12.4 CYLINDRICAL & SPHERICAL COORDINATE SYSTEMS IN 3D

Most of our work in two dimensions used the rectangular coordinate system, but we also examined the polar coordinate system (Fig. 1), and for some uses the polar coordinate system was more effective and efficient. A similar situation occurs in three dimensions. Mostly we use the 3–dimensional xyz–coordinate system, but there are two alternate systems, called cylindrical coordinates and spherical coordinates, that are sometimes better. In two dimensions, the rectangular and the polar coordinate systems each located a point by means of two numbers, but each system used those two numbers in different ways. In three dimensions, each of the coordinate systems locates a point using three numbers, and each system uses those three numbers in different ways. Fig. 2 illustrates how three numbers are used to locate the point P in each of the different systems, and the rest of this section examines the cylindrical and spherical coordinate systems.

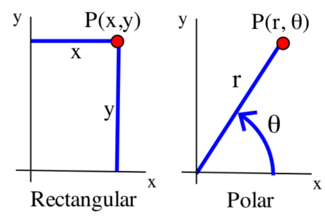


Fig. 1

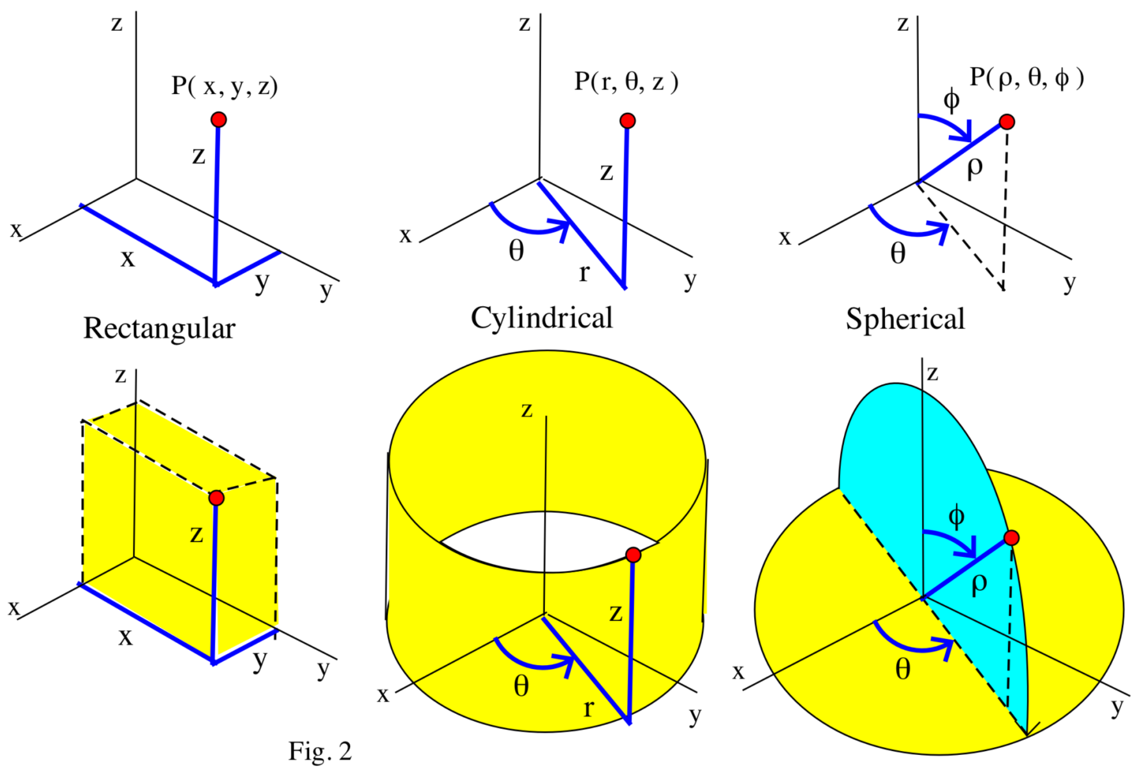


Fig. 2

CYLINDRICAL COORDINATES

Cylindrical coordinates are basically "polar coordinates with altitude." The cylindrical coordinates (r, θ, z) specify the point P that is z units above the point on the xy–plane whose polar coordinates are r and θ (Fig. 3).

