11.1 Selected Answers

5. (b) $|\mathbf{U}| = \sqrt{17}$, $|\mathbf{V}| = \sqrt{13}$

direction of **U** is $\langle 1/\sqrt{17}\ ,\ 4/\sqrt{17}\rangle$, direction of **V** is $\langle 3/\sqrt{13}\ ,\ 2/\sqrt{13}\rangle$

(c) slope of **U** is 4/1 = 4, slope of **V** is 2/3

angle of **U** with x-axis is $\theta = \arctan(4) = 1.326\ (\approx 76^{\circ})$

angle of **V** with x-axis is $\theta = \arctan(2/3) = 0.588\ (\approx 33.7^{\circ})$

7. (b) $|\mathbf{U}| = \sqrt{29}$, $|\mathbf{V}| = \sqrt{58}$

direction of **U** is $\langle -2/\sqrt{29}\ ,\ 5/\sqrt{29}\rangle$, direction of **V** is $\langle 3/\sqrt{58}\ ,\ -7/\sqrt{58}\rangle$

(c) slope of **U** is -5/2 , slope of **V** is -7/3

angle of **U** with x-axis is $\theta = \arctan(-5/2) = -1.190\ (\approx -68.8^{\circ})$

angle of **V** with x-axis is $\theta = \arctan(-7/3) = -1.166\ (\approx -66.8^{\circ})$

9. (b) $|\mathbf{U}| = \sqrt{25} = 5$, $|\mathbf{V}| = \sqrt{25} = 5$

direction of **U** is $\langle -4/5\ ,\ -3/5\rangle$, direction of **V** is $\langle 3/5\ ,\ -4/5\rangle$

(c) slope of **U** is 3/4 , slope of **V** is -4/3

angle of **U** with x-axis is $\theta = \arctan(3/4) = 0.643\ (\approx 36.9^{\circ})$

angle of **V** with x-axis is $\theta = \arctan(-4/3) = -0.927\ (\approx -53.1^{\circ})$

13. $\mathbf{U} = \mathbf{A} + \mathbf{B} - \mathbf{C} = \langle -1,\ 3\rangle$, $\mathbf{V} = \mathbf{A} - \mathbf{B} + \mathbf{C} = \langle 3,\ 5\rangle$

15. $\mathbf{V} = 3\langle\ 0.6,\ 0.8\rangle\ =\ \langle\ 1.8,\ 2.4\rangle$

17. $\mathbf{V} = \langle\ 4.10,\ 2.87\rangle$

19. $\mathbf{V} = \left\langle\ 7/\sqrt{10},\ 21/\sqrt{10}\ \right\rangle$ or $\left\langle\ -7/\sqrt{10},\ -21/\sqrt{10}\ \right\rangle$

21. $\mathbf{V} = \left\langle\ 1/\sqrt{26},\ 5/\sqrt{26}\ \right\rangle$ or $\left\langle\ -1/\sqrt{26},\ -5/\sqrt{26}\ \right\rangle$

23. $\mathbf{V} = \langle\ 1,\ 0\ \rangle$ or $\langle\ -1,\ 0\ \rangle$

25. $\mathbf{V} = \langle\ 1,\ 0\ \rangle$ or $\langle\ -1,\ 0\ \rangle$

31. shadow on x-axis $1\mathbf{i} + 0\mathbf{j}$, on y-axis is $0\mathbf{i} + 4\mathbf{j}$

33. shadow on x-axis $5\mathbf{i} + 0\mathbf{j}$, on y-axis is $0\mathbf{i} - 2\mathbf{j}$

37. $\mathbf{C} = \langle\ -4,\ -6\ \rangle$

38. $\mathbf{C} = \langle\ 3,\ 7\ \rangle$

39. $\langle\ 25.36,\ 0\ \rangle$ and $\langle\ 0,\ 54.37\ \rangle$

40. $\langle\ 96.59,\ 0\ \rangle$ and $\langle\ 0,\ 25.88\ \rangle$

41. magnitude = 119.5 pounds, angle $\approx 39.2^{o}$

42. magnitude = 139 pounds, angle $\approx 21.4^{o}$

43. magnitude = 268.45 pounds, angle $\approx 23.9^{o}$

44. (a) path = $\langle\ 230,\ 40\rangle$ (b) aim 11.5^{o} south of east

46. (a) You are 19.6 miles from home (b) You should hike in the direction 34.6^{o} west of south

47. (a) The tension in each rope is 90.45 pounds.

(b) The tension in the short rope is 97.1 pounds, The tension in the long rope is 59.2 pounds.

11.2 Selected Answers

13. dist(A,B)= $\sqrt{5}$, dist(A,C)= $\sqrt{11}$, dist(A,d)= 5 , dist(B,C)= $\sqrt{6}$, dist(B,D)= $\sqrt{8}$, dist(C,D)= $\sqrt{18}$
No three of these points are colinear.

15. dist(A,B)= 6 , dist(A,C)= 3 , dist(A,d)= $\sqrt{6}$, dist(B,C)= 9 , dist(B,D)= $\sqrt{50}$, dist(C,D)= $\sqrt{11}$
The points A, B, and C are colinear.

