11.3 VECTORS IN THREE DIMENSIONS

Once you understand the 3–dimensional coordinate system, 3–dimensional vectors are a straightforward extension of vectors in two dimensions. Vectors in three dimensions are more difficult to visualize and sketch, but all of the 2–dimensional algebraic techniques extend very naturally, with just one more component.

A vector in any setting is a **quantity that has both a direction and a magnitude**, and in three dimensions vectors can be represented geometrically as directed line segments, (arrows). The vector **V** given by the line segment from the starting point (tail) P = (1, 2, 3) to the ending point (head) Q = (4, 1, 4) is shown in Fig. 1. The vector **V** from P to Q is represented algebraically by the ordered triple enclosed in "bent" brackets: $\mathbf{V} = \langle 3, -1, 1 \rangle$ with each component representing the displacement from P to Q. (We continue to reserve the "round brackets" ( ) to represent points.) Fig. 1 also shows several other geometric representations of vector **V**.

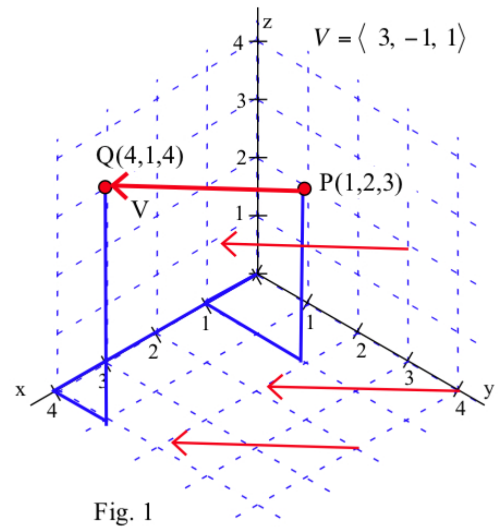

Fig. 1

Definitions and Properties

The following definitions, arithmetic operations, and properties of vectors in 3–dimensional space are straightforward generalizations from two dimensions.

Definition: Equality of Vectors

Geometrically, two vectors are equal if their lengths are equal and their directions are the same.

Algebraically, two vectors are equal if their respective components are equal:

if $\mathbf{U} = \langle a, b, c \rangle$ and $\mathbf{V} = \langle x, y, z \rangle$,

then $\mathbf{U} = \mathbf{V}$ if and only if $a = x$, $b = y$, and $c = z$.

The definitions of scalar multiplication, vector addition and vector subtraction in three dimensions are similar to the definitions in two dimensions, but each vector has one more component.

Definitions: Vector Arithmetic

If $\mathbf{A} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{B} = \langle b_1, b_2, b_3 \rangle$ are vectors and k is a scalar, then

then $k\mathbf{A} = \langle ka_1, ka_2, ka_3 \rangle$

$\mathbf{A} + \mathbf{B} = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$

$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-1)\mathbf{B} = \langle a_1 - b_1, a_2 - b_2, a_3 - b_3 \rangle$.

Example 1: For $\mathbf{A} = \langle 3, -4, 2 \rangle$ and $\mathbf{B} = \langle 5, 1, -3 \rangle$, calculate $\mathbf{C} = 3\mathbf{B}$, $\mathbf{D} = 2\mathbf{A} + 3\mathbf{B}$, and $\mathbf{E} = 5\mathbf{A} - 2\mathbf{B}$.

Solution: $\mathbf{C} = 3\mathbf{B} = 3\langle 5, 1, -3 \rangle = \langle 15, 3, -9 \rangle$.

$\mathbf{D} = 2\mathbf{A} + 3\mathbf{B} = 2\langle 3, -4, 2 \rangle + 3\langle 5, 1, -3 \rangle = \langle 6+15, -8+3, 4-9 \rangle = \langle 21, -5, -5 \rangle$.

$\mathbf{E} = 5\mathbf{A} - 2\mathbf{B} = 5\langle 3, -4, 2 \rangle - 2\langle 5, 1, -3 \rangle = \langle 5, -22, 16 \rangle$.

