10.2 INFINITE SERIES

Our goal in this section is to add together the numbers in a sequence. Since it would take a "very long time" to add together the infinite number of numbers, we first consider finite sums, look for patterns in these finite sums, and take limits as more and more numbers are included in the finite sums.

What does it mean to add together an infinite number of terms? We will define that concept carefully in this section. Secondly, is the sum of all the terms a finite number? In the next few sections we will examine a variety of techniques for determining whether an infinite sum is finite. Finally, if we know the sum is finite, can we determine the value of the sum? The difficulty of finding the exact value of the sum varies from very easy to very, very difficult.

Example 1: A golf ball is thrown 9 feet straight up into the air, and on each bounce it rebounds to two thirds of its previous height (Fig. 1). Find a sequence whose terms give the distances the ball travels during each successive bounce. Represent the **total** distance traveled by the ball as a sum.

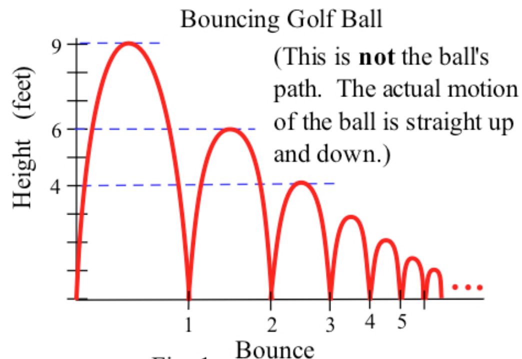

Fig. 1

Solution: The heights of the successive bounces are 9 feet, $(\frac{2}{3})\cdot 9$ feet, $(\frac{2}{3})\cdot\left[(\frac{2}{3})\cdot 9\right]$ feet, $(\frac{2}{3})^3\cdot 9$ feet, and so forth. On each bounce, the ball rises and falls so the distance traveled is twice the height of that bounce:

$$18 \text{ feet}, (\tfrac{2}{3})\cdot 18 \text{ feet}, (\tfrac{2}{3})\cdot(\tfrac{2}{3})\cdot 18 \text{ feet}, (\tfrac{2}{3})^3\cdot 18 \text{ feet}, (\tfrac{2}{3})^4\cdot 18 \text{ feet}, \ldots .$$

The total distance traveled is the sum of the bounce–distances:

$$\begin{aligned}\text{total distance} &= 18 + (\tfrac{2}{3})\cdot 18 + (\tfrac{2}{3})\cdot(\tfrac{2}{3})\cdot 18 + (\tfrac{2}{3})^3\cdot 18 + (\tfrac{2}{3})^4\cdot 18 + \ldots \\ &= 18\left\{ 1 + \tfrac{2}{3} + (\tfrac{2}{3})^2 + (\tfrac{2}{3})^3 + (\tfrac{2}{3})^4 + \ldots \right\}\end{aligned}$$

At the completion of the first bounce the ball has traveled 18 feet. After the second bounce, it has traveled 30 feet, a total of 38 feet after the third bounce, $43\frac{1}{3}$ feet after the fourth, and so on. With a calculator and some patience, we see that after the 20^{th} bounce the ball has traveled a total of approximately 53.996 feet, after the 30^{th} bounce approximately 53.99994 feet, and after the 40th bounce approximately 53.9999989 feet.

Practice 1: A tennis ball is thrown 10 feet straight up into the air, and on each bounce it rebounds to 40% of its previous height. Represent the total distance traveled by the ball as a sum, and find the total distance traveled by the ball after the completion of its third bounce. (Fig. 2)

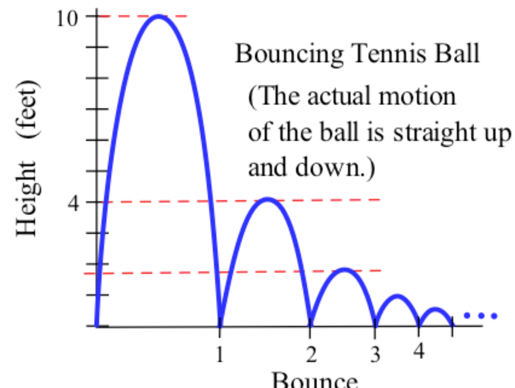

Fig. 2

Infinite Series

The infinite sums in the Example and Practice are called infinite series, and they are the objects we will start to examine in this section.

