Section 4.0

1. (a) $(10)(12)+(8)(4)=152$
   (b) $(10)(20)-(3)(8)=176$

3. $bh+\frac{1}{2}b(H-h)=bh+\frac{1}{2}bH-\frac{1}{2}bh=b\left(\frac{h+H}{2}\right)$

5. (a) $1\cdot 3+1\cdot 2=5$ (b) area <5

7. Answers will vary from 5 to 13.

9. $A(1)=1$, $A(2)=2.5$, $A(3)=4.5$, $A(4)=6$, $A(5)=7$

11. $C(1)=1.5$, $C(2)=4$, $C(3)=7.5$
    $C(x)$ is sum of rectangular and triangular areas:
    $$C(x)=x+\frac{1}{2}x\cdot x=x+\frac{1}{2}x^2$$

13. $(20)(30)+\frac{1}{2}(10)(30)=600+150=750$ feet

15. (a) A: 20 seconds to stop; B: 40 seconds to stop
    (b) A: $\frac{1}{2}(20)(80)=800$ ft; B: $\frac{1}{2}(40)(40)=800$ ft

17. miles, \$, ft^3, kilowatt-hours, people, square meals

Section 4.1

1. $2^2+3^2+4^2=29$

3. $(1+1)^2+(1+2)^2+(1+3)^2=29$

5. $\cos(0)+\cos(\pi)+\cos(2\pi)+\cos(3\pi)+\cos(4\pi)+\cos(5\pi)=1+(-1)+1+(-1)+1+(-1)=0$

7. $\displaystyle\sum_{k=3}^{94}k$ 9. $\displaystyle\sum_{k=3}^{12}k^2$ 11. $\displaystyle\sum_{k=1}^{7}k\cdot 2^k$

13. (a) $(1+2)+(2+2)+(3+2)=3+4+5=12$
    (b) $(1+2+3)+(2+2+2)=12$

15. (a) $5\cdot 1+5\cdot 2+5\cdot 3=5+10+15=30$
    (b) $5\cdot(1+2+3)=5\cdot 6=30$

17. (a) $1^2+2^2+3^2=1+4+9=14$
    (b) $(1+2+3)^2=6^2=36$

19. $f(0)+f(1)+f(2)+f(3)=0^2+1^2+2^2+3^2=14$

21. $2\cdot f(0)+2\cdot f(1)+2\cdot f(2)+2\cdot f(3)=2\cdot 0+2\cdot 1+2\cdot 4+2\cdot 9=28$

23. $g(1)+g(2)+g(3)=3+6+9=18$

25. $g^2(1)+g^2(2)+g^2(3)=3^2+6^2+9^2=126$

27. $h(2)+h(3)+h(4)=\frac{2}{2}+\frac{2}{3}+\frac{2}{4}=\frac{13}{6}$

29. $(1)(2)+(4)(1)+(9)\left(\frac{2}{3}\right)=12$

31. $(1^2-0^2)+(2^2-1^2)+(3^2-2^2)+(4^2-3^2)+\cdots+(7^2-6^2)=7^2-0^2=49$

33. $\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{6}\right)=1-\frac{1}{6}=\frac{5}{6}$

35. $\left(\sqrt{1}-\sqrt{0}\right)+\left(\sqrt{2}-\sqrt{1}\right)+\left(\sqrt{3}-\sqrt{2}\right)+\left(\sqrt{4}-\sqrt{3}\right)+\cdots+\left(\sqrt{9}-\sqrt{8}\right)=3-0=3$

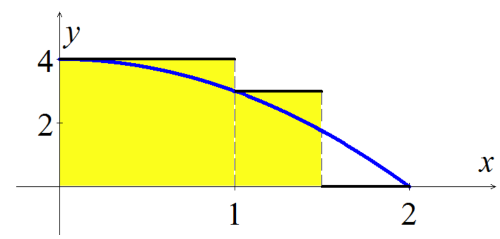
37. (a) $[2,3]$, $[3,4.5]$, $[4.5,6]$, $[6,7]$ (b) 1, 1.5, 1.5, 1
    (c) mesh $=1.5$ (d) $1+1.5+1.5+1=5$

39. (a) $[-3,-1]$, $[-1,0]$, $[0,1.5]$, $[1.5,2]$
    (b) 2, 1, 1.5, 0.5 (c) mesh $=2$
    (d) $2+1+1.5+0.5=5$

41. (a) $[3,3.8]$, $[3.8,4.5]$, $[4.5,5.2]$, $[5.2,7]$
    (b) 0.8, 0.7, 0.7, 1.8 (c) mesh $=1.8$
    (d) $0.8+0.7+0.7+1.8=4$

43. $\Delta x_1+\Delta x_2+\cdots+\Delta x_n=(x_1-x_0)+(x_2-x_1)+(x_3-x_2)+\cdots+(x_n-x_{n-1})=x_n-x_0$