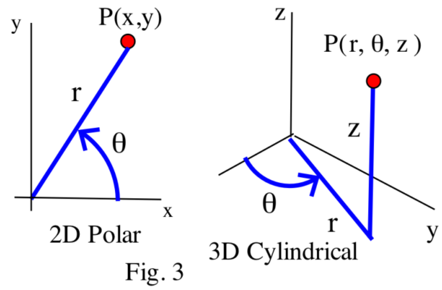


Fig. 3

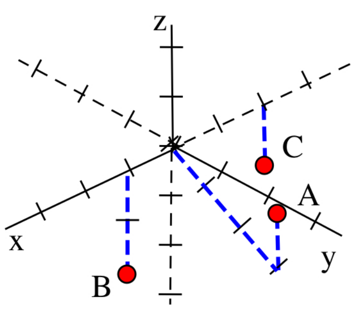


Fig. 4

Example 1: Plot the points given by the cylindrical coordinates $A(3, \pi/3, 1)$, $B(1, 0, -2)$, and $C(2, 180^{o}, -1)$.

Solution: The points are plotted in Fig. 4.

Practice 1: On Fig. 4, plot the points given by the cylindrical coordinates $P(3, \pi/6, -1)$, $Q(3, \pi/2, 2))$, and $R(0, \pi, 3)$.

We can start to develop an understanding of the effect of each variable in the ordered triple by holding two of the variables fixed and letting the other one vary. Fig. 5 shows the results in the rectangular coordinate system of fixing x and y $(x = 1, y = 2)$ and letting z vary: we get a vertical line parallel to the z–axis. Similarly, in the rectangular coordinate system, when we fix x and z $(x = 1, z = 3)$ and let y vary we get a line parallel to the y–axis. We can try the same process in the cylindrical coordinate system.

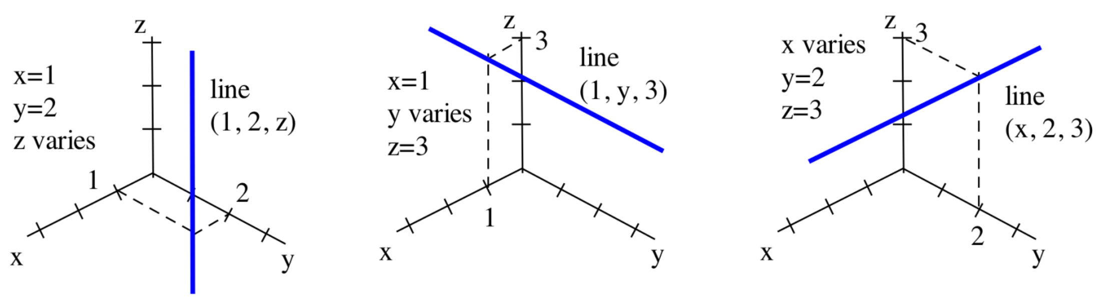


Fig. 5: Rectangular coordinates, two variables fixed

Fixing two of the variables and letting the other one vary:

In the cylindrical coordinate system, if we fix r and θ $(r = 2, \theta = \pi/3 = 60^{o})$ and let z vary (Fig. 6a) we get a line parallel to the z–axis.

If we fix r and z $(r = 2, z = 3)$ and let θ vary (Fig. 6b) we get a circle of radius 1 centered around the z–axis at a height of 3 units above the xy plane.

It we fix θ and z $(= \pi/3 = 60^{o}, z = 3)$ and let r vary (Fig. 6c) we get a line that is always 3 units above the xy plane.