17. corners: (1,2,3), (4,2,3), (4,4,3), (4,4,1), (1,4,1). volume = (3)(2)(2) = 12

19. corners: (1,4,0), (1,5,0), (4,4,0), (4,4,3), (4,5,3). volume = (3)(1)(3) = 9

25. $(x-4)^2+(y-3)^2+(z-5)^2=9$

26. $x^2+(y-3)^2+(z-6)^2=4$

27. $(x-5)^2+(y-1)^2+z^2=25$

29. center (3, -4, 1), radius = 4

30. center (-2, 0, 4), radius = 5

31. center (2, 3, 4), radius = 10

33. empty set (no intersection), a point, a line

34. empty set (no intersection), a line, a plane

35. empty set (no intersection), a point, a circle

36. empty set (no intersection), a point, a circle, a sphere

45. (a) $\frac{16\pi}{3}$ (for half sphere) (b) $\frac{8\pi}{3}$ (for quarter sphere) (c) $\frac{4\pi}{3}$ (for 1/8 sphere)

46. (a) 18π (b) 9π (c) $\frac{9\pi}{2}$

S1. (1, 2, 0) on xy-plane, (1, 0, 3) on xz-plane, (0, 2, 3) on yz-plane

S2. (4, 1, 0) on xy-plane, (4, 0, 2) on xz-plane, (0, 1, 2) on yz-plane

S3. (a, b, 0) on xy-plane, (a, 0, c) on xz-plane, (0, b, c) on yz-plane

S4. (4, 2, 0) to (1, 3, 0) on xy-plane, (4, 0, 1) to (1, 0, 3) on xz-plane, (0, 2, 1) to (0, 3, 3) on yz-plane

S7. xy-plane: line segment from (0,0,0) to (4,0,0). yz-plane: line segment from (0,0,0) to (0,0,3),
xz-plane: triangle with vertices (0,0,0), (4,0,3), and (4,0,2)

S8. xy-plane: triangle with vertices (1,2,0), (4,3,0), and (2,3,0)
xz-plane: triangle with vertices (1,0,3), (4,0,1), and (2,0,4)
yz-plane: triangle with vertices (0,2,3), (0,3,1), and (0,3,4)

S10. (a) 0 (b) 10

S11. (a) 0 (b) 12

11.3 Selected Answers

5. $\mathbf{W} = \langle 6, -3, 18 \rangle$, $|\mathbf{U}| = 7$, $|\mathbf{V}| = 11$, $|\mathbf{W}| = \sqrt{369} \approx 19.21$

{ dir. of $\mathbf{U}$} = $\mathbf{U}/|\mathbf{U}| = \langle 2/7, 3/7, 6/7 \rangle$, { dir. of $\mathbf{V}$} = $\mathbf{V}/|\mathbf{V}| = \langle 2/11, -9/11, 6/11 \rangle$

{ dir. of $\mathbf{W}$} = $\mathbf{W}/|\mathbf{W}| = \langle 6/\sqrt{369}, -3/\sqrt{369}, 18/\sqrt{369} \rangle \approx \langle 0.31, -0.16, 0.94 \rangle$

7. $\mathbf{W} = \langle 14, -3, 32 \rangle$, $|\mathbf{U}| = 15$, $|\mathbf{V}| = 9$, $|\mathbf{W}| = \sqrt{1229} \approx 35.06$

{ dir. of $\mathbf{U}$} = $\mathbf{U}/|\mathbf{U}| = \langle 5/15, 2/15, 14/15 \rangle$, { dir. of $\mathbf{V}$} = $\mathbf{V}/|\mathbf{V}| = \langle 4/9, -7/9, 4/9 \rangle$

{ dir. of $\mathbf{W}$} = $\mathbf{W}/|\mathbf{W}| = \langle 14/\sqrt{1229}, -3/\sqrt{1229}, 32/\sqrt{1229} \rangle \approx \langle 0.40, -0.09, 0.91 \rangle$

9. $\mathbf{W} = \langle 21, 18, -2 \rangle$, $|\mathbf{U}| = 11$, $|\mathbf{V}| = 9$, $|\mathbf{W}| = \sqrt{769} \approx 27.73$

{ dir. of $\mathbf{U}$} = $\mathbf{U}/|\mathbf{U}| = \langle 9/11, 6/11, 2/11 \rangle$, { dir. of $\mathbf{V}$} = $\mathbf{V}/|\mathbf{V}| = \langle 1/3, 2/3, -2/3 \rangle$

{ dir. of $\mathbf{W}$} = $\mathbf{W}/|\mathbf{W}| = \langle 21/\sqrt{769}, 18/\sqrt{769}, -2/\sqrt{769} \rangle \approx \langle 0.76, 0.65, -0.07 \rangle$

11. $\mathbf{W} = \langle 26, 25, -2 \rangle$, $|\mathbf{U}| = 15$, $|\mathbf{V}| = 9$, $|\mathbf{W}| = \sqrt{1305} \approx 36.12$