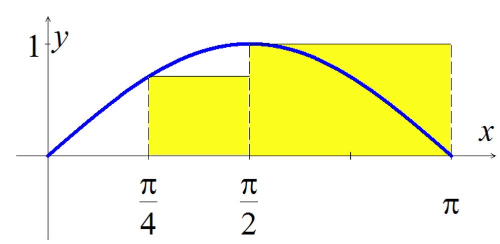
45. (a)



$f(0)(1)+f(1)(0.5)+f(2)(0.5)=(4)(1)+(3)(0.5)+(0)(0.5)=5.5$

(b) $f(1)(1)+f(1.5)(0.5)+f(1.5)(0.5)=(3)(1)+(1.75)(0.5)+(1.75)(0.5)=4.75$

47. (a)



$\left(\frac{\pi}{4}\right)(0)+\left(\frac{\pi}{4}\right)\left(\frac{1}{\sqrt{2}}\right)+\left(\frac{\pi}{2}\right)(1)\approx 2.13$

(b) $\left(\frac{\pi}{4}\right)\left(\frac{1}{\sqrt{2}}\right)+\left(\frac{\pi}{4}\right)(1)+\left(\frac{\pi}{2}\right)(0)\approx 1.34$

49. (a) $(2)(1)+(5)(2)+(17)(1) \le \text{RS} \le (5)(1)+(17)(2)+(26)(1) \;\Rightarrow\; 29 \le \text{RS} \le 65$

(b) $(2)(1)+(5)(1)+(10)(1)+(17)(1) \le \text{RS} \le (5)(1)+(10)(1)+(17)(1)+(26)(1) \;\Rightarrow\; 34 \le \text{RS} \le 58$

(c) $2(0.5)+3.25(0.5)+5(1)+10(1)+17(1) \le \text{RS} \le 3.25(0.5)+5(0.5)+10(1)+17(1)+26(1)$
$\Rightarrow 34.625 \le \text{RS} \le 57.125$

51. (a) $(0)\left(\frac{\pi}{2}\right)+(0)\left(\frac{\pi}{2}\right) \le \text{RS} \le (1)\left(\frac{\pi}{2}\right)+(1)\left(\frac{\pi}{2}\right) \;\Rightarrow\; 0 \le \text{RS} \le \pi$

(b) $0\left(\frac{\pi}{4}\right)+\frac{1}{\sqrt{2}}\left(\frac{\pi}{2}\right)+0\left(\frac{\pi}{2}\right) \le \text{RS} \le \frac{1}{\sqrt{2}}\left(\frac{\pi}{4}\right)+1\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\pi}{4}\right)+1\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\pi}{2}\right) \Rightarrow\; 0.56 \le \text{RS} \le 2.91$

(c) $0\left(\frac{\pi}{4}\right)+\frac{1}{\sqrt{2}}\left(\frac{\pi}{4}\right)+\frac{1}{\sqrt{2}}\left(\frac{\pi}{4}\right)+0\left(\frac{\pi}{4}\right) \le \text{RS} \le \frac{1}{\sqrt{2}}\left(\frac{\pi}{2}\right)+1\left(\frac{\pi}{4}\right)+1\left(\frac{\pi}{4}\right)+\frac{1}{\sqrt{2}}\left(\frac{\pi}{4}\right) \Rightarrow\; 1.11 \le \text{RS} \le 2.68$

53. (a) $|7.402-7.362| = 0.04$ (b) $|7.390-7.372| = 0.018$

55. $|\text{error}| = (\text{base})(\text{height}) = \frac{4-2}{50}(65-9) = \frac{56}{25} = 2.24$

57. (a) $\frac{100(101)}{2} = 5050$

(b)
$$\begin{array}{rcccccccccc} S = & 1 & + & 2 & + & 3 & + & \cdots & + & 100 \\ +\,S = & 100 & + & 99 & + & 98 & + & \cdots & + & 1 \\ \hline 2S = & 101 & + & 101 & + & 101 & + & \cdots & + & 101 \end{array} = 100(101) = 10100 \;\Rightarrow\; S = 5050$$

59. $10+11+12+\cdots+20 = (1+2+3+\cdots+20)-(1+2+3+\cdots+9) = \frac{20(21)}{2} - \frac{9(10)}{2} = 210-45 = 165$

61. $\sum_{k=1} 10\left(k^3+k\right) = \sum_{k=1} 10\,k^3 + \sum_{k=1} 10\,k = \left[\frac{10(11)}{2}\right]^2 + \left[\frac{10(11)}{2}\right] = (55)^2+55 = 3080$