Definitions

An **infinite series** is an expression of the form

$$a_1 + a_2 + a_3 + a_4 + \ldots \quad \text{or} \quad \sum_{k=1}^{\infty} a_k \ .$$

The numbers $a_1, a_2, a_3, a_4, \ldots$ are called the **terms** of the series. (Fig. 3)

Example 2: Represent the following series using the sigma notation. (a) $1 + 1/3 + 1/9 + 1/27 + \ldots$, (b) $-1 + 1/2 - 1/3 + 1/4 - 1/5 + \ldots$, (c) $18(2/3 + 4/9 + 8/27 + 16/81 + \ldots)$ (d) $0.777 \ldots = 7/10 + 7/100 + 7/1000 + \ldots$, and (e) $0.222\ldots$

Solution: (a) $1 + 1/3 + 1/9 + 1/27 + \ldots = \sum_{k=0}^{\infty} (\frac{1}{3})^k$ or $\sum_{k=1}^{\infty} (\frac{1}{3})^{k-1}$

(b) $-1 + 1/2 - 1/3 + 1/4 - 1/5 + \ldots = \sum_{k=1}^{\infty} (-1)^k \frac{1}{k}$ (c) $18 \sum_{k=1}^{\infty} (\frac{2}{3})^k$

(d) $0.777 \ldots = 7/10 + 7/100 + 7/1000 + \ldots = \sum_{k=1}^{\infty} \frac{7}{10^k}$ (e) $\sum_{k=1}^{\infty} \frac{2}{10^k}$

Practice 2: Represent the following series using the sigma notation. (a) $1 + 2 + 3 + 4 + \ldots$, (b) $-1 + 1 - 1 + 1 - \ldots$ (c) $2 + 1 + 1/2 + 1/4 + \ldots$ (d) $1/2 + 1/4 + 1/6 + 1/8 + 1/10 + \ldots$ (e) $0.111\ldots$

In order to determine if the infinite series adds up to a finite value, we examine the sums as more and more terms are added.

Definition

The **partial sums** $\mathbf{s_n}$ of the infinite series $\sum_{k=1}^{\infty} a_k$

are the numbers

$s_1 = a_1$,

$s_2 = a_1 + a_2$,

$s_3 = a_1 + a_2 + a_3$,

. . .

In general, $s_n = a_1 + a_2 + a_3 + \ldots + a_n = \sum_{k=1}^{n} a_k$

or , recursively, as $s_n = s_{n-1} + a_n$.

The partial sums form the **sequence of partial sums** $\{ \mathbf{s_n} \}$.

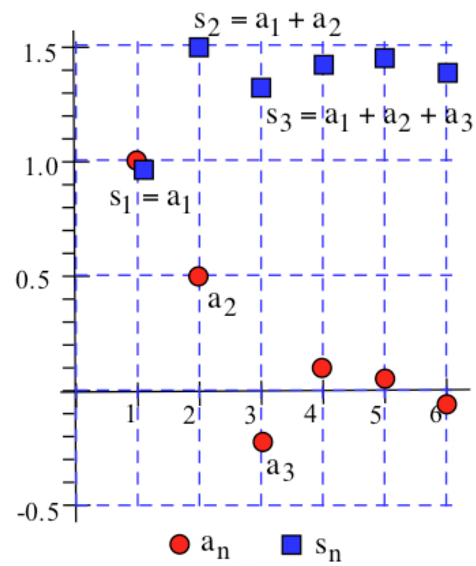

Fig. 3

Example 3: Calculate the first 4 partial sums for the following series.

(a) 1 + 1/2 + 1/4 + 1/8 + 1/16 + . . . , (b) $\sum_{k=1}^{\infty} (-1)^k$, and (c) $\sum_{n=1}^{\infty} \frac{1}{n}$.

Solution: (a) $s_1 = 1$, $s_2 = 1 + 1/2 = 3/2$, $s_3 = 1 + 1/2 + 1/4 = 7/4$, $s_4 = 1 + 1/2 + 1/4 + 1/8 = 15/8$

It is usually easier to use the recursive version of s_n :

$s_3 = s_2 + a_3 = 3/2 + 1/4 = 7/4$ and $s_4 = s_3 + a_4 = 7/4 + 1/8 = 15/8$.