Practice 1: For $\mathbf{A} = \langle 5, -4, 1 \rangle$ and $\mathbf{B} = \langle 2, -3, 4 \rangle$, calculate $\mathbf{C} = 5\mathbf{A}$, $\mathbf{D} = 3\mathbf{A} + 4\mathbf{B}$, and $\mathbf{E} = 2\mathbf{B} - 3\mathbf{A}$.

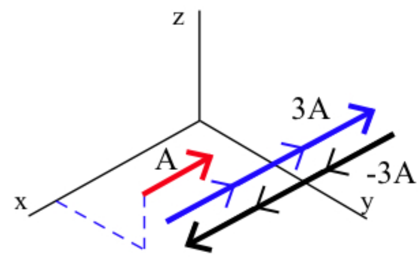

Fig. 2

Each of the given vector arithmetic operations also has a geometric interpretation:

Multiplying by a scalar k results in a vector that is $|k|$ times as long as the original vector, $|k\mathbf{A}| = |k|\,|\mathbf{A}|$. If k is positive, then $\mathbf{A}$ and $k\mathbf{A}$ have the same direction. If k is negative, then $\mathbf{A}$ and $k\mathbf{A}$ point in opposite directions (Fig. 2)

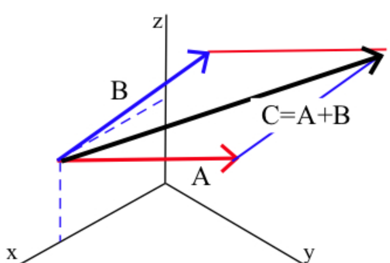

Fig. 3

The sum $\mathbf{C} = \mathbf{A} + \mathbf{B}$ vector can be found geometrically by using the parallelogram or head–to–tail methods described in Section 11.1. (Fig. 3)

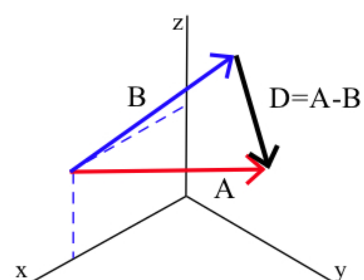

Fig. 4

The difference $\mathbf{D} = \mathbf{A} - \mathbf{B}$ vector can be sketched by adding $\mathbf{A}$ and $-\mathbf{B}$ or by drawing $\mathbf{A}$ and $\mathbf{B}$ with a common starting point and then drawing the line segment from the head of $\mathbf{B}$ to the head of $\mathbf{A}$. (Fig. 4)

Because it is more difficult to make precise drawings and measurements in three dimensions, the geometric methods are seldom used to perform vector arithmetic in three dimensions. These geometric interpretations are still very powerful and are important for understanding the meaning of various arithmetic operations and for understanding how certain algorithms are developed.

Visualizing vector arithmetic in three dimensions: If you took the time in Section 11.2 to build a small model of a 3–dimensional coordinate system, you can use it now to see and handle some 3–dimensional vectors. Sharpened pencils or skewer sticks make good physical "vectors."

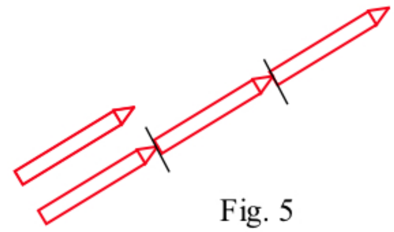

Fig. 5

Visualizing the vectors **A** and 3**A** is easy — just tape together a short pencil and one three times as long (Fig. 5). How could you modify this arrangement to illustrate **A** and –3**A**?

Addition and subtraction are more difficult, but the following vectors fit together nicely:
C = **A** + **B** with **A** = $\langle 2, 3, 6 \rangle$ and **B** = $\langle 4, 0, 0 \rangle$ (Fig. 6), and **D** = **A** – **B** with **A** = $\langle 4, 7, 4 \rangle$ and **B** = $\langle 4, 2, 4 \rangle$.