Section 4.2

1. $\int_0^4 [2+3x]\;dx$

3. $\int_2^5 \cos(5x)\,dx$

5. $\int_1^5 x^3\,dx$

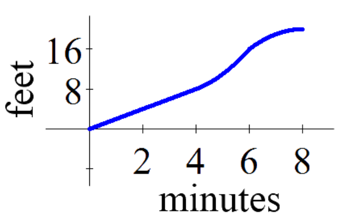
7. $\int_{0.5}^2 x\cdot\sin(x)\,dx$

9. $\int_1^3 \ln(x)\,dx$

11. $\int_1^3 2x\,dx = 8$

13. $\int_{-1}^0 |x|\;dx = \frac{1}{2}$

15. $\int_0^4 \left[3-\frac{x}{2}\right]\;dx = 8$



17. (a) 3 (b) -1 (c) 6 (d) 8 (e) 7

19. (a) See margin figure. (b) 24 ft (c) 24 feet from starting point

21. meters

23. ft^3

25. gram-meters

27. ft/sec

29. $\Delta x = \frac{2-0}{n} = \frac{2}{n}$, $m_k = \frac{2}{n}(k-1)$, $M_k = \frac{2}{n}k$, so $f(m_k) = \left[\frac{2}{n}(k-1)\right]^3$ and $f(M_k) = \left[\frac{2}{n}k\right]^3$

(a)
$$\text{LS} = \sum_{k=1}^{n} f(m_k)\,\Delta x = \sum_{k=1}^{n}\left[\frac{2}{n}(k-1)\right]^3\Delta x = \frac{2}{n}\cdot\frac{8}{n^3}\left[\sum_{k=1}^{n}k^3 - 3\sum_{k=1}^{n}k^2 + 3\sum_{k=1}^{n}k - \sum_{k=1}^{n}1\right]$$
$$= \frac{16}{n^4}\left[\left(\frac{1}{4}n^4+\frac{1}{2}n^3+\frac{3}{12}n^2\right) - 3\left(\frac{1}{3}n^3+\frac{1}{2}n^2+\frac{2}{12}n\right) + 3\left(\frac{1}{2}n^2+\frac{1}{2}n\right) - n\right]$$
$$= \frac{16}{n^4}\left[\frac{1}{4}n^4 - \frac{1}{2}n^3 + \frac{1}{4}n^2\right] = 4 - \frac{8}{n} + \frac{4}{n^2} \longrightarrow 4$$

(b)
$$\text{US} = \sum_{k=1}^{n} f(M_k)\,\Delta x = \sum_{k=1}^{n}\left[\frac{2}{n}(k)\right]^3\frac{2}{n} = \frac{16}{n^4}\left[\sum_{k=1}^{n}k^3\right] = \frac{16}{n^4}\left[\frac{1}{4}n^4+\frac{1}{2}n^3+\frac{3}{12}n^2\right] = 4+\frac{8}{n}+\frac{4}{n^2} \longrightarrow 4$$

Section 4.3

1. 5 3. 0 5. 3 7. 0 9. -5

11. -5 13. 0 15. $4.5 + 5 = 9.5$

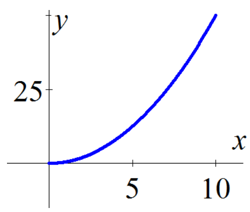
17. $10 + 3 = 13$ 19. $5 + 2 = 7$

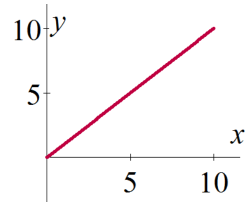
21. 1 23. -1 25. 2 27. 1 29. 1

31. (a) $8 \cdot 6 = 48$ (b) 24 33. (a) 32 (b) $8^2 = 64$

35. 8 37. 2.5 39. 3 41. 7

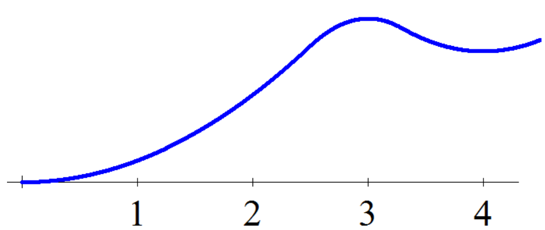
43. (a) $y = A(x) = \frac{1}{2}x^2$ (below left)



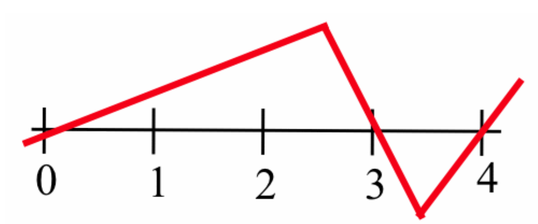


(b) $y = A'(x) = x$ (above right)

45. (a)



(b)



47. (a) f is continuous on $[1,4]$ (b) f is not differentiable on $[1,4]$ (not differentiable at $x \approx 2.5$ and $x \approx 3.3$) (c) f is integrable on $[1,4]$