(b) $s_1 = (-1)^1 = -1$, $s_2 = s_1 + a_2 = -1 + (-1)^2 = 0$, $s_3 = s_2 + a_3 = 0 + (-1)^3 = -1$, $s_4 = 0$.

(c) $s_1 = 1$, $s_2 = 3/2$, $s_3 = 11/6$, $s_4 = 25/12$.

Practice 3: Calculate the first 4 partial sums for the following series.

(a) 1 – 1/2 + 1/4 – 1/8 + 1/16 – . . . , (b) $\sum_{k=1}^{\infty} \left(\frac{1}{3}\right)^k$, and (c) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n}$.

If we know the values of the partial sums s_n , we can recover the values of the terms a_n used to build the s_n.

Example 4: Suppose $s_1 = 2.1$, $s_2 = 2.6$, $s_3 = 2.84$, and $s_4 = 2.87$ are the first partial sums of $\sum_{k=1}^{\infty} a_k$. Find the values of the first four terms of $\{ a_n \}$.

Solution: $s_1 = a_1$ so $a_1 = 2.1$. $s_2 = a_1 + a_2$ so $2.6 = 2.1 + a_2$ and $a_2 = 0.5$.

Similarly, $s_3 = a_1 + a_2 + a_3$ so $2.84 = 2.1 + 0.5 + a_3$ and $a_3 = 0.24$. Finally, $a_4 = 0.03$.

An alternate solution method starts with $a_1 = s_1$ and then uses the fact that $s_n = s_{n-1} + a_n$ so $a_n = s_n - s_{n-1}$. Then

$$a_2 = s_2 - s_1 = 2.6 - 2.1 = 0.5 \ .$$
$$a_3 = s_3 - s_2 = 2.84 - 2.6 = 0.24, \text{ and}$$
$$a_4 = s_4 - s_3 = 2.87 - 2.84 = 0.03 \ .$$

Practice 4: Suppose $s_1 = 3.2$, $s_2 = 3.6$, $s_3 = 3.5$, $s_4 = 4$, $s_{99} = 7.3$, $s_{100} = 7.6$, and $s_{101} = 7.8$ are partial sums of $\sum_{k=1}^{\infty} a_k$. Find the values of a_1, a_2, a_3, a_4 , and a_{100} .

Example 5: Graph the first five **terms** of the series $\sum_{k=1}^{\infty} (\frac{1}{2})^k$. Then graph the first five **partial sums**.

Solution: $a_1 = \left(\frac{1}{2}\right)^1 = \frac{1}{2},\ a_2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4},\ a_3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8},\ a_4 = \frac{1}{16},\ a_4 = \frac{1}{32}$

$$s_1 = \frac{1}{2},\ s_2 = \frac{1}{2} + \frac{1}{4} = \frac{3}{4},\ s_3 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{7}{8},\ s_4 = \frac{15}{16},\ \text{and } s_5 = \frac{31}{32}.$$

These values are graphed in Fig. 4.

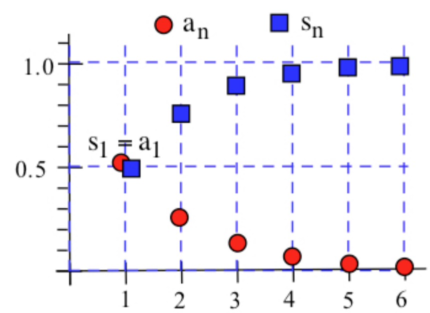

Fig. 4

Practice 5: Graph the first five terms of the series $\sum_{k=1}^{\infty} (-\frac{1}{2})^k$. Then graph the first five partial sums.

Convergence of a Series

The convergence of a series is defined in terms of the behavior of the sequence of partial sums. If the partial sums, the sequence obtained by adding more and more of the terms of the series, approaches a finite number, we say the **series** converges to that finite number. If the sequence of partial sums diverges (does not approach a single finite number), we say that the **series** diverges.