{ dir. of $\mathbf{U}$} = $\mathbf{U}/|\mathbf{U}| = \langle 10/15, 11/15, 2/15 \rangle$, { dir. of $\mathbf{V}$} = $\mathbf{V}/|\mathbf{V}| = \langle 2/3, 1/3, -2/3 \rangle$

{ dir. of $\mathbf{W}$} = $\mathbf{W}/|\mathbf{W}| = \langle 26/\sqrt{1305}, 25/\sqrt{1305}, -2/\sqrt{1305} \rangle \approx \langle 0.72, 0.69, -0.06 \rangle$

13. $\mathbf{C} = \langle -8, -6, 6 \rangle$ 14. $\mathbf{C} = \langle 4, 1, -4 \rangle$ 15. $\mathbf{C} = \langle -3 - e, -9 - \pi, -1 \rangle$

17. smallest magnitude is $\mathbf{C}$, largest magnitude is $\mathbf{D}$

18. smallest magnitude is $\mathbf{D}$, largest magnitude is $\mathbf{C}$

23. $\langle 0, 1, 0 \rangle, \langle 0, 0, 1 \rangle, \langle 0, 3, -4 \rangle$ are all perpendicular to $\mathbf{A}$ as is ever non-zero vector with x-coordinate equal to 0 (= vectors that lie in the yz-plane). There are an infinite number of nonparallel vectors that are perpendicular to $\mathbf{A}$.

24. $\langle 1, 0, 0 \rangle, \langle 0, 3, 0 \rangle, \langle 5, -4, 0 \rangle$ are all perpendicular to $\mathbf{B}$ as is ever non-zero vector with z-coordinate equal to 0 (= vectors that lie in the xy-plane). There are an infinite number of nonparallel vectors that are perpendicular to $\mathbf{B}$.

25. $\langle 0, 0, 2 \rangle, \langle -2, 1, 0 \rangle, \langle 2, -1, 7 \rangle$ are all perpendicular to $\mathbf{C}$**.** There are an infinite number of nonparallel vectors that are perpendicular to $\mathbf{C}$.

31. $x(t) = 3 + 4t$, $y(t) = 5 - t$, $z(t) = 1$

32. $x(t) = 1 + 4t$, $y(t) = 2 - 2t$, $z(t) = 3 + 2t$

33. $x(t) = 2 + 3t$, $y(t) = 3$, $z(t) = 6 - 5t$

11.4 Selected Answers

1. $\mathbf{A}\bullet\mathbf{B} = 3$, $\mathbf{B}\bullet\mathbf{A} = 3$, $\mathbf{A}\bullet\mathbf{A} = 14$, $\mathbf{A}\bullet(\mathbf{B}+\mathbf{A}) = 17$, and $(2\mathbf{A}+3\mathbf{B})\bullet(\mathbf{A}-2\mathbf{B}) = -101$

2. $\mathbf{A}\bullet\mathbf{B} = 2$, $\mathbf{B}\bullet\mathbf{A} = 2$, $\mathbf{A}\bullet\mathbf{A} = 41$, $\mathbf{A}\bullet(\mathbf{B}+\mathbf{A}) = 43$, and $(2\mathbf{A}+3\mathbf{B})\bullet(\mathbf{A}-2\mathbf{B}) = -94$

3. $\mathbf{U}\bullet\mathbf{V} = 2$, $\mathbf{U}\bullet\mathbf{U} = 41$, $\mathbf{U}\bullet\mathbf{i} = 6$, $\mathbf{U}\bullet\mathbf{j} = -1$, $\mathbf{U}\bullet\mathbf{k} = 2$, and $(\mathbf{V}+\mathbf{k})\bullet\mathbf{U} = 8$

4. $\mathbf{U}\bullet\mathbf{V} = 0$, $\mathbf{U}\bullet\mathbf{U} = 22$, $\mathbf{V}\bullet\mathbf{i} = 2$, $\mathbf{V}\bullet\mathbf{j} = 4$, $\mathbf{V}\bullet\mathbf{k} = -3$, and $(\mathbf{V}+\mathbf{k})\bullet\mathbf{U} = 2$

5. $\mathbf{S}\bullet\mathbf{T} = -3$, $\mathbf{T}\bullet\mathbf{U} = -4$, $\mathbf{T}\bullet\mathbf{T} = 35$, $(\mathbf{S}+\mathbf{T})\bullet(\mathbf{S}-\mathbf{T}) = -14$, and $(\mathbf{S}\bullet\mathbf{T})\mathbf{U} = \langle -3, -9, -6\rangle$

7. angle between **A** and **B** is 1.39 ($\approx 79.9^o$), angle between **A** and x-axis is 1.30 ($\approx 74.5^o$)
   angle between **A** and y-axis is 1.01 ($\approx 57.7^o$), angle between **A** and z-axis is 0.641 ($\approx 36.7^o$)