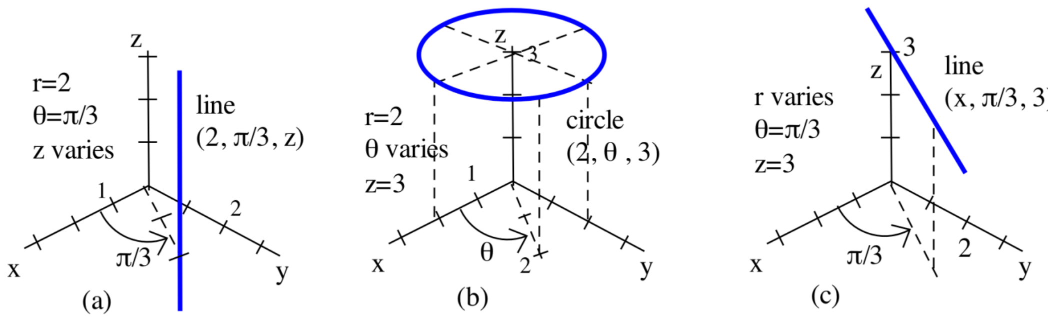


Fig. 6: Cylindrical coordinates, two variables fixed

Fixing the value of only one of the variables and letting the other two vary can also be informative.

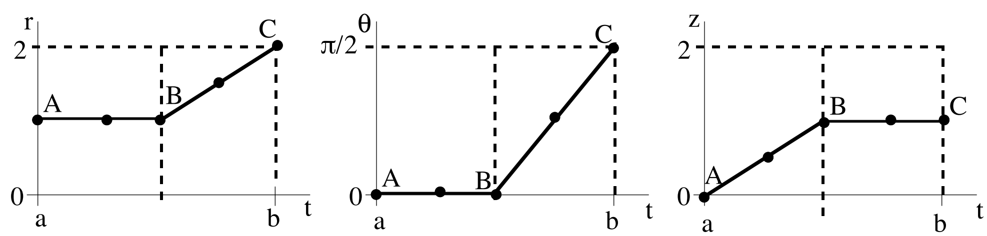


Fig. 8

If we fix r (r = 2) and let θ and z vary, the result (Fig. 7a) is a cylinder, the reason this is called the cylindrical coordinate system.

If we fix θ (θ = π/3) and let r and z vary, the result (Fig. 7b) is a plane.

If we fix z (z = 3) and let r and θ vary, the result (Fig. 7c) is a plane parallel to the xy plane.

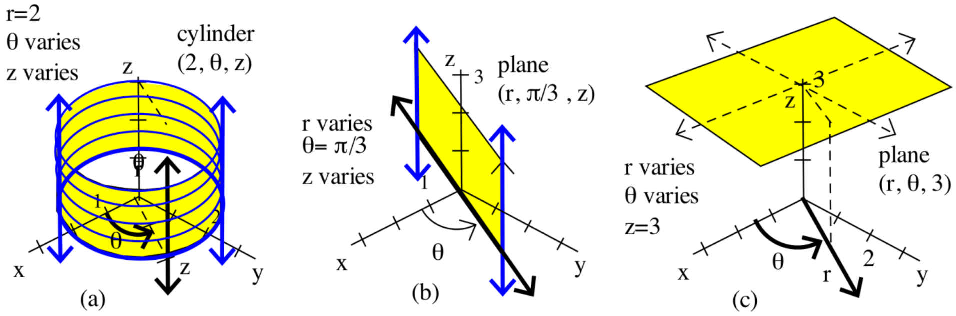


Fig. 7: Cylindrical coordinates, one variable fixed

Practice 2: An Alternate View: Cylindrical coordinates are "rectangular coordinates on a door." We began the discussion of cylindrical coordinates by describing them as "polar coordinates with altitude." A student said she found it easier to think of cylindrical coordinates as "rectangular coordinates on a door." By means of a labeled sketch show how this new description makes good sense.

Parametric Curves Using Cylindrical Coordinates

If we consider each of the rectangular coordinate variables x, y, and z to be a function of a parameter t, then $(x, y, z) = (x(t), y(t), z(t))$ describes the location of a point at time t. When the values of t are an interval of numbers, the graph of $(x(t), y(t), z(t))$ is a path or curve in three dimensional space. Similarly, if we consider each of the cylindrical coordinate variables r, θ, and z to be a function of a parameter t, then $(r, \theta, z) = (r(t), \theta(t), z(t))$ describes the location of a point, in cylindrical coordinates, at time t. When the values of t are an interval of numbers, the graph of $(r(t), \theta(t), z(t))$ is again a path or curve in three dimensional space.