49. (a) f is not continuous on $[1,4]$ (not continuous at $x = 2$) (b) f is not differentiable on $[1,4]$ (not differentiable at $x = 2$) (c) f is integrable on $[1,4]$

51. $\int_1^4 v(t)\,dt = \int_1^2 v(t)\,dt + \int_2^4 v(t)\,dt$
$= 35 + 50 = 85$ miles

53. (a) The rectangle associated with the interval containing $x = 2$ has width w and height 5, so its area is $5w$; all of the other rectangles have the same height (7) and the sum of their widths is $(4-1) - w = 3 - w$, so their total area is $7(3-w)$: $\text{RS} = 5w + 7(3-w)$

(b) Because $0 < w \le \|\mathcal{P}\|$, as $\|\mathcal{P}\| \to 0 \Rightarrow w \to 0$ so $\lim\limits_{\|\mathcal{P}\|\to 0} \text{RS} = \lim\limits_{w\to 0} [5w + 7(3-w)] = 21$

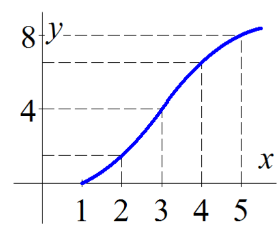
(c) $\int_1^4 g(x)\,dx = \lim\limits_{\|\mathcal{P}\|\to 0} \text{RS} = 21$

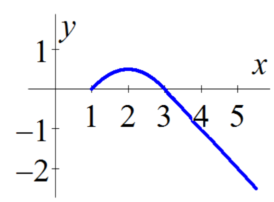
$\int_1^4 7\,dx = 7(4-1) = 7(3) = 21$

(d) A (very) similar argument shows that redefining any constant function f at a single point does not alter the value of $\int_a^b f(x)\,dx$. A (somewhat) similar argument can extend this result to all integrable functions.

Section 4.4

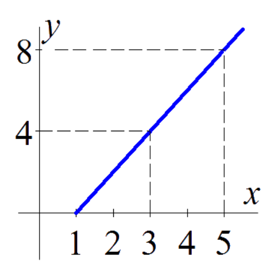
1. (a) See figure below left.
(b) $A(1) = 0$, $A(2) = 1.5$, $A(3) = 4$, $A(4) = 6.5$
(c) $A'(1) = 1$, $A'(2) = 2$, $A'(3) = 3$, $A'(4) = 2$

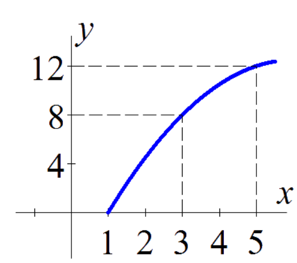




3. (a) See figure above right.
(b) $A(1) = 0$, $A(2) = 0.5$, $A(3) = 0$, $A(4) = -1$
(c) $A'(1) = 1$, $A'(2) = 0$, $A'(3) = -1$, $A'(4) = -1$

5. (a) See figure below left.
(b) $A(1) = 0$, $A(2) = 2$, $A(3) = 4$, $A(4) = 6$
(c) $A'(1) = 2$, $A'(2) = 2$, $A'(3) = 2$, $A'(4) = 2$





7. (a) See figure above right.

(b) $A(1) = 0$, $A(2) = 4.5$, $A(3) = 8$, $A(4) = 10.5$
(c) $A'(1) = 5$, $A'(2) = 4$, $A'(3) = 3$, $A'(4) = 2$

9. (a) $x^2\Big|_0^3 = 9$ (b) $x^2\Big|_1^3 = 8$ (c) $x^2\Big|_0^1 = 1$

11. (a) $2x^3\Big|_1^3 = 52$ (b) $2x^3\Big|_1^2 = 14$ (c) $2x^3\Big|_0^3 = 54$

13. (a) $x^4\Big|_0^3 = 81$ (b) $x^4\Big|_1^3 = 80$ (c) $x^4\Big|_0^1 = 1$

15. (a) $x^3\Big|_{-3}^3 = 54$ (b) $x^3\Big|_{-3}^0 = 27$ (c) $x^3\Big|_0^3 = 27$

17. (a) $x^3\Big|_0^2 = 8$ (b) $x^3\Big|_1^3 = 26$ (c) $x^3\Big|_3^1 = -26$

19. (a) $\int_0^{10} 2t\,dt = t^2\Big|_0^{10} = 100$ ft
(b) $50 = \int_0^T 2t\,dt = T^2 \Rightarrow T = \sqrt{50} \approx 7.07$ sec

21. (a) $\int_0^{10} 4t^3\,dt = t^4\Big|_0^{10} = 10000$ ft
(b) $5000 = \int_0^T 4t^3\,dt = T^4 \Rightarrow T =\approx 8.41$ sec

