ of a.