Example 2: The graphs in Fig. 8 show values of r, θ, and z as functions of t. Use those values to graph the path $(r(t), \theta(t), z(t))$ for $a \le t \le b$.

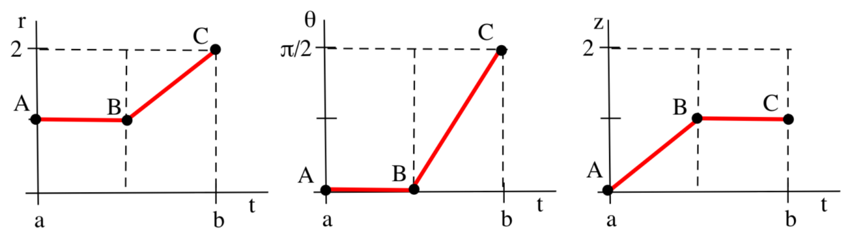


Fig. 8

Solution: Fig. 9 shows the result of plotting several points $(r(t), \theta(t), z(t))$ and connecting them.

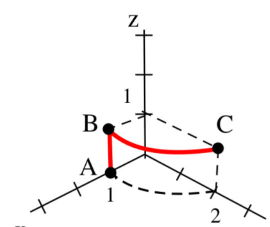


Fig. 9

Practice 3: The graphs in Fig. 10 show values of r, θ, and z as functions of t. Use those values to graph the path $(r(t), \theta(t), z(t))$ for $a \le t \le b$.

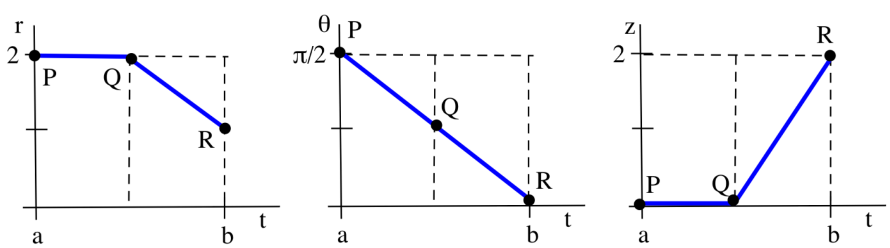


Fig. 10

Converting Cylindrical To And From Rectangular Coordinates

Typically we stay in one coordinate system for any particular use, but occasionally it may be necessary to convert information from one system to another, and it is rather straightforward to convert between rectangular and cylindrical coordinates. The important conversion information is contained in Fig. 11 and is summarized below.

Converting Between Cylindrical and Rectangular Systems

If $P(x, y, z)$ and $P(r, \theta, z)$ represent the same point in rectangular and cylindrical coordinates (Fig. 11),

then (cylindrical to rectangular) $x = r \cdot \cos(\theta)$ $y = r \cdot \sin(\theta)$

(rectangular to cylindrical) $r^2 = x^2 + y^2$ $\tan(\theta) = y/x$

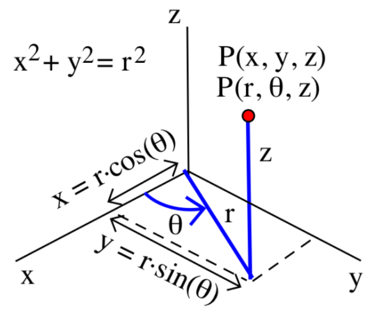


Fig. 11

Example 3: (a) Write the rectangular coordinate location $A(3, 4, 2)$ in the cylindrical coordinate system.

(b) Write the cylindrical coordinate location $B(2, \pi/6, 3)$ in the rectangular coordinate system.

Solution: (a) $r^2 = x^2 + y^2 = 3^2 + 4^2 = 5^2$. $\tan(\theta) = y/x = 4/3$ so $\theta = \arctan(4/3) \approx 0.927$ (53.1°) .