Definitions

Let $\{ s_n \}$ be the sequence of partial sums of the series $\sum_{k=1}^{\infty} a_k$: $s_n = \sum_{k=1}^{n} a_k$

If $\{ s_n \}$ is a convergent sequence, (Fig. 5) we say the series $\sum_{k=1}^{\infty} a_k$ **converges**.

If the sequence of partial sums $\{ s_n \}$ converges to A, we say the series $\sum_{k=1}^{\infty} a_k$ **converges to A**

or **the sum of the series is A**, and we write $\sum_{k=1}^{\infty} a_k = A$.

If the sequence of partial sums $\{ s_n \}$ diverges, we say the series $\sum_{k=1}^{\infty} a_k$ **diverges**.

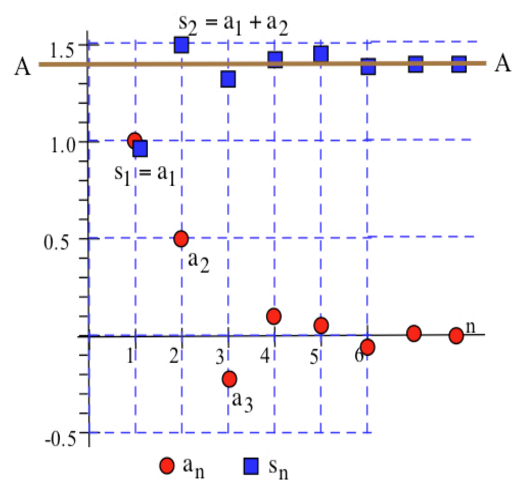

Fig. 5

Example 6: In the next section we present a method for determining that the n^{th} partial sum of $\sum_{k=1}^{\infty} (\frac{1}{2})^k$ is $s_n = \frac{2^n - 1}{2^n} = 1 - \frac{1}{2^n}$. Use this result to evaluate the limit of $\{ s_n \}$.

Does the series $\sum_{k=1}^{\infty} (\frac{1}{2})^k$ converge? If so, to what value?

Solution: $\lim_{n\to\infty} s_n = \lim_{n\to\infty} 1 - \frac{1}{2^n} = 1$ (Fig. 6), so $\sum_{k=1}^{\infty} (\frac{1}{2})^k$ converges to 1: $\sum_{k=1}^{\infty} (\frac{1}{2})^k = 1$.

Practice 6: The n^{th} partial sum of $\sum_{k=1}^{\infty} (-\frac{1}{2})^k$ is $s_n = -\frac{1}{3} + \frac{1}{3} \cdot (-\frac{1}{2})^n$. Use this result to evaluate the limit of $\{ s_n \}$. Does the series $\sum_{k=1}^{\infty} (-\frac{1}{2})^k$ converge? If so, to what value?

The next theorem says that if a series converges, then the terms of the series must approach 0. When a series is convergent then the partial sums s_n approach a finite limit (Fig. 3) so all of the s_n must be close to that limit when n is large. Then s_n and s_{n-1} must be close to each other (why?) and $a_n = s_n - s_{n-1}$ must be close to 0.

Theorem: If the series $\sum_{k=1}^{\infty} a_k$ converges, then $\lim_{k\to\infty} a_k = 0$.

We can NOT use this theorem to conclude that a series converges. If the terms of the series do approach 0, then the series may or may not converge — more information is needed to draw a conclusion. However, an alternate form of the theorem, called the n^{th} Term Test for Divergence, is **very useful for quickly concluding that some series diverge.**

n^{th} Term Test for Divergence of a Series

If the terms a_n of a series do not approach 0 (as "$n\to\infty$")

then the series diverges:

if $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ diverges.

Example 7: Which of these series diverge **by the n^{th} Term Test**?

(a) $\sum_{n=1}^{\infty} (-1)^n$ (b) $\sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n$ (c) $\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^n$ (d) $\sum_{n=1}^{\infty} \frac{1}{n}$

Solution: (a) $a_n = (-1)^n$ oscillates between -1 and $+1$ and does not approach 0. $\sum_{n=1}^{\infty} (-1)^n$ **diverges**.

(b) $a_n = \left(\frac{3}{4}\right)^n$ approaches 0 so $\sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n$ **may or may not** converge.

(c) $a_n = \left(1+\frac{1}{n}\right)^n$ approaches $e \neq 0$, so $\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^n$ **diverges**.