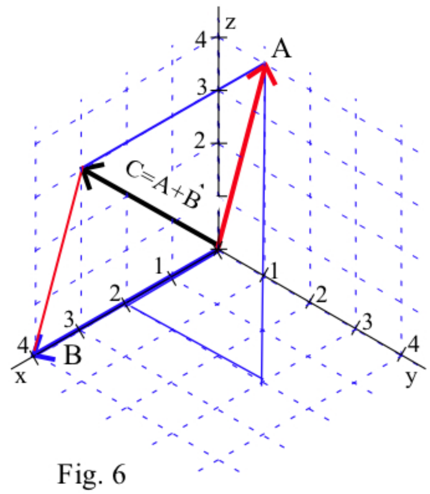

Fig. 6

The length of a vector in three dimensions follows directly from the formula for the distance between points in 3–dimensional space.

> The **magnitude** or **length** of a vector **V** = $\langle a, b, c \rangle$ is $|\mathbf{V}| = \sqrt{a^2 + b^2 + c^2}$.

Since the components a, b, and c of the vector **V** = $\langle a, b, c \rangle$ represent the displacements of the ending point from the starting point in the x, y, and z directions, we can represent **V** as a line segment from the point (0, 0, 0) to the point (a, b, c). Then the length of **V** is the distance from the point (0, 0, 0) to the point (a, b, c): length = { distance from (0, 0, 0) to (a, b, c) } = $\sqrt{a^2 + b^2 + c^2}$.
The only vector in 3–dimensional space with magnitude 0 is the zero vector **0** = $\langle 0, 0, 0 \rangle$. The zero vector has no specific direction.

Example 2: Determine the lengths of **A** = $\langle 2, 8, 16 \rangle$, **B** = $\langle -4, 8, 8 \rangle$, **C** = {vector represented by the line segment from (1,2,3) to (7,–1,9) }, **D** = **A** – **B** , and **E** = **B** + **C** .

Solution: $|\mathbf{A}| = \sqrt{2^2 + 8^2 + 16^2} = \sqrt{324} = 18$. $|\mathbf{B}| = \sqrt{(-4)^2 + 8^2 + 8^2} = \sqrt{144} = 12$.

$\mathbf{C} = \langle 7-1, -1-2, 9-3 \rangle = \langle 6, -3, 6 \rangle$ so $|\mathbf{C}| = \sqrt{81} = 9$.

$\mathbf{D} = \mathbf{A} - \mathbf{B} = \langle 2-(-4), 8-8, 16-8 \rangle = \langle 6, 0, 8 \rangle$ so $|\mathbf{D}| = \sqrt{100} = 10$.

$\mathbf{E} = \mathbf{B} + \mathbf{C} = \langle -4+6, 8+(-3), 8+6 \rangle = \langle 2, 5, 14 \rangle$ so $|\mathbf{E}| = \sqrt{225} = 15$.

Practice 2: Determine the lengths of $\mathbf{A} = \langle 2, 3, 6 \rangle$, $\mathbf{B} = \langle 2, 1, 2 \rangle$, and $\mathbf{C} = \mathbf{A} - 2\mathbf{B}$.

Definitions:

The **direction** of a nonzero vector **A** is the unit vector

$$\mathbf{U} = \frac{1}{|\mathbf{A}|}\mathbf{A} = \frac{\mathbf{A}}{|\mathbf{A}|} .$$

The **standard basis vectors** in the plane are

$$\mathbf{i} = \langle 1, 0, 0 \rangle, \mathbf{j} = \langle 0, 1, 0 \rangle, \text{ and } \mathbf{k} = \langle 0, 0, 1 \rangle . \quad \text{(Fig. 7)}$$

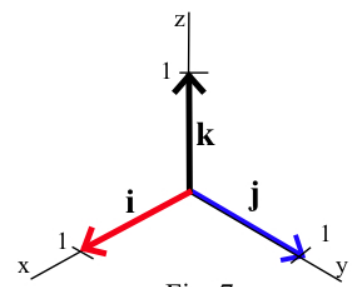

Fig. 7

Example 3: Determine the lengths and directions of $\mathbf{A} = 6\mathbf{i} + 2\mathbf{j} + 9\mathbf{k}$, $\mathbf{B} = 6\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$, $\mathbf{C} = 3\mathbf{A} + 2\mathbf{B}$, and $\mathbf{D} = \mathbf{B} - 2\mathbf{A}$.