23. (a) $75 - 3t^2 = 0 \Rightarrow t = 5$ sec
(b) $\int_0^5 \left[75 - 3t^2\right]\,dt = 75t - t^3\Big|_0^5 = 250$ ft
(c) $125 = \int_0^T \left[75 - 3t^2\right]\,dt = 75T - T^3$, so use a graph of $y = x^3 - 75x + 125$ (or Newton's method) to solve for $T \approx 1.74$ sec

25. The total area is $\int_0^3 x^2\,dx = \frac{1}{3}x^3\Big|_0^3 = \frac{1}{3} \cdot 27 = 9$.
(a) $\frac{1}{2} \cdot 9 = \frac{1}{3}x^3\Big|_0^T = \frac{1}{3}T^3 \Rightarrow T = \sqrt[3]{\frac{27}{2}} \approx 2.38$
(b) $\frac{1}{3} \cdot 9 = \int_0^T x^2\,dx = \frac{1}{3}T^3 \Rightarrow T = \sqrt[3]{9} \approx 2.08$
$\frac{2}{3} \cdot 9 = \int_0^T x^2\,dx = \frac{1}{3}T^3 \Rightarrow T = \sqrt[3]{18} \approx 2.62$

Section 4.5

1. (a) $A(x) = x^3 \Rightarrow A'(x) = 3x^2$ so $A'(1) = 3$, $A'(2) = 12$ and $A'(3) = 27$
(b) $A'(x) = \mathbf{D}\left[\int_0^x 3t^2\,dt\right] = 3x^2$ so $A'(1) = 3$, $A'(2) = 12$ and $A'(3) = 27$

3. $A'(x) = 2x$ so $A'(1) = 2$, $A'(2) = 4$, $A'(3) = 6$

5. $A'(x) = 2x$ so $A'(1) = 2$, $A'(2) = 4$, $A'(3) = 6$

7. $A'(1) \approx 0.84$, $A'(2) \approx 0.91$, $A'(3) \approx 0.14$

9. $A'(x) = f(x)$, $A'(1) = 2$, $A'(2) = 1$, $A'(3) = 2$

11. $A'(x) = f(x)$, $A'(1) = 1$, $A'(2) = 2$, $A'(3) = 2$

13. $F(1) - F(0) = 6 - 5 = 1$

15. $F(3) - F(1) = 9 - \frac{1}{3} = \frac{26}{3}$

17. $F(5) - F(1) \approx 1.61 - 0 = 1.61$

19. $F(3) - F\left(\frac{1}{2}\right) \approx 1.10 - (-0.69) = 1.79$

21. $F\left(\frac{\pi}{2}\right) - F(0) = 1 - 0 = 1$

23. $F(1) - F(0) \approx 0.67 - 0 = 0.67$

25. $F(7) - F(1) = \frac{2}{3}(7)^{\frac{3}{2}} - \frac{2}{3} \approx 11.68$

27. $F(9) - F(1) = 3 - 1 = 2$

29. $F(3) - F(-2) \approx 20.09 - 0.14 = 19.95$

31. $F\left(\frac{\pi}{4}\right) - F(0) = 1 - 0 = 1$

33. $F(3) - F(0) = \frac{2}{3}(10)^{\frac{3}{2}} - \frac{2}{3} \approx 20.42$

35. $F(x) = \frac{1}{3}x^3 \Rightarrow F(2) - F(-1) = \frac{8}{3} - \left(-\frac{1}{3}\right) = 3$

37. $F(x) = \ln(x) \Rightarrow F(e) - F(1) = 1 - 0 = 1$

39. $F(x) = \frac{2}{3}x^{\frac{3}{2}} \Rightarrow F(100) - F(25) = \frac{2000}{3} - \frac{250}{3} = \frac{1750}{3}$

41. $F(x) = -\frac{1}{x} \Rightarrow F(10) - F(1) = -0.1 - (-1) = 0.9$

43. $F(x) = e^x \Rightarrow F(1) - F(0) = e - 1 \approx 1.718$

45. $F(x) = \tan(x) \Rightarrow F\left(\frac{\pi}{4}\right) - F\left(\frac{\pi}{6}\right) = 1 - \frac{1}{\sqrt{3}} \approx 0.423$

47. $\int_3^3 f(x)\,dx = 0$ for any integrable f

49. $\int_0^\pi \sin(x)\,dx = -\cos(x)\Big|_0^\pi = -(-1) - (-1) = 2$

51. $\int_0^{3.5} \lfloor x \rfloor\,dx = 0 + 1 + 2 + \frac{1}{2}(3) = 4.5$

53. $\int_0^3 (x-2)^2\,dx = \int_0^3 (x^2 - 4x + 4)\,dx =$
$\frac{1}{3}x^3 - 2x^2 + 4x\Big|_0^3 = 3$