(d) $a_n = \frac{1}{n}$ approaches 0 so $\sum_{n=1}^{\infty} \frac{1}{n}$ **may or may not** converge.

We can be certain that (a) and (c) diverge. We don't have enough information yet to decide about (b) and (d). (In the next section we show that (b) converges and (d) diverges.)

Practice 7: Which of these series diverge by the n^{th} Term Test?

(a) $\sum_{n=1}^{\infty} (-0.9)^n$ (b) $\sum_{n=1}^{\infty} (1.1)^n$ (c) $\sum_{n=1}^{\infty} \sin(n\pi)$ (d) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

New Series From Old

If we know about the convergence of a series, then we also know about the convergence of several related series.

- Inserting or deleting a "few" terms, any **finite** number of terms, does not change the convergence or divergence of a series. The insertions or deletions typically change the sum (the limit of the partial sums), but they do not change whether or not the series converges. (Inserting or deleting an infinite number of terms can change the convergence or divergence.)

- Multiplying each term in a series by a nonzero constant does not change the convergence or divergence of a series:

  Suppose $c \neq 0$. $\sum_{n=1}^{\infty} a_n$ converges if and only if $\sum_{n=1}^{\infty} c \cdot a_n$ converges.

- Term–by–term addition and subtraction of the terms of two convergent series result in convergent series. (Term by term multiplication and division of series do not have such nice results.)

Theorem:

If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converge with $\sum_{n=1}^{\infty} a_n = A$ and $\sum_{n=1}^{\infty} b_n = B$,

then $\sum_{n=1}^{\infty} C \cdot a_n = C \cdot A$,

$\sum_{n=1}^{\infty} (a_n + b_n) = A + B$, and

$\sum_{n=1}^{\infty} (a_n - b_n) = A - B$.

The proofs of these statements follow directly from the definition of convergence of a series and from results about convergence of sequences (of partial sums).

PROBLEMS

In problems 1 – 6, rewrite each sum using sigma notation starting with k = 1.

1. $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + ...$

2. $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + ...$

3. $\frac{2}{3} + \frac{2}{6} + \frac{2}{9} + \frac{2}{12} + \frac{2}{15} + \frac{2}{18} + ...$

4. sin(1) + sin(8) + sin(27) + sin(64) + sin(125) + ...

5. $(-\frac{1}{2}) + (\frac{1}{4}) + (-\frac{1}{8}) + (\frac{1}{16}) + (-\frac{1}{32}) + ...$

6. $(-\frac{1}{3}) + (\frac{1}{9}) + (-\frac{1}{27}) + (\frac{1}{81}) + (-\frac{1}{243}) + ...$

In problems 7 – 14, calculate and graph the first four partial sums s_1 to s_4 of the given series $\sum_{n=1}^{\infty} a_n$.

7. $\sum_{n=1}^{\infty} n^2$

8. $\sum_{n=1}^{\infty} (-1)^n$

9. $\sum_{n=1}^{\infty} \frac{1}{n+2}$

10. $\sum_{n=1}^{\infty} \left\{ \frac{1}{n} - \frac{1}{n+1} \right\}$

11. $\sum_{n=1}^{\infty} \frac{1}{2^n}$

12. $\sum_{n=1}^{\infty} (-\frac{1}{2})^n$

In problems 13 – 18, the first five partial sums s_1 to s_5 are given. Find the first four terms a_1 to a_4 of the series.

13. $s_1 = 3$, $s_2 = 2$, $s_3 = 4$, $s_4 = 5$, $s_5 = 3$

14. $s_1 = 3$, $s_2 = 5$, $s_3 = 4$, $s_4 = 6$, $s_5 = 5$

15. $s_1 = 4$, $s_2 = 4.5$, $s_3 = 4.3$, $s_4 = 4.8$, $s_5 = 5$

16. $s_1 = 4$, $s_2 = 3.7$, $s_3 = 3.9$, $s_4 = 4.1$, $s_5 = 4$

17. $s_1 = 1, s_2 = 1.1, s_3 = 1.11, s_4 = 1.111, s_5 = 1.1111$

18. $s_1 = 1, s_2 = 0.9, s_3 = 0.93, s_4 = 0.91, s_5 = 0.92$

In problems 19 – 28 , represent each repeating decimal as a series using the sigma notation.