The cylindrical coordinates of A are approximately $(5, 0.927, 2)$.

(b) $x = r \cdot \cos(\pi/6) = 2\left(\sqrt{3}/2\right) \approx 1.73$, $y = r \cdot \sin(\pi/6) = 2(0.5) = 1$. The rectangular coordinates of B are approximately $(1.73, 1, 3)$.

Practice 4: (a) Write the rectangular coordinate location $P(-5, 12, 1)$ in the cylindrical coordinate system.

(b) Write the cylindrical coordinate location $Q(4, 3\pi/4, 3)$ in the rectangular coordinate system.

Example 4: (a) Rewrite the rectangular coordinate equation $x^2 + y^2 = 6z$ as an equation in cylindrical coordinates.

(b) Rewrite the cylindrical coordinate equation $r = 6 \cdot \cos(\theta)$ as an equation in rectangular coordinates.

Solution: (a) $x^2 + y^2 = r^2$ so $x^2 + y^2 = 6z$ becomes $r^2 = 6z$.

(b) Multiplying each side of $r = 6 \cdot \cos(\theta)$ by r we have $r^2 = 6r \cdot \cos(\theta)$ so $x^2 + y^2 = 6x$ or, moving the 6x to the left side and completing the square, $(x - 3)^2 + y^2 = 9$. The graph of $(x - 3)^2 + y^2 = 9$ is a circular cylinder generated by moving the circle with radius 3 and center $(3, 0, 0)$ parallel to the z–axis.

SPHERICAL COORDINATES

A point P with spherical coordinates (ρ, θ, φ) (the Greek letters ρ = rho and φ = phi are pronounced as "row" and "fee" or "fie") is located in three dimensions as shown in Fig. 12: ρ is the distance of P from the origin, and φ is the angle the segment from the origin to P makes with the positive z–axis.

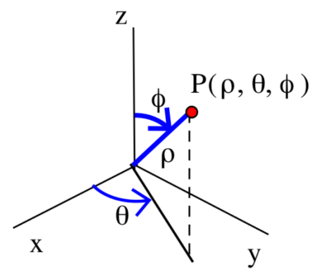


Fig. 12

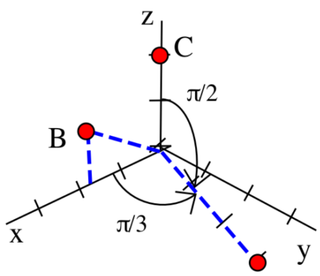


Fig. 13

Example 5: Plot the points given by the spherical coordinates $A(3, \pi/3, \pi/2)$, $B(2, 0, \pi/3)$, and $C(2, 180^{o}, 0)$.

Solution: The points are plotted in Fig. 13.

Practice 5: On Fig. 13, plot the points given by the spherical coordinates $P(3, 3\pi/4, \pi/6)$, $Q(3, 0, \pi/4)$, and $R(2, \pi/6, 0)$.

Fixing two of the variables and letting the other one vary:

In the spherical coordinate system, if we fix ρ and θ $(\rho = 2, \theta = \pi/3)$ and let φ vary (Fig. 14a) we get a circle with radius 2 and center at the origin.

If we fix ρ and φ $(\rho = 2, \varphi = \pi/4)$ and let θ vary (Fig. 14b) we get a circle with radius $\sqrt{2}$ and center at $(0, 0, \sqrt{2})$.

If we fix θ and φ $(\theta = \pi/3, \varphi = \pi/4)$ and let ρ vary (Fig. 14c) we get a straight line through the origin.