9. angle between **U** and **V** is 1.51 ($\approx 86.7^o$), angle between **U** and x-axis is 0.36 ($\approx 20.4^o$)
   angle between **U** and y-axis is 1.73 ($\approx 98.98^o$), angle between **U** and z-axis is 1.25 ($\approx 71.8^o$)

11. angle between **S** and **T** is 1.68 ($\approx 96.3^o$), angle between **S** and x-axis is 1.12 ($\approx 64.1^o$)
    angle between **S** and y-axis is 2.63 ($\approx 150.5^o$), angle between **S** and z-axis is 1.35 ($\approx 77.4^o$)

13. angle between **A** and **B** is 1.57 ($\approx 90^o$), angle between **A** and x-axis is 1.24 ($\approx 71.1^o$)
    angle between **A** and y-axis is 2.08 ($\approx 119.1^o$), angle between **A** and z-axis is 0.624 ($\approx 35.8^o$)

15. angle between **A** and **B** is 2.59 ($\approx 148.7^o$), angle between **A** and x-axis is 0.38 ($\approx 21.8^o$)
    angle between **A** and y-axis is 1.95 ($\approx 111.8^o$), angle between **A** and z-axis is 1.57 ($\approx 90^o$)

17. angle between **U** and **V** is 1.43 ($\approx 81.9^o$), angle between **U** and x-axis is 1.25 ($\approx 71.6^o$)
    angle between **U** and y-axis is 1.57 ($\approx 90^o$), angle between **U** and z-axis is 0.322 ($\approx 18.4^o$)

19. 0.124 ($\approx 7.13^o$) 20. 2.48 ($\approx 142.1^o$) 21. 0.785 ($\approx 45^o$)

22. 1.23 ($\approx 70.5^o$) 23. 0.897 ($\approx 51.4^o$) 24. 0 ($= 0^o$)

25. 0 ($= 0^o$) 26. 1.57 ($\approx 90^o$) 27. 0.32 ($\approx 18.4^o$)

29. $\mathbf{N} = \langle -2, -1, 0\rangle$ is one correct answer. 30. $\mathbf{N} = \langle 3, 0, 5\rangle$ is one correct answer.

31. $\mathbf{N} = \langle 3, -7\rangle$ is one correct answer. 32. $\mathbf{N} = \langle 3, 7\rangle$ is one correct answer.

33. $\mathbf{N} = \langle -1, 1, 1\rangle$ is one correct answer. 34. $\mathbf{N} = \langle -5, 0, 1\rangle$ is one correct answer.

36. $\mathbf{N} = \langle 0, 2, -3\rangle$ is one correct answer. 37. $\mathbf{N} = \langle 1, 1, 2\rangle$ is one correct answer.

38. $\mathbf{N} = \langle 1, 0, 6\rangle$ is one correct answer. 39. $\mathbf{N} = \langle 3, 2\rangle$ is one correct answer.

40. $\mathbf{N} = \langle -2, 3\rangle$ is one correct answer. 41. $\mathbf{N} = \langle 2, 3\rangle$ is one correct answer.

43. $\mathbf{N} = \langle 3,\ 2\rangle$ is one correct answer.

45. $\mathbf{N} = \langle 1,\ -4\rangle$ is one correct answer.

46. $\mathbf{N} = \langle 5,\ 1\rangle$ is one correct answer.

47. $\mathbf{N} = \langle 0,\ 3\rangle$ is one correct answer.

57. $\mathbf{Proj_B}\ \mathbf{A} = \langle 25/34, 0, -15/34\rangle$, $\mathbf{Proj_A}\ \mathbf{B} = \langle -1, 2, 0\rangle$

59. $\mathbf{Proj_B}\ \mathbf{A} = \langle 0,0,0\rangle$, $\mathbf{Proj_A}\ \mathbf{B} = \langle 0,0,0\rangle$. **A** and **B** are perpendicular.

61. $\mathbf{Proj_B}\ \mathbf{A} = \langle 0,0,0\rangle$, $\mathbf{Proj_A}\ \mathbf{B} = \langle 0,0,0\rangle$. **A** and **B** are perpendicular.

63. $\mathbf{Proj_B}\ \mathbf{A} = \langle 0, -2, 0\rangle$, $\mathbf{Proj_A}\ \mathbf{B} = \langle -1/7, 2/7, -3/7\rangle$

65. $|\ \mathbf{Proj_B}\ \mathbf{A}\ | = \left| \dfrac{\mathbf{A}\cdot\mathbf{B}}{|\mathbf{B}|} \right|$, $\mathbf{Proj_A}\ \mathbf{B} = \left| \dfrac{\mathbf{A}\cdot\mathbf{B}}{|\mathbf{A}|} \right| = \left| \dfrac{\mathbf{A}\cdot\mathbf{B}}{3|\mathbf{B}|} \right| = \dfrac{1}{3}\left| \dfrac{\mathbf{A}\cdot\mathbf{B}}{|\mathbf{B}|} \right|$. $|\ \mathbf{Proj_B}\ \mathbf{A}\ |$ is larger.