Solution: $|\mathbf{A}| = \sqrt{6^2 + 2^2 + 9^2} = \sqrt{121} = 11$. Direction of **A** is $\frac{\mathbf{A}}{|\mathbf{A}|} = \frac{6}{11}\mathbf{i} + \frac{2}{11}\mathbf{j} + \frac{9}{11}\mathbf{k}$.

$|\mathbf{B}| = \sqrt{6^2 + 3^2 + (-6)^2} = \sqrt{81} = 9$. Direction of **B** is $\frac{\mathbf{B}}{|\mathbf{B}|} = \frac{2}{3}\mathbf{i} + \frac{1}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}$.

$\mathbf{C} = 3\mathbf{A} + 2\mathbf{B} = 30\mathbf{i} + 12\mathbf{j} + 15\mathbf{k}$. $|\mathbf{C}| = \sqrt{1269} \approx 35.6$.

The direction of **C** is $\frac{\mathbf{C}}{|\mathbf{C}|} = \frac{30}{\sqrt{1269}}\mathbf{i} + \frac{12}{\sqrt{1269}}\mathbf{j} + \frac{15}{\sqrt{1269}}\mathbf{k} \approx 0.84\,\mathbf{i} + 0.34\,\mathbf{j} + 0.42\,\mathbf{k}$.

$\mathbf{D} = \mathbf{B} - 2\mathbf{A} = -6\mathbf{i} - 1\mathbf{j} - 24\mathbf{k}$. $|\mathbf{D}| = \sqrt{613} \approx 24.8$.

The direction of **D** is $\frac{\mathbf{D}}{|\mathbf{D}|} = \frac{-6}{\sqrt{613}}\mathbf{i} - \frac{1}{\sqrt{613}}\mathbf{j} - \frac{24}{\sqrt{613}}\mathbf{k} \approx 0.24\mathbf{i} - 0.04\mathbf{j} - 0.97\mathbf{k}$.

Practice 3: Determine the lengths and directions of $\mathbf{A} = 3\mathbf{i} + 2\mathbf{j} - 6\mathbf{k}$, $\mathbf{B} = 6\mathbf{j} - 8\mathbf{k}$, $\mathbf{C} = \mathbf{A} + 3\mathbf{B}$, and $\mathbf{D} = 2\mathbf{B} - 3\mathbf{A}$.

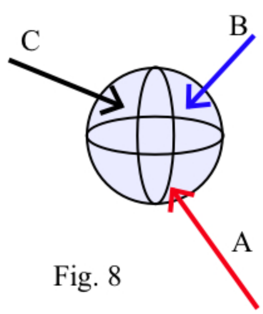

Fig. 8

Example 4: Three players are pushing on a ball, but the ball is not moving (Fig. 8). Player A is pushing with a force of 45 pounds in the direction $\frac{1}{9}\mathbf{i} - \frac{4}{9}\mathbf{j} + \frac{8}{9}\mathbf{k}$, and player B is pushing with a force of 60 pounds in the direction $\frac{4}{6}\mathbf{i} + \frac{4}{6}\mathbf{j} + \frac{2}{6}\mathbf{k}$. How hard and in what direction is player C pushing?

Solution: Since we know the magnitude and direction of the force vectors for players A and B, we can determine the force vector for each player:

$\mathbf{A} = \{\text{ magnitude }\}\{\text{ direction }\} = 45\left\{ \frac{1}{9}\mathbf{i} - \frac{4}{9}\mathbf{j} + \frac{8}{9}\mathbf{k} \right\} = 5\mathbf{i} - 20\mathbf{j} + 40\mathbf{k}$, and

$\mathbf{B} = \{\text{ magnitude }\}\{\text{ direction }\} = 60\left\{ \frac{4}{6}\mathbf{i} + \frac{4}{6}\mathbf{j} + \frac{2}{6}\mathbf{k} \right\} = 40\mathbf{i} + 40\mathbf{j} + 20\mathbf{k}$.