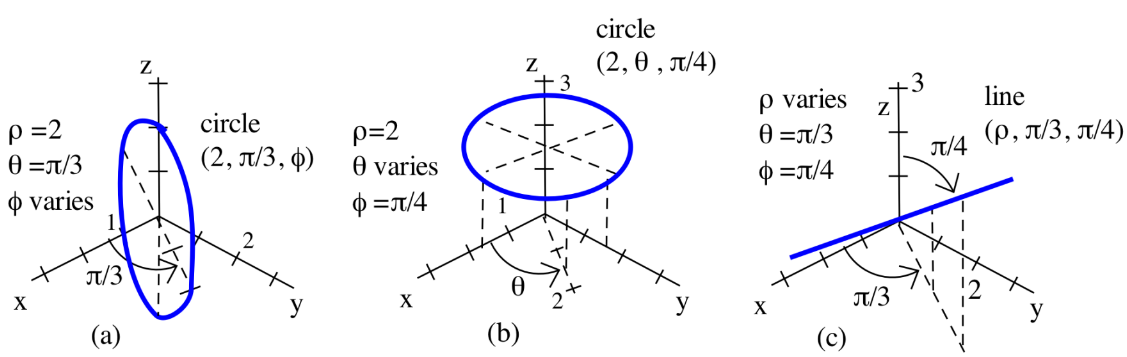


Fig. 14: Spherical coordinates, two variables fixed

Fixing one of the variables and letting the other two vary:

If we fix ρ ($\rho = 2$) and let θ and φ vary (Fig. 15a), we get a sphere, the reason this is called the spherical coordinate system, with radius 2 and center at the origin.

If we fix θ ($\theta = \pi/3$) and let ρ and φ vary (Fig. 15b), we get a plane.

If we fix φ ($\varphi = \pi/4$) and let ρ and θ vary (Fig. 15c), we get a cone around the z–axis.

It is possible to sketch the path of a point when the variables ρ, θ and φ are functions of a parameter t, but it is difficult to do "by hand" and we omit it. The Appendix at the end of this section illustrates some commands in the language Maple to create these graphs.

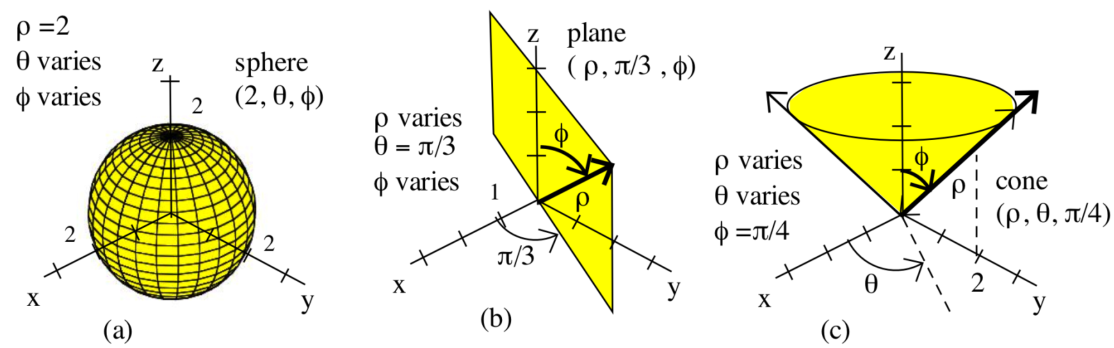


Fig. 15: Spherical coordinates, one variable fixed

Converting Spherical To And From Rectangular Coordinates

The conversions between rectangular and spherical coordinates are mostly a matter of applied trigonometry, and the essential conversion information is contained in Fig. 16.

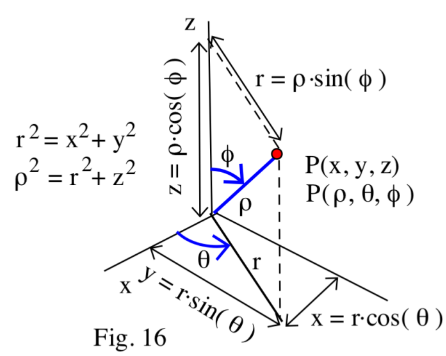


Fig. 16

The conversion from rectangular coordinates (x, y, z) to spherical coordinates (ρ, θ, φ) is straightforward:

ρ is the distance of the point (x, y, z) from the origin so

$$\rho^2 = x^2 + y^2 + z^2 ,$$

$$\tan(\theta) = \frac{y}{x} , \text{ and}$$

$$\cos(\varphi) = \frac{z}{\sqrt{x^2+y^2+z^2}} = \frac{z}{\rho} .$$

For the spherical to rectangular conversion it helps to calculate the value of $r = \rho \cdot \sin(\varphi)$ in Fig. 16. Then

$$x = r \cdot \cos(\theta) = \rho \cdot \sin(\varphi) \cdot \cos(\theta), \quad y = r \cdot \sin(\theta) = \rho \cdot \sin(\varphi) \cdot \sin(\theta), \text{ and } z = \rho \cdot \cos(\varphi) .$$

Converting Between Spherical and Rectangular Systems

If $P(x, y, z)$ and $P(\rho, \theta, \varphi)$ represent the same point in rectangular and spherical coordinates (Fig. 16),

then (spherical to rectangular) $x = \rho \cdot \sin(\varphi) \cdot \cos(\theta)$ $\quad y = \rho \cdot \sin(\varphi) \cdot \sin(\theta)$ $\quad z = \rho \cdot \cos(\varphi)$

(rectangular to spherical) $\rho^2 = x^2 + y^2 + z^2$ $\quad \tan(\theta) = \frac{y}{x}$ $\quad \cos(\varphi) = \frac{z}{\sqrt{x^2+y^2+z^2}} = \frac{z}{\rho}$

Example 6: (a) Write the rectangular coordinate location $A(3, 6, 2)$ in the spherical coordinate system.

(b) Write the spherical coordinate location $B(2, \pi/6, \pi/4)$ in the rectangular coordinate system.

Solution: (a) $\rho^2 = x^2 + y^2 + z^2 = 3^2 + 6^2 + 2^2 = 7^2$. $\tan(\theta) = 6/3 = 2$ so $\theta = \arctan(2) \approx 1.107$ (63.4°). $\cos(\varphi) = z/\rho = 2/7$ so $\varphi = \arccos(2/7) \approx 1.281$ (73.4°). The spherical coordinates of A are approximately $(7, 1.107, 1.281)$.

(b) $x = 2 \cdot \sin(\pi/4) \cdot \cos(\pi/6) = 2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{6}}{2}$

$y = 2 \cdot \sin(\pi/4) \cdot \sin(\pi/6) = 2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{2}}{2} \cdot$ $\quad z = 2\cos(\pi/4) = 2\left(\frac{\sqrt{2}}{2}\right) = \sqrt{2}$

The rectangular coordinates of B are approximately $(\sqrt{6}/2, \sqrt{2}/2, \sqrt{2}) \approx (1.225, 0.707, 1.414)$.

Practice 6: (a) Write the rectangular coordinate location $P(2, 9, 6)$ in the spherical coordinate system.

(b) Write the spherical coordinate location $Q(4, 3\pi/4, \pi/2)$ in the rectangular coordinate system.

Example 7: (a) Rewrite the rectangular coordinate equation $x^2 + y^2 = 6z$ as an equation in spherical coordinates.

(b) Rewrite the spherical coordinate equation $\rho = 6 \cdot \cos(\varphi)$ as an equation in rectangular coordinates.

Solution: (a) $x^2 + y^2 = 6z$ becomes $\rho^2 \cdot \sin^2(\varphi) \cdot \cos^2(\theta) + \rho^2 \cdot \sin^2(\varphi) \cdot \sin^2(\theta) = 6\rho \cdot \cos(\varphi)$ or $\rho^2 \cdot \sin^2(\varphi) = 6\rho \cdot \cos(\varphi)$.

(b) Multiplying each side of $\rho = 6 \cdot \cos(\varphi)$ by ρ we have $\rho^2 = 6\rho \cdot \cos(\theta)$ so $x^2 + y^2 + z^2 = 6z$ or, moving the 6z to the left side and completing the square, $x^2 + y^2 + (z-3)^2 = 9$. The graph of $x^2 + y^2 + (z-3)^2 = 9$ is a sphere with radius 3 and center at $(0, 0, 3)$.

PROBLEMS

Cylindrical coordinates

In problems 1 – 4, plot the points whose cylindrical coordinates are given.