19. 0.888 ...
20. 0.333 ...
21. 0.555 ...
22. 0.111 ...
23. 0.aaa ...
24. 0.232323 ...
25. 0.171717 ...
26. 0.838383 ...
27. 0.070707 ...
28. 0.ababab ...

29. Find a pattern for a fraction representation of the repeating decimal 0.abcabcabc ... .

30. A golf ball is thrown 20 feet straight up into the air, and on each bounce it rebounds to 60% of its previous height. Represent the total distance traveled by the ball as a sum.

31. A "super ball" is thrown 15 feet straight up into the air, and on each bounce it rebounds to 80% of its previous height. Represent the total distance traveled by the ball as a sum.

32. Each special washing of a pair of overalls removes 80% of the radioactive particles attached to the overalls. Represent, as a sequence of numbers, the percent of the original radioactive particles that remain after each washing.

33. Each week, 20% of the argon gas in a container leaks out of the container. Represent, as a sequence of numbers, the percent of the original argon gas that remains in the container at the end of the 1^{st}, 2^{nd}, 3^{rd}, and n^{th} weeks.

34. Eight people are going on an expedition by horseback through desolate country. The people and scientific equipment (fishing gear) require 12 horses, and additional horses are needed to carry food for the horses. Each horse can carry enough food to feed 2 horses for the trip. Represent the number of horses needed to carry food as a sum. (Start of a solution: The original 12 horses will require 6 new horses to carry their food. The 6 new horses require 3 additional horses to carry their food. The 3 additional horses require another 1.5 horses to carry food for them, etc.)

Which of the series in problems 35 – 43, definitely diverge by the n^{th} Term Test? What can we conclude about the other series in these problems?

35. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^n$

36. $\displaystyle\sum_{n=1}^{\infty} \frac{7}{n}$

37. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{4}{3}\right)^n$

38. $\displaystyle\sum_{n=1}^{\infty} \left(-\frac{7}{4}\right)^2$

39. $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(n)}{n}$

40. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$

41. $\displaystyle\sum_{n=1}^{\infty} \cos\left(\frac{1}{n}\right)$

42. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2-20}{n^2+4}$

43. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2-20}{n^5+4}$

Practice Answers

Practice 1: The heights of the bounces are $10, (0.4)\cdot 10, (0.4)\cdot(0.4)\cdot 10, (0.4)^3\cdot 10, \ldots$ so the distances traveled (up and down) by the ball are $20, (0.4)\cdot 20, (0.4)\cdot(0.4)\cdot 20, (0.4)^3\cdot 20, \ldots$

The total distance traveled is

$$20 + (0.4)\cdot 20 + (0.4)^2\cdot 20 + (0.4)^3\cdot 20 + \ldots = 20\{ 1 + 0.4 + (0.4)^2 + (0.4)^3 + \ldots \} = 20 \sum_{k=0}^{\infty} (0.4)^k$$

After 3 bounces the ball has traveled $20 + (0.4)(20) + (0.4)^2(20) = 20 + 8 + 3.2 = 31.2$ feet.

Practice 2: (a) $\displaystyle\sum_{k=1}^{\infty} k$ (b) $\displaystyle\sum_{k=1}^{\infty} (-1)^k$

(c) $2(1 + \frac{1}{2} + \frac{1}{4} + \ldots) = 2 \sum_{k=0}^{\infty} (\frac{1}{2})^k$ or $\sum_{k=0}^{\infty} (\frac{1}{2})^{k-1}$ or $\sum_{k=1}^{\infty} (\frac{1}{2})^{k-2}$

(d) $\sum_{k=1}^{\infty} \frac{1}{2k}$ (e) $\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \ldots = \frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} + \ldots = \sum_{k=1}^{\infty} (\frac{1}{10})^k$ or $\sum_{k=1}^{\infty} \frac{1}{10^k}$

Practice 3: (a) Partial sums: 1, 1/2, 3/4, 5/8 (b) $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots$; partial sums: $\frac{1}{3}, \frac{4}{9}, \frac{13}{27}, \frac{40}{81}$

(c) $\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \ldots$; partial sums: $\frac{1}{2}, \frac{1}{6}, \frac{10}{24} = \frac{5}{12}, \frac{13}{60}$