Pushing together, their force vector is $\mathbf{A} + \mathbf{B} = 45\mathbf{i} + 20\mathbf{j} + 60\mathbf{k}$. Since the ball is not moving, the force vectors of A, B, and C equal the zero vector, and we can solve for C's force vector:

$$\mathbf{C} = -(\mathbf{A} + \mathbf{B}) = -45\mathbf{i} - 20\mathbf{j} - 60\mathbf{k} \,.$$

C's force is $|\mathbf{C}| = \sqrt{(-45)^2 + (-20)^2 + (-60)^2} = \sqrt{6025} \approx 77.6$ pounds. C is pushing in

the direction $\dfrac{\mathbf{C}}{|\mathbf{C}|} = \dfrac{-45}{\sqrt{6025}}\mathbf{i} - \dfrac{20}{\sqrt{6025}}\mathbf{j} - \dfrac{60}{\sqrt{6025}}\mathbf{k} \approx -0.60\mathbf{i} - 0.26\mathbf{j} - 0.77\mathbf{k}$.

At this point, you should find the arithmetic of vectors in three dimensions is not much different or harder than in two dimensions. Angles, however, are another story, one we consider in Section 11.4. Fortunately, there is a straightforward process for determining the angle between two 3–dimensional vectors and it is useful in a variety of applications.

Beyond Three Dimensions

Just as we can represent points in 4, 5, or n–dimensional space, we can also work with n–dimensional vectors, $\langle a_1, a_2, a_3, \ldots a_n \rangle$. Even though it is no longer easy (or possible?) to work geometrically with these vectors, the arithmetic operations of scalar multiplication, vector addition, vector subtraction, and length are still defined component–by–component and are still easy.

Example 5: Write the vector **V** for the directed line segment from P = (1,2,–3,5,6) to Q = (5,–1,4,0,7), and find the length and direction of **V**.

Solution: $\mathbf{V} = \langle 5-1, -1-2, 4--3, 0-5, 7-6 \rangle = \langle 4, -3, 7, -5, 1 \rangle$.

$|\mathbf{V}| = \sqrt{4^2 + (-3)^2 + 7^2 + (-5)^2 + 1^2} = \sqrt{100} = 10$.

Direction of **V** is $\dfrac{\mathbf{V}}{|\mathbf{V}|} = \langle 0.4, -0.3, 0.7, -0.5, 0.1 \rangle$.

Practice 4: Calculate the lengths of $\mathbf{A} = \langle 4, 2, -5, 1, 0 \rangle$, $\mathbf{B} = \langle 6, 0, 2, -3, 6 \rangle$, and $\mathbf{C} = 2\mathbf{A} - 3\mathbf{B}$.

We can even determine formulas for some collections of points in higher dimensions.

Example 6: Find a formula for the set of points (w,x,y,z) in 4–dimensional space that are at a distance of 5 units from the point (5,3,–2,1). (This set is "4–sphere" with radius 5 and center (5,3,–2,1).)

Solution: We want the distance from (w,x,y,z) to (5,3,–2,1), $\sqrt{(w-5)^2 + (x-3)^2 + (y+2)^2 + (z-1)^2}$, to be 5 so

$$\sqrt{(w-5)^2 + (x-3)^2 + (y+2)^2 + (z-1)^2} = 5 \text{ and}$$

$$(w-5)^2 + (x-3)^2 + (y+2)^2 + (z-1)^2 = 25 \,.$$

PROBLEMS

In problems 1 – 4 the vectors **A** and **B** are shown. Sketch and label $\mathbf{C} = 2\mathbf{A}$, $\mathbf{D} = -\mathbf{B}$, and $\mathbf{E} = \mathbf{A} - \mathbf{B}$.

