12.0 INTRODUCTION TO VECTOR–VALUED FUNCTIONS

So far, our excursion into 3–dimensional space has been rather static — we examined points, lines, planes, and vectors, but they did not move (except for points along lines). Those ideas and techniques are important for representing the positions of objects, but objects change position, and calculus is the study of "change." This chapter begins our extension of the ideas of calculus beyond two dimensions, and that extension is the focus for most of the rest of the book.

In earlier chapters, the functions we worked with generally had the form $y = f(x)$ — a single input value x resulted in a single output value y. As we move to higher dimensions, we expand the notion of function keeping the idea that each input produces a single output: f(input) = output. Now, however, we expand the types of objects that can be valid outputs, the range of f, and we expand the types of objects that can be valid inputs, the domain of f.

> Chapter 12 focuses on functions whose domains consist of numbers, but whose ranges consist of vectors: functions of the form f(number) = vector such as $\mathbf{r}(t) = \langle \cos(t), \sin(t), t \rangle$. These are called vector–valued functions, and typically their graphs are curves in space (Fig. 1).
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> Chapters 13 and 14 focus on functions whose domains consist of more than one variable and whose ranges consist of numbers: functions of the type f(number, number) = number or $f(x,y) = z$. These are called functions of several variables, and typically their graphs are surfaces in space (Fig. 2).

$r(t) = \langle \cos(t), \sin(t), t \rangle$

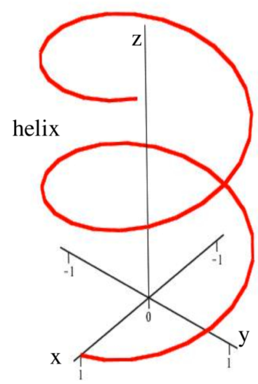

Fig. 1

In this chapter we examine vector–valued functions, and the discussion of vector–valued functions is similar in outline to our discussion of functions $y = f(x)$ in the early chapters of this book. First we discuss the meaning of vector–valued functions and their graphs. Then we look at the calculus ideas of limit, derivative and integral as they apply to vector–valued functions and examine some applications of these calculus ideas. As before, the meaning of these topics is inherently geometric, and you need to be able to work "visually" as well as "analytically."

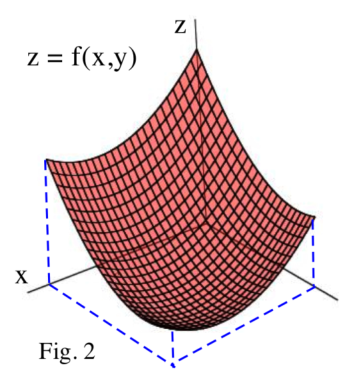

Fig. 2