67. distance is $3/\sqrt{2} \approx 2.12$

68. distance is 3.88

69. distance is 3.06

70. distance is 0.95

71. distance is 1.40

73. distance is 15.4 feet

74. distance is 2.78 inches

75. 9.18 m

77. work = $\dfrac{5000}{\sqrt{141}} \approx 421$ foot-pounds

78. work = 842 foot-pounds

79. work = 480 foot-pounds

80. work = 240 foot-pounds

81. work = 1212.1 foot-pounds

82. A work = 2349 ft-lbs, B work = 2441 ft-lbs

83. $\mathbf{A} \bullet \mathbf{B} = 15$, angle $\approx 1.05\ (= 60^{\circ})$

84. $\mathbf{A} \bullet \mathbf{B} = 16$, angle $\approx 1.21\ (\approx 69.3^{\circ})$

85. angle $\approx 112^{\circ}$, "different"

86. angle $\approx 19.9^{\circ}$, "very alike"

87. angle $\approx 9.45^{\circ}$, "very alike"

88. angle = 90°, "different"

11.5 Selected Answers

1. 7 2. 7 3. 2x – 5y 4. 15 – ab

5. 1 6. –1 7. –6 8. –42

9. x + 7y – 5z 10. 4a + 10b – 6c 11. –77 12. $3x - 3x^2$

13. (a) $\langle -10, -5, 10 \rangle$ (b) 0 (c) 0 (d) 15

15. (a) $\langle -24, -8, 0 \rangle$ (b) 0 (c) 0 (d) $\sqrt{640}$

17. (a) $\langle 3, -11, -5 \rangle$ (b) 0 (c) 0 (d) ≈ 12.45

19. scalar 20. not defined 21. not defined 22. not defined

25. The angle between a vector **A** and itself is 0 so **AxA** = |**A**| |**A**| sin(0) = 0. Alternately, | **AxA** | = the area of the parallelogram determined by **A** and **A**, and that area is 0.

26. | **AxB** | = |**A**| |**B**| | sin(θ) | which is maximum when | sin(θ) | = 1, and | sin(θ) |=1 when $\theta = \pm\pi/2$.

31. (a) torque = **AxB** = $\langle 12\cos(-30^o),\ 12\sin(-30^o),\ 0 \rangle x \langle 0,\ 70,\ 0 \rangle$

$= \{840\cos(-30^o)\}\mathbf{k} \approx 727.46\mathbf{k}$ inch-pounds

(b) torque = **AxB** = $\langle 8\cos(-30^o),\ 8\sin(-30^o),\ 0 \rangle x \langle 20\cos(40^o),\ 20\sin(40^o),\ 0 \rangle$

$= \{160\cos(-30^o)\sin(40^o) - 160\sin(-30^o)\cos(40^o)\}\mathbf{k} \approx 150.35\mathbf{k}$ inch-pounds

33. Yes. {torque on **A** by **B**} + {torque on **A** by **C**} = **AxB** + **AxC** = **Ax**(**B**+**C**) = {torque on **A** by (**B**+**C**) }

34. parallelogram area = | **AxB** | = 18 , triangle area = 9

35. parallelogram area = | **AxB** | = 6 , triangle area = 3

37. triangle area = $\frac{\sqrt{180}}{2} \approx 6.71$ 38. triangle area = $\frac{\sqrt{(bc)^2 + (ac)^2 + (ab)^2}}{2}$

39. parallelpiped volume = | (**AxB**) • **C** | = 17 40. parallelpiped volume = | (**AxB**) • **C** | = 78

41. parallelpiped volume = | (**AxB**) • **C** | = | a |· | b |· | c | = | abc | cubic units

42. tetrahedron volume = $\frac{1}{6}$ | 36**k** | = 6 cubic units 43. tetrahedron volume = $\frac{15}{6}$ cubic units

44. tetrahedron volume = 8 cubic units 45. tetrahedron volume = $\frac{|\ abc\ |}{6}$ cubic units

47. $A_{xy} = 4$, $A_{xz} = 4$, $A_{yz} = 8$, and $A_{xyz} = \sqrt{96}$ 48. $A_{xy} = 4$, $A_{xz} = 6$, $A_{yz} = 12$, and $A_{xyz} = \sqrt{196} = 14$

11.6 Selected Answers

1. $x(t) = 2 + 3t$, $y(t) = -3 + 4t$, $z(t) = 1 + 2t$

3. $x(t) = -2 + 5t$, $y(t) = 1$, $z(t) = 4 - 3t$

5. $x(t) = 2 + t$, $y(t) = -1 + 5t$, $z(t) = 3 - 5t$

7. $x(t) = 3$, $y(t) = -2 + 6t$, $z(t) = 1 - 2t$

9. Lines intersect at the point $(2, -1, 3)$ when $t = 0$, $\theta = \arccos\left(\frac{9}{\sqrt{6}\sqrt{21}}\right) \approx \arccos(0.802) \approx 0.604\ (\approx 36.7^{\circ})$

11. $L(0) = (1, 5, -2) = K(-2)$. The lines intersect at the point $(1, 5, -2)$.
$\theta = \arccos\left(\frac{18}{5\sqrt{14}}\right) \approx \arccos(0.962) \approx 0.277\ (\approx 15.8^{\circ})$