1. See Fig. 9.
2. See Fig. 10.
3. See Fig. 11.
4. See Fig. 12.

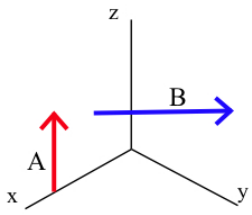

Fig. 9

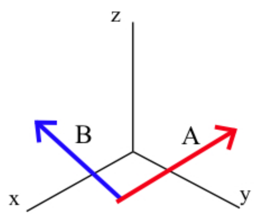

Fig. 10

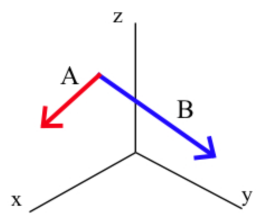

Fig. 11

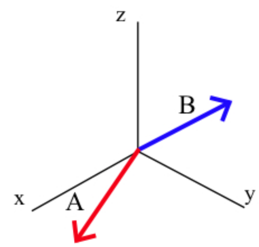

Fig. 12

In problems 5 – 12, vectors **U** and **V** are given. Calculate $\mathbf{W} = 2\mathbf{U} + \mathbf{V}$, $|\mathbf{U}|$, $|\mathbf{V}|$, and $|\mathbf{W}|$, and the directions of **U**, **V**, and **W**.

5. $\mathbf{U} = \langle 2, 3, 6 \rangle$, $\mathbf{V} = \langle 2, -9, 6 \rangle$
6. $\mathbf{U} = \langle 6, 3, 6 \rangle$, $\mathbf{V} = \langle 2, 4, 4 \rangle$
7. $\mathbf{U} = \langle 5, 2, 14 \rangle$, $\mathbf{V} = \langle 4, -7, 4 \rangle$
8. $\mathbf{U} = \langle 8, 4, 1 \rangle$, $\mathbf{V} = \langle 4, 4, -2 \rangle$
9. $\mathbf{U} = 9\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}$, $\mathbf{V} = 3\mathbf{i} + 6\mathbf{j} - 6\mathbf{k}$
10. $\mathbf{U} = 24\mathbf{i} + 2\mathbf{j} + 24\mathbf{k}$, $\mathbf{V} = 10\mathbf{i} - 25\mathbf{j} + 2\mathbf{k}$
11. $\mathbf{U} = 10\mathbf{i} + 11\mathbf{j} + 2\mathbf{k}$, $\mathbf{V} = 6\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$
12. $\mathbf{U} = 8\mathbf{i} + 1\mathbf{j} + 4\mathbf{k}$, $\mathbf{V} = 3\mathbf{i} + 12\mathbf{j} + 4\mathbf{k}$
13. $\mathbf{A} = \langle 3, 6, -2 \rangle$, $\mathbf{B} = \langle 5, 0, -4 \rangle$. Find **C** so $\mathbf{A} + \mathbf{B} + \mathbf{C} = \mathbf{0}$.
14. $\mathbf{A} = \langle 1, -3, 7 \rangle$, $\mathbf{B} = \langle -5, 2, -3 \rangle$. Find **C** so $\mathbf{A} + \mathbf{B} + \mathbf{C} = \mathbf{0}$.
15. $\mathbf{A} = \langle 9, \pi, -3 \rangle$, $\mathbf{B} = \langle e, 7, 4 \rangle$, and $\mathbf{C} = \langle -6, 2, 0 \rangle$. Find **D** so $\mathbf{A} + \mathbf{B} + \mathbf{C} + \mathbf{D} = \mathbf{0}$.
16. $\mathbf{A} = \langle -4, 3, 1 \rangle$, $\mathbf{B} = \langle 5, 5, -\pi \rangle$, and $\mathbf{C} = \langle 0, -2, 1 \rangle$. Find **D** so $\mathbf{A} + \mathbf{B} + \mathbf{C} + \mathbf{D} = \mathbf{0}$.
17. Which of the following vectors has the smallest magnitude and which has the largest:
    $\mathbf{A} = \langle 6, 9, 2, 3 \rangle$, $\mathbf{B} = \langle -8, 0, 6, 0 \rangle$, $\mathbf{C} = \langle -6, 3, 3, 6 \rangle$, and $\mathbf{D} = \langle 9, 1, 8, -5 \rangle$?
18. Which of the following vectors has the smallest magnitude and which has the largest:
    $\mathbf{A} = \langle 11, 2, -10, 1 \rangle$, $\mathbf{B} = \langle -5, 14, 2, 6 \rangle$, $\mathbf{C} = \langle 2, 8, 16, -5 \rangle$, and $\mathbf{D} = \langle 12, 3, 7, 3 \rangle$?
19. $\mathbf{A} = \langle 2, 4, 3 \rangle$. Sketch **A** and find the "shadows" of **A** on the coordinate planes (e.g., on the xy, xz, and yz planes).
20. $\mathbf{B} = \langle 4, 1, 2 \rangle$. Sketch **B** and find the "shadows" of **B** on the coordinate planes (e.g., on the xy, xz, and yz planes).
21. $\mathbf{C} = \langle 5, 2, 3 \rangle$. Sketch **C** and find the "shadows" of **C** on the coordinate planes.