55. $\mathbf{D}\left(A(3x)\right) = 3\tan(3x)$, $\mathbf{D}\left(A(x^2)\right) = 2x\tan(x^2)$, $\mathbf{D}\left(A(\sin(x))\right) = \cos(x)\tan(\sin(x))$

57. $\sqrt{1+5x}(5)$

59. $\sqrt{1+\sin(x)} \cdot \cos(x)$

61. $\left[3(1-2x)^2 + 2\right](-2)$

63. $-\cos(3x)$

65. $\tan(x^2) \cdot 2x - \tan(x)$

67. $5\ln(x)\cos(3\ln(x)) \cdot \frac{1}{x}$

Section 4.6

1. $\frac{1}{4}x^4\Big|_1^2 = \frac{15}{4} \neq \frac{7}{2} = \left[\frac{1}{3}x^3\Big|_1^2\right]\left[\frac{1}{2}x^2\Big|_1^2\right]$

3. $\frac{1}{4} \neq \frac{1}{3} \cdot \frac{1}{2}$

5. $\frac{1}{3}\sin(3x) + C$

7. $-\cos(2 + e^x) + C$

9. $\tan(\sin(x)) + C$

11. $\frac{5}{2}\ln|3 + 2x| + C$

13. $-\frac{1}{3}\cos(1 + x^3) + C$

15. $\frac{1}{4}\sin(4x) + C$

17. $\frac{1}{48}\left(5 + x^4\right)^{12} + C$

19. $\ln\left|2 + x^3\right| + C$

21. $\frac{1}{2}(\ln(x))^2 + C$

23. $\frac{1}{24}(1 + 3x)^8 + C$

25. $\sec(e^x) + C$

27. $\frac{1}{3}\sin(3x)\Big|_0^{\frac{\pi}{2}} = -\frac{1}{3}$

29. $-\cos(2 + e^x)\Big|_0^1 \approx -0.996$

31. $\frac{1}{18}\left(1 + x^3\right)^6\Big|_{-1}^1 = \frac{32}{9}$

33. $\frac{5}{2}\ln|3 + 2x|\Big|_0^2 = \frac{5}{2}\ln\left(\frac{7}{3}\right)$

35. $-\frac{1}{3}\left(1 - x^2\right)^{\frac{3}{2}}\Big|_0^1 = \frac{1}{3}$

37. $\frac{2}{9}(1 + 3x)^{\frac{3}{2}}\Big|_0^1 = \frac{16}{9} - \frac{2}{9} = \frac{14}{9}$

39. $\frac{1}{2}x - \frac{1}{20}\sin(10x) + C$

41. $\frac{1}{4}\sin(2x) + C$

43. $\frac{1}{2}x - \frac{1}{4}\sin(2x)\Big|_0^{\pi} = \frac{\pi}{2}$

45. $\frac{1}{7}x^7 + \frac{3}{5}x^5 + x^3 + x + C$

47. $\frac{1}{2}e^{2x} + 2e^x + x + C$

49. $\frac{1}{6}x^6 + \frac{1}{4}x^4 + \frac{5}{3}x^3 + 5x + C$

51. $\frac{1}{2}e^{2x} + \frac{1}{4}e^{4x} + C$

53. $\frac{2}{7}x^{\frac{7}{2}} + \frac{6}{5}x^{\frac{5}{2}} - \frac{4}{3}x^{\frac{3}{2}} + C$

55. $3x - 3\ln|x + 1| + C$

57. $\frac{1}{2}x^2 - x + C$

59. $x^2 - 11x + 7\ln|x - 1| + C$

61. $x + 3\ln|x - 1| + C$

63. $\frac{2}{3}x^{\frac{3}{2}} + 8x^{\frac{1}{2}} + C$

65. area of semicircle with radius 1 $= \frac{1}{2}\pi(1)^2 = \frac{\pi}{2}$

67. area of semicircle with radius 3 $= \frac{1}{2}\pi(3)^2 = \frac{9\pi}{2}$

69. $(2)(2) + \frac{1}{2}\pi(1)^2 = 4 + \frac{\pi}{2}$

Section 4.7

1. Answers will vary between 11 (using left endpoints) and 6 (using right endpoints).

3. Between 4 (left endpoints) and 6 (right).

5. Using left-hand widths:

$$(40)\,[0 + 70 + 55 + 90 + 130 + 115] = 18400 \text{ ft}^2$$

Right-hand widths (70, 55,...) and average widths ($\frac{70}{2}$, $\frac{125}{2}$, ...) yield the same result.