13. $5(x-2) + (-2)(y-3) + 4(z-1) = 0$ or $5x - 2y + 4z = 8$

14. $3x + y - 5z = 22$

15. $0(x + 3) + 3(y - 5) + 0(z-6) = 0$ or $3y = 15$

16. $2x - 2y + z = 0$

17. $(-6)(x-1) + (-6)(y-2) + (-12)(z-3) = 0$ or $x + y + 2z = 9$

18. $y = 5$

19. $20x + 28y + 25z = 101$

20. $-x + 2y - z = 0$

21. $z = 7$

22. $x = 2$

23. $3x - 2y + 5z = 23$

24. $2x + 3y - z = 0$

25. $5x - 3y + 2z = 23$

26. $y = 7$

27. They intersect along the y-axis

29. Plane intersects the x-axis at $x=10$

31. They intersect at the point $(4, 2, 1)$

32. y-axis never intersects the plane $z = 3$

33. $x(t) = -26 + t$, $y(t) = -57 + 3t$, $z(t) = t$. $\mathbf{N_1} = \langle 4, -2, 2\rangle$, $\mathbf{N_2} = \langle 3, -2, 3\rangle$
$\theta = \arccos\left(\frac{22}{\sqrt{24}\sqrt{22}}\right) \approx \arccos(0.957) \approx 0.294\ (\approx 16.8^{\circ})$

34. $x(t) = 9 + 6t$, $y^*(t) = -9t$, $z(t) = t$. $\theta \approx 1.066\ (\approx 61.1^{\circ})$

35. $x(t) = 12 - \frac{22}{5}t$, $y(t) = 2 + \frac{1}{5}t$, $z(t) = t$. $\mathbf{N_1} = \langle 0,\ 5,\ -1\rangle$, $\mathbf{N_2} = \langle 1,\ 2,\ 4\rangle$
$\theta = \arccos\left(\frac{6}{\sqrt{26}\sqrt{21}}\right) \approx 1.311\ (\approx 75.1^{\circ})$

37. They intersect at the point $(-11/3, -1/3, -58/3)$. $\arccos\left(\frac{3}{\sqrt{30}\sqrt{21}}\right) \approx 1.451\ (\approx 83.1^{\circ})$ so the angle of intersection is $\theta = \pi/2 - 1.451 \approx 0.120\ \ (\approx 6.9^{\circ})$

38. They intersect at the point $(0, 6, -7)$. Angle of intersection is approximately $0.222\ \ (\approx 12.7^{\circ})$

39. They intersect at the point $(0, 8, 5)$. Angle of intersection is approximately $0.271\ \ (\approx 15.5^{\circ})$

41. Yes

42. Yes

43. $x(t) = -7t$, $y(t) = 4t$, $z(t) = 3 + 9t$

44. (a) $\approx 1.30\ \ (\approx 74.5^{\circ})$ (b) $\approx 1.01\ \ (\approx 57.7^{\circ})$ (c) $\approx 0.64\ \ (\approx 36.7^{\circ})$

45. $\theta = \arccos\left(\dfrac{c}{\sqrt{a^2+b^2+c^2}}\right)$ with the xy-plane. $\theta = \arccos\left(\dfrac{b}{\sqrt{a^2+b^2+c^2}}\right)$ with the xz-plane.

$\theta = \arccos\left(\dfrac{a}{\sqrt{a^2+b^2+c^2}}\right)$ with the yz-plane.

47. (a) $\theta = \arctan(481/378) \approx 0.905 \quad (\approx 51.8^o)$ (b) $\cos(\varphi) \approx 0.382$ so $\varphi \approx 1.179 \quad (\approx 84.8^o)$

(c) $\alpha = \arctan(481/534.6) \approx 0.733 \quad (\approx 42.0^o)$

51. distance $\approx$ 4.879 52. distance $\approx$ 2.145 53. distance $\approx$ 1.18 54. distance $\approx$ 6.164

55. distance $\approx$ 1.32 56. distance $\approx$ 1.35

57. (a) The objects "crash" at the point (15, 24, 13) when t=6. (b) Paths intersect fo {min. dist.} = 0

(c) No, the objects crash, and their paths intersect.

58. (a) The objects "crash" at the point (2, 3, 4) when t=3.

59. (a) The objects do not crash. They are never at the same point at the same time.

(b) The paths of the objects intersect: object A is at (0,1,5) when t=1 and B is at (0,1,5) when t=2.

(c) {minimum distance between objects} ≈ 0.85 , {min. distance between paths} = 0 since paths intersect.

61. (a) The objects do not crash. They are never at the same point at the same time.

(b) The paths of the objects do not intersect.

(c) {minimum distance between objects when t=2.2} ≈ 1.67 , {min. distance between paths} = 1

63. (a) Shortest distance between the airplane and the car is $\sqrt{58} \approx 7.62$.

(b) Shortest distance between paths is $13/\sqrt{5} \approx 5.81$.