Practice 4: $a_1 = s_1 = 3.2$, $a_2 = s_2 - s_1 = (3.6) - (3.2) = 0.4$, $a_3 = s_3 - s_2 = (3.5) - (3.6) = -0.1$,
$a_4 = s_4 - s_3 = (4) - (3.5) = 0.5$, $a_{100} = s_{100} - s_{99} = (7.6) - (7.3) = 0.3$

Practice 5: $a_1 = -1/2, a_2 = 1/4, a_3 = -1/8, a_4 = 1/16, a_5 = -1/32$

$s_1 = a_1 = -\frac{1}{2}$, $s_2 = a_1 + a_2 = -\frac{1}{2} + \frac{1}{4} = -\frac{1}{4}$,

$s_3 = s_2 + a_3 = -\frac{1}{4} - \frac{1}{8} = -\frac{3}{8} \approx -0.375$,

$s_4 = s_3 + a_4 = -\frac{3}{8} + \frac{1}{16} = -\frac{5}{16} \approx -0.3125$,

$s_5 = s_4 + a_5 = -\frac{5}{16} - \frac{1}{32} = -\frac{11}{32} \approx -0.34375$,

The graphs of a_n and s_n are shown in Fig. 6.

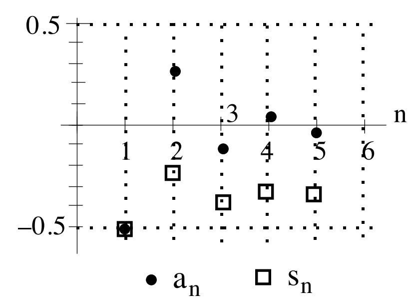

Fig. 6

Practice 6: $\lim_{n\to\infty} s_n = \lim_{n\to\infty} -\frac{1}{3} + \frac{1}{3} \cdot (-\frac{1}{2})^n = -\frac{1}{3} + \frac{1}{3} \cdot 0$
$= -\frac{1}{3}$.

The limit, as "$n\to\infty$", of s_n is a finite number, so $\sum_{k=1}^{\infty} (-\frac{1}{2})^k$ **converges to $-\frac{1}{3}$.**

Practice 7: (a) $\sum_{n=1}^{\infty} (-0.9)^n$: $a_n = (-0.9)^n$ approaches 0 so $\sum_{n=1}^{\infty} (-0.9)^n$ **may converge**.

(b) $\sum_{n=1}^{\infty} (1.1)^n$: $a_n = (1.1)^n$ "approaches infinity" so $\sum_{n=1}^{\infty} (1.1)^n$ **diverges.**

(c) $\sum_{n=1}^{\infty} \sin(n\pi) = 0 + 0 + 0 + \ldots$: $\sin(n\pi) = 0$ "approaches 0" so $\sum_{n=1}^{\infty} \sin(n\pi)$ **may converge**.

(d) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$: $a_n = \frac{1}{\sqrt{n}}$ approaches 0 so $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ **may converge**.

(Later in this chapter we will show that series (a) and (c) converge and series (d) diverges.)

Appendix: Programming Partial Sums of Numerical Series

MAPLE commands for $\sum_{n=1}^{100} \frac{1}{n}$, $\sum_{n=1}^{100} \frac{1}{n^2}$, $\sum_{n=1}^{100} \frac{1}{n!}$:

```
> sum(1/n , n=1..100) ;   (then press ENTER key)

> sum(1/n^2 , n=1..100) ;

> sum(1/(n!) , n=1..100) ;
```

TI–85 program for $\sum_{n=1}^{M} \frac{1}{n}$

Prgm1:NUMSUM		
Disp "NUM TERMS ="		
Input M		
1 → A		
0 → N		
Lbl ONE		
N+1→ N		
1/N → A	(value of the new term)	**1/N → A**
S + A → S	(add new term to partial sum)	S + A → S
If N<M	(test if need next term)	If N<M
Goto ONE		
Disp S		
Stop		Stop

For $\sum_{n=1}^{M} \frac{1}{n^2}$ change the bold line to **1/(N*N) → A .** For $\sum_{n=1}^{M} \frac{1}{n!}$ change the bold line to **A/N → A .**