7. $\int_{-1}^{2}\left[(x^2 + 3) - 1\right]\,dx = 9$

9. $\int_0^1\left[x - x^2\right]\,dx + \int_1^2\left[x^2 - x\right]\,dx = 1$

11. $\int_1^e\left[x - \frac{1}{x}\right]\,dx = \frac{1}{2}e^2 - \frac{3}{2}$

13. $\int_0^{\frac{\pi}{4}}\left[(x + 1) - \cos(x)\right]\,dx = \frac{1}{32}\pi^2 + \frac{1}{4}\pi - \frac{\sqrt{2}}{2}$

15. $\int_0^2\left[e^x - x\right]\,dx = e^2 - 3$

17. $\int_0^1\left[3 - \sqrt{1 - x^2}\right]\,dx = 3 - \frac{\pi}{4}$

19. Using $\mathcal{P} = \{0.5, 1.5, 2.5, 3.5, 4.5\}$, so that $\Delta x = 1$, and $c_1 = 1$, $c_2 = 2$, $c_3 = 3$, $c_4 = 4$:

$$\frac{1}{4.5 - 0.5}\int_{0.5}^{4.5} f(x)\,dx \approx \frac{1}{4}\,[6 + 6 + 4 + 3]\,(1) = \frac{19}{4}$$

21. With $\mathcal{P} = \{1.5, 2.5, 3.5\}$, $\Delta x = 1$, $c_1 = 2$, $c_2 = 3$:

$$\frac{1}{3.5 - 1.5}\int_{0.5}^{3.5} f(x)\,dx \approx \frac{1}{2}\,[6 + 4]\,(1) = \frac{10}{2} = 5$$

23. $\frac{1}{2 - 0}\int_0^2 f(x)\,dx = \frac{2}{2} = 1$

25. $\frac{1}{6 - 1}\int_1^6 f(x)\,dx = \frac{11}{5}$

27. $\frac{1}{4 - 0}\int_0^2\left[2x + 1\right]\,dx = 5$

29. $\frac{1}{3 - 1}\int_1^3 x^2\,dx = \frac{13}{3}$

31. $\frac{1}{\pi - 0}\int_0^{\pi}\sin(x)\,dx = \frac{2}{\pi}$

33. $C = 1$: $\overline{f} = \frac{2}{3}$; $C = 9$: $\overline{f} = 2$; $C = 81$: $\overline{f} = 6$; $C = 100$: $\overline{f} = \frac{20}{3}$. In general, $\overline{f} = \frac{2}{3}\sqrt{C}$.

35. (a) $\approx 3000 \cdot 1000\frac{\text{calls}}{\text{hour}} = \frac{3000000\text{ calls}}{60\text{ min}} = 50000\frac{\text{calls}}{\text{min}}$
(b) About 58,333 calls per minute.

37. (a) 1,950 foot-pounds (b) 1,312.5 foot-pounds

39. (a) 1,200 ft-lbs (b) 600 ft-lbs (c) 400 foot-lbs

41. 1,275 foot-pounds

Section 4.8

1. $\frac{1}{2}\arctan\left(\frac{x}{2}\right)+C$
3. $x^2+\frac{2}{5}\arctan\left(\frac{x}{5}\right)+C$
5. $\frac{1}{3}\ln\left|\frac{x+3}{x-3}\right|+C$
7. $\frac{1}{\sqrt{3}}\arctan\left(\frac{x}{\sqrt{3}}\right)+C$
9. $e^x+\frac{7}{\sqrt{2}}\arctan\left(\frac{x}{\sqrt{2}}\right)+C$
11. $3\arcsin\left(\frac{x}{\sqrt{5}}\right)+C$
13. $\frac{1}{10}\arctan\left(\frac{5x}{2}\right)+C$
15. $\frac{5}{2}\arcsin(2x)+C$
17. $\frac{2}{3}\ln\left|3x+\sqrt{1+9x^2}\right|+C$
19. $(x+1)\ln(x+1)-x+K$
21. $\frac{3}{10}(5x^2+7)\left[\ln\left(5x^2+7\right)-1\right]+C$
23. $\sin(x)\left[\ln|\sin(x)|-1\right]+C$
25. $\frac{x}{2}\sqrt{x^2+4}+2\ln\left|x+\sqrt{x^2+4}\right|+C$
27. $\frac{x}{2}\sqrt{x^2+16}+8\ln\left|x+\sqrt{x^2+16}\right|+C$
29. $8+\frac{2}{5}\left[\arctan\left(\frac{3}{5}\right)-\arctan\left(\frac{1}{5}\right)\right]$
31. $\frac{1}{\sqrt{3}}\arctan\left(\frac{x}{\sqrt{3}}\right)\Big|_{-1}^{1}=\frac{\pi}{3\sqrt{3}}$
33. $3\left[\arcsin\left(\frac{2}{\sqrt{5}}\right)-\arcsin\left(\frac{1}{\sqrt{5}}\right)\right]$
35. $\frac{5}{2}\arcsin(2x)\Big|_{0}^{0.1}=\frac{5}{2}\arcsin(0.2)$
37. $7\ln(7)-6$
39. $3\ln(3)-2\ln(2)-1$
41. $3\sqrt{18}+\frac{9}{2}\ln\left(\frac{3+\sqrt{18}}{-3+\sqrt{18}}\right)$
43. $-\frac{1}{3}\sin^2(x)\cos(x)-\frac{2}{3}\cos(x)+C$
45. $\frac{1}{5}\cos^4(x)\sin(x)+\frac{4}{5}\int\cos^3(x)\,dx$
47. $x^2\sin(x)+2x\cos(x)-2\sin(x)+C$
49. average of $\sin(x)=\frac{2}{\pi}>\frac{1}{2}=$ average of $\sin^2(x)$
51. $C=e$: $\frac{1}{e-1}$; $C=10$: $\frac{1}{9}\left[10\ln(10)-9\right]$;
$C=100$: $\frac{1}{99}\left[100\ln(100)-99\right]$;
$C=1000$: $\frac{1}{999}\left[1000\ln(1000)-999\right]$
53. (a) $e^2-e\approx 4.67$ (b) $e^2\approx 7.39$ (c) $2e^2-e\approx 12.06$
55. $2\arctan(C)\approx 1.57, 2.94, 3.04, 3.07, 3.09$