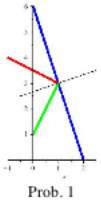
11.7 Selected Answers

1. $\mathbf{V} = \langle 2, -1\rangle$, $\mathbf{N} = \langle 3, 1\rangle$, $\text{Proj}_{\mathbf{N}}\mathbf{V} = \left\langle \frac{3}{2}, \frac{1}{2}\right\rangle$, $\mathbf{R} = \mathbf{V} - 2\cdot\text{Proj}_{\mathbf{N}}\mathbf{V} = \langle -1, -2\rangle$.

Point is (1, 3) and the line is $x(t) = 1 - t$, $y(t) = 3 - 2t$.

2. $\mathbf{V} = \langle -1, 1\rangle$, $\mathbf{N} = \langle 3, 1\rangle$, $\text{Proj}_{\mathbf{N}}\mathbf{V} = \left\langle -\frac{3}{5}, -\frac{1}{5}\right\rangle$, $\mathbf{R} = \left\langle \frac{1}{5}, \frac{7}{5}\right\rangle$.

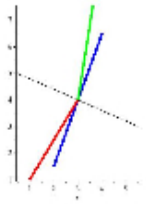
Point is (1, 3) and the line is $x(t) = 1 + \frac{1}{5}t$, $y(t) = 3 + \frac{7}{5}t$.



Prob. 1

3. $\mathbf{V} = \langle\, 2,\ 3\,\rangle$, $\mathbf{N} = \langle 5,\ -2\rangle$, $\text{Proj}_{\mathbf{N}}\mathbf{V} = \left\langle \frac{20}{29},\ -\frac{8}{29} \right\rangle$, $\mathbf{R} = \mathbf{V} - 2\cdot\text{Proj}_{\mathbf{N}}\mathbf{V} = \left\langle \frac{18}{29},\ \frac{103}{29} \right\rangle$.

Point is (3, 4) and the line is $x(t) = 3 + \frac{18}{29}t$, $y(t) = 4 + \frac{103}{29}t$.



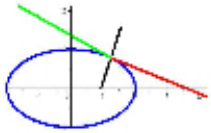
Prob. 3

4. $\mathbf{V} = \langle\, 0,\ -2\,\rangle$, $\mathbf{N} = \langle 5,\ -2\rangle$, $\text{Proj}_{\mathbf{N}}\mathbf{V} = \left\langle \frac{20}{29},\ -\frac{8}{29} \right\rangle$, $\mathbf{R} = \left\langle -\frac{40}{29},\ -\frac{42}{29} \right\rangle$.

Point is (3, 4) and the line is $x(t) = 3 - \frac{40}{29}t$, $y(t) = 4 - \frac{42}{29}t$.

5. $\mathbf{V} = \langle -3,\ 2\rangle$, $\mathbf{N} = \langle\, 1, 0\rangle$, $\text{Proj}_{\mathbf{N}}\mathbf{V} = \langle -3, 0\rangle$, $\mathbf{R} = \mathbf{V} - 2\cdot\text{Proj}_{\mathbf{N}}\mathbf{V} = \langle\, 3,\ 2\rangle$.

Point is (0, 3) and the line is $x(t) = 3t$, $y(t) = 3 + 2t$.



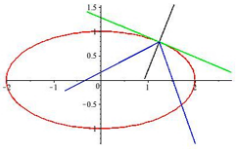
Prob. 7

6. $\mathbf{V} = \langle\, 3, -1\rangle$, $\mathbf{N} = \langle\, 0,\ 1\rangle$, $\text{Proj}_{\mathbf{N}}\mathbf{V} = \langle\, 0,\ -1\rangle$, $\mathbf{R} = \langle\, 3,\ 2\rangle$.

Point is (2, 0) and the line is $x(t) = 2 + 3t$, $y(t) = 2t$.

7. $\mathbf{V} = \langle -3,\ 1\rangle$, $\mathbf{N} = \langle -0.622, -1.567\rangle$, $\text{Proj}_{\mathbf{N}}\mathbf{V} = \langle -0.065,\ -0.164\rangle$,

$\mathbf{R} = \mathbf{V} - 2\cdot\text{Proj}_{\mathbf{N}}\mathbf{V} = \langle -2.869,\ 1.329\rangle$. Point is (1.243, 0.783) and the line is $x(t) = 1.243 - 2.869t$, $y(t) = 0.783 + 1.329t$.



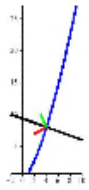
Prob. 8

8. $\mathbf{V} = \langle\, 2,\ 1\rangle$, $\mathbf{N} = \langle\, 0.622,\ 1.527\ \rangle$, $\text{Proj}_{\mathbf{N}}\mathbf{V} = \langle\, 0.615,\ 1.55\ \rangle$,

$\mathbf{R} = \langle\, 0.77,\ -2.1\ \rangle$. Point is (1.243, 0.783) and the line is $x(t) = 1.243 + 0.77t$, $y(t) = 0.783 - 2.1t$.