22. $\mathbf{D} = \langle 3, 4, 0 \rangle$. Sketch $\mathbf{D}$ and find the "shadows" of $\mathbf{D}$ on the coordinate planes.

23. $\mathbf{A} = \langle 1, 0, 0 \rangle$. Sketch $\mathbf{A}$ and find three nonparallel vectors that are perpendicular to $\mathbf{A}$. How many vectors are perpendicular to $\mathbf{A}$?

24. $\mathbf{B} = \langle 0, 0, 1 \rangle$. Sketch $\mathbf{B}$ and find three nonparallel vectors that are perpendicular to $\mathbf{B}$. How many vectors are perpendicular to $\mathbf{B}$?

25. $\mathbf{C} = \langle 1, 2, 0 \rangle$. Sketch $\mathbf{C}$ and find two nonparallel vectors that are perpendicular to $\mathbf{C}$.

26. $\mathbf{C} = \langle 0, 2, 3 \rangle$. Sketch $\mathbf{C}$ and find two nonparallel vectors that are perpendicular to $\mathbf{C}$.

In problems 27 – 30, you are asked to sketch smooth curves that go through given points with given directions. Later, in Section 12.1, we will discuss how to find parametric equations for curves that satisfy conditions of this type.

27. Sketch a smooth curve that goes through the point (0,0,1) with direction $\langle 1, 0, 0 \rangle$ and then bends and goes through the point (0,1,0) with direction $\langle 0, 1, 0 \rangle$.

28. Sketch a smooth curve that goes through the point (0,0,1) with direction $\langle 0.8, 0, 0.6 \rangle$ and then bends and goes through the point (0,2,0) with direction $\langle 0, 0, 1 \rangle$.

29. Sketch a smooth curve that goes through the point (2,0,0) with direction $\langle 0, 0, 1 \rangle$ and then bends and goes through the point (0,1,2) with direction $\langle -1, 0, 0 \rangle$.

30. Sketch a smooth curve that goes through the point (4, 1, 2) with direction $\langle 0, 0, -1 \rangle$ and then bends and goes through the point (0,0,0) with direction $\langle 0, -1, 0 \rangle$.

31. Find a linear equation for $x(t)$ so $x(0) = 3$ and $x(1) = 7$, for $y(t)$ so $y(0) = 5$ and $y(1) = 4$, and for $z(t)$ so $z(0) = 1$ and $z(1) = 1$.

32. Find a linear equation for $x(t)$ so $x(0) = 1$ and $x(1) = 5$, for $y(t)$ so $y(0) = 2$ and $y(1) = 0$, and for $z(t)$ so $z(0) = 3$ and $z(1) = 5$.

33. Find a linear equation for $x(t)$ so $x(0) = 2$ and $x(1) = 5$, for $y(t)$ so $y(0) = 3$ and $y(1) = 3$, and for $z(t)$ so $z(0) = 6$ and $z(1) = 1$.