Section 4.9

1. $T_4=\frac{1}{2}\left[2.1+2(3.8)+2(0.3)+2(-0.9)+2.2\right]=5.35$
$S_4=\frac{1}{3}\left[2.1+4(3.8)+2(0.3)+4(-0.9)+2.2\right]=5.5$
3. $T_8=7.35$, $S_8=\frac{22}{3}\approx 7.3333$
5. (a) $T_4=4$ (b) $S_4=4$ (c) 4
7. (a) $T_4=0.75$ (b) $S_4=\frac{2}{3}\approx 0.67$ (c) $\frac{2}{3}$
9. (a) $T_4=1.896118898$ (b) $S_4=2.004559755$ (c) 2
11. (a) $T_6=1.088534906$ (b) $S_6=1.090560447$
13. (a) $T_6=3.815780054$ (b) $S_6=3.826350295$
15. (a) $T_6=0.8159928163$ (b) $S_6=0.8120491229$
17. (a) $f(x)=x\Rightarrow f''(x)=0\Rightarrow B_2=0$, so the error bound is 0 (the Trapezoidal approximation is exact) (b) $f^{(4)}(x)=0\Rightarrow B_4=0$, so the error bound is 0 (the Simpson's Rule approximation is exact) (c) $n=1$ (d) $n=2$ (must be an even integer)
19. (a) $f(x)=x^3\Rightarrow f''(x)=6x$ so when $|x|\le 1$, $|f''(x)|\le 6$; taking $B_2=6$, $|\text{error}|\le\frac{2^3\cdot 6}{12\cdot 4^2}=0.25$ (b) $f^{(4)}(x)=0\Rightarrow B_4=0$, so error bound is 0 (c) $\frac{2^3\cdot 6}{12\cdot n^2}\le 0.001\Rightarrow n^2\ge 4000\Rightarrow n\ge 63.25$, so take $n=64$ (d) $n=2$
21. (a) $f''(x)=-\sin(x)\Rightarrow|f''(x)|\le 1\Rightarrow B_2=1$, so $|\text{error}|\le\frac{\pi^3\cdot 1}{12\cdot 4^2}\approx 0.1612$ (b) $f^{(4)}(x)=\sin(x)\Rightarrow\left|f^{(4)}(x)\right|\le 1=B_4$, so $|\text{error}|\le\frac{\pi^5\cdot 1}{180\cdot 4^4}\approx 0.0066$ (c) $\frac{\pi^3\cdot 1}{12\cdot n^2}\le 0.001\Rightarrow n^2\ge\frac{1000\pi^3}{12}\Rightarrow n\ge 50.83$, so take $n=51$ (d) $\frac{\pi^5\cdot 1}{180\cdot n^4}\le 0.001\Rightarrow n^4\ge\frac{1000\pi^3}{180}\Rightarrow n\ge 6.42$, so take $n=8$
23. $S_6=\frac{30}{3}[50+4(62)+2(92)+4(86)+2(74)+4(50)+40]=12140\text{ ft}^2$
25. area: $S_6\approx 37166.7\text{ ft}^2$
volume: $(37166.7)(22)=817{,}667\text{ ft}^3$
27. distance $\approx T_{10}=4{,}010$ ft
29. On your own.
31. On your own.
33. (a) $L_4=3.5$ (b) $R_4=4.5$ (c) $M_4=4$ (d) 4
35. (a) $L_4=0.75$ (b) $R_4=0.75$ (c) $M_4=0.625$ (d) $\frac{2}{3}$
37. (a) 1.8961 (b) 1.8961 (c) 2.0523 (d) 2
39. On your own.
41. $S_{10}=6.12572$; $S_{40}=6.12573$
43. $S_{10}=22.1035$; $S_{40}=22.1035$