9. $\mathbf{V} = \langle\, 2,\ 1\rangle$, $\mathbf{N} = \langle -12,\ 4\rangle$, $\text{Proj}_{\mathbf{N}}\mathbf{V} = \langle\, 1.5,\ -0.5\rangle$,

$\mathbf{R} = \mathbf{V} - 2\cdot\text{Proj}_{\mathbf{N}}\mathbf{V} = \langle -1,\ 2\rangle$. Point is (4, 8) and the line is $x(t) = 4 - t$, $y(t) = 8 + 2t$.



Prob. 9

10. $\mathbf{V} = \langle -1,\ 1\rangle$, $\mathbf{N} = \langle -12,\ 4\rangle$, $\text{Proj}_{\mathbf{N}}\mathbf{V} = \langle\ -1.2, 0.4\rangle$, $\mathbf{R} = \langle\, 1.4,\ 0.2\rangle$.

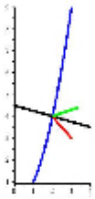
Point is (4, 8) and the line is $x(t) = 4 + 1.4t$, $y(t) = 8 + 0.2t$.

11. $\mathbf{V} = \langle -1,\ 1\rangle$, $\mathbf{N} = \langle -4,\ 1\rangle$, $\text{Proj}_{\mathbf{N}}\mathbf{V} = \langle\ -1.176, 0.294\rangle$, $\mathbf{R} = \mathbf{V} - 2\cdot\text{Proj}_{\mathbf{N}}\mathbf{V} = \langle\, 1.353,\ 0.412\rangle$.

Point is (2, 4) and the line is $x(t) = 2 + 1.353t$, $y(t) = 4 + 0.412t$.

12. $\mathbf{V} = \langle\, 2,\ 1\rangle$, $\mathbf{N} = \langle -4,\ 1\rangle$, $\text{Proj}_{\mathbf{N}}\mathbf{V} = \langle\, 1.647,\ -0.412\rangle$, $\mathbf{R} = \langle -1.294,\ 1.824\rangle$.

Point is (2, 4) and the line is $x(t) = 2 - 1.294t$, $y(t) = 4 + 1.824t$.

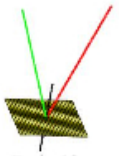


Prob. 11

13. $\mathbf{V} = \langle\, 2,\ 6,\ 3 \rangle$, $\mathbf{N} = \langle\, 1,\ 2,\ 3 \rangle$, $\text{Proj}_{\mathbf{N}}\mathbf{V} = \left\langle \frac{23}{14},\ -\frac{23}{7},\ \frac{69}{14} \right\rangle$,

$\mathbf{R} = \mathbf{V} - 2 \cdot \text{Proj}_{\mathbf{N}}\mathbf{V} = \left\langle -\frac{9}{7},\ -\frac{4}{7}, -\frac{48}{7} \right\rangle$. Point is (2, 4, 1) and the

line is $x(t) = 2 - \frac{9}{7}t$, $y(t) = 4 - \frac{4}{7}t$, $z(t) = 1 - \frac{48}{7}t$.


Prob. 13

14. $\mathbf{V} = \langle\, 4,\ 1,\ 3 \rangle$, $\mathbf{N} = \langle\, 3, -2,\ 4 \rangle$, $\text{Proj}_{\mathbf{N}}\mathbf{V} = \left\langle \frac{66}{29},\ -\frac{44}{29},\ \frac{88}{29} \right\rangle$,

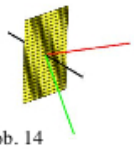
$\mathbf{R} = \left\langle -\frac{16}{29},\ \frac{117}{29},\ -\frac{89}{29} \right\rangle$.

Point is (1, 3, 2) and the line is $x(t) = 1 - \frac{16}{29}t$, $y(t) = 3 + \frac{117}{29}t$, $z(t) = 2 - \frac{89}{29}t$.

15. $\mathbf{V} = \langle\, 3,\ 2,\ 1 \rangle$, $\mathbf{N} = \langle\, 1,\ 0,\ 0 \rangle$, $\text{Proj}_{\mathbf{N}}\mathbf{V} = \langle\, 3,\ 0,\ 0 \rangle$,

$\mathbf{R} = \mathbf{V} - 2 \cdot \text{Proj}_{\mathbf{N}}\mathbf{V} = \langle\, -3,\ 2,\ 1 \rangle$.

Point is (0, 4, 2) and line is $x(t) = -3t$, $y(t) = 4 + 2t$, $z(t) = 2 + t$


Prob. 14

16. $\mathbf{V} = \langle\, 2, -3, -1 \rangle$, $\mathbf{N} = \langle\, 0,\ 0,\ 1 \rangle$, $\text{Proj}_{\mathbf{N}}\mathbf{V} = \langle\, 0,\ 0, -1 \rangle$, $\mathbf{R} = \langle\, 2, -3,\ 1 \rangle$.

Point is (2, –3, –1) and line is $x(t) = 2 + 2t$, $y(t) = -3 - 3t$, $z(t) = -1 + t$.