34. Find a linear equation for $x(t)$ so $x(0) = 0$ and $x(1) = -3$, for $y(t)$ so $y(0) = -2$ and $y(1) = 1$, and for $z(t)$ so $z(0) = 4$ and $z(1) = 1$.

Practice Answers

Practice 1: $\mathbf{C} = 5\mathbf{A} = 5\langle 5, -4, 1 \rangle = \langle 25, -20, 5 \rangle$.

$\mathbf{D} = 3\mathbf{A} + 4\mathbf{B} = 3\langle 5, -4, 1 \rangle + 4\langle 2, -3, 4 \rangle = \langle 15+8, -12-12, 3+16 \rangle = \langle 23, -24, 19 \rangle$.

$\mathbf{E} = 2\mathbf{B} - 3\mathbf{A} = 2\langle 2, -3, 4 \rangle - 3\langle 5, -4, 1 \rangle = \langle 4-15, -6+12, 8-3 \rangle = \langle -11, 6, 5 \rangle$.

Practice 2: $|\mathbf{A}| = \sqrt{2^2 + 3^2 + 6^2} = \sqrt{49} = 7$. $|\mathbf{B}| = \sqrt{2^2 + 1^2 + 2^2} = \sqrt{9} = 3$.

$\mathbf{C} = \mathbf{A} - 2\mathbf{B} = \langle 2, 3, 6 \rangle - 2\langle 2, 1, 2 \rangle = \langle -2, 1, 2 \rangle$ so $|\mathbf{C}| = \sqrt{9} = 3$.

Practice 3: $|\mathbf{A}| = \sqrt{3^2 + 2^2 + (-6)^2} = \sqrt{49} = 7$. Direction of $\mathbf{A}$ is $\frac{\mathbf{A}}{|\mathbf{A}|} = \frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$.

$|\mathbf{B}| = \sqrt{0^2 + 6^2 + (-8)^2} = \sqrt{100} = 10$. Direction of $\mathbf{B}$ is $\frac{\mathbf{B}}{|\mathbf{B}|} = \frac{6}{10}\mathbf{j} - \frac{8}{10}\mathbf{k}$.

$\mathbf{C} = \mathbf{A} + 3\mathbf{B} = 3\mathbf{i} + 20\mathbf{j} - 30\mathbf{k}$. $|\mathbf{C}| = \sqrt{1309} \approx 36.18$.

Direction of $\mathbf{C}$ is $\frac{\mathbf{C}}{|\mathbf{C}|} = \frac{3}{\sqrt{1309}}\mathbf{i} + \frac{20}{\sqrt{1309}}\mathbf{j} - \frac{30}{\sqrt{1309}}\mathbf{k} \approx 0.08\,\mathbf{i} + 0.55\,\mathbf{j} - 0.83\,\mathbf{k}$.

$\mathbf{D} = 2\mathbf{B} - 3\mathbf{A} = -9\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}$. $|\mathbf{D}| = \sqrt{121} = 11$.

Direction of $\mathbf{D}$ is $\frac{\mathbf{D}}{|\mathbf{D}|} = \frac{-9}{11}\mathbf{i} + \frac{6}{11}\mathbf{j} + \frac{2}{11}\mathbf{k} \approx -0.82\mathbf{i} + 0.55\mathbf{j} + 0.18\mathbf{k}$.

Practice 4: $|\mathbf{A}| = \sqrt{4^2 + 2^2 + (-5)^2 + 1^2 + 0} = \sqrt{46} \approx 6.8$.

$|\mathbf{B}| = \sqrt{6^2 + 0^2 + 2^2 + (-3)^2 + 6^2} = \sqrt{85} \approx 9.2$.

$\mathbf{C} = 2\mathbf{A} - 3\mathbf{B} = 2\langle 4, 2, -5, 1, 0 \rangle - 3\langle 6, 0, 2, -3, 6 \rangle = \langle -10, 4, -16, 11, -18 \rangle$.

$|\mathbf{C}| = \sqrt{(-10)^2 + 4^2 + (-16)^2 + 11^2 + (-18)^2} = \sqrt{817} \approx 28.6$.