1. $A(3, \pi/2, -2)$, $B(2, \pi/6, 3)$, $C(0, 30^\circ, 3)$
2. $D(3, 3\pi/2, 2)$, $E(4, 0, -2)$, $F(2, \pi, 1)$
3. $P(1, 45^\circ, -2)$, $Q(3, 3\pi/2, 2)$, $R(3, \pi/3, 0)$
4. $S(1, 2\pi/3, 2)$, $T(0, 30^\circ, -2)$, $U(2, \pi/4, 3)$

In problems 5 – 10, values are given for two of the r, θ, z variables. Plot the graph as the third variable takes all possible values.

5. $r = 3, \theta = 0$ 6. $r = 1, \theta = \pi/6$ 7. $r = 1, z = 3$

8. $r = 3, z = -2$ 9. $\theta = \pi/4, z = 2$ 10. $\theta = \pi, z = 3$

In problems 11 – 16, a value is given for one of the r, θ, z variables. Plot the graph as the other two variables take all possible values.

11. $r = 1$ 12. $r = 3$ 13. $\theta = \pi/2$

14. $\theta = \pi/6$ 15. $z = 2$ 16. $z = -1$

In Problems 17 – 20, separate parametric graphs of the r, θ, z variables are given. Using the information in these graphs, sketch the graph of (r, θ, z) in the cylindrical coordinate system.

17. See Fig. 17. 18. See Fig. 18. 19. See Fig. 19. 20. See Fig. 20.

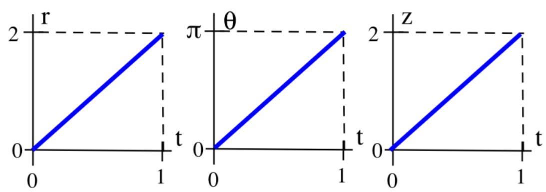


Fig. 17

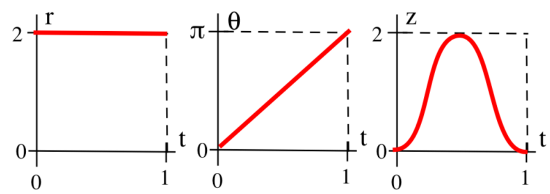


Fig. 18

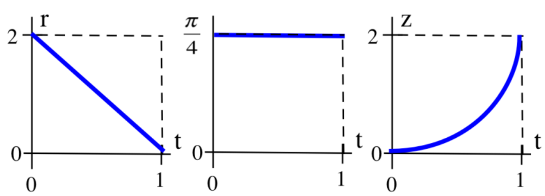


Fig. 19

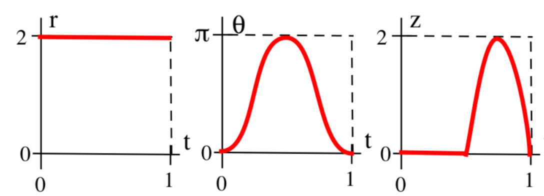


Fig. 20

In problems 21 – 24 sketch the set of points that satisfy the given conditions.

21. $1 \le r \le 3,\ 0 \le \theta \le 90^o$, and $z = 0$ 22. $0 \le r \le 3,\ \theta = 90^o$, and $0 \le z \le 2$

23. $r = 2,\ 0 \le \theta \le 30^o$, and $0 \le z \le 2$ 24. $0 \le r \le 3,\ 0 \le \theta \le 90^o$, and $0 \le z \le 1$

In problems 25 – 28, convert the cylindrical coordinates of the given point to rectangular coordinates.

25. $(5, \pi/6, 3)$ 26. $(4, \pi/3, 5)$ 27. $(3, 35^{o}, -2)$ 28. $(6, 75^{o}, -4)$

In problems 25 – 28, convert the rectangular coordinates of the given point to cylindrical coordinates.

29. $(1, 2, 3)$ 30. $(5, 2, -3)$ 31. $(-4, 3, -1)$ 32. $(7, -5, 3)$

In problems 33 – 36, convert the cylindrical coordinate equations to equations in rectangular coordinates.

33. (a) $r^2 = 4r \cdot \sin(\theta) - 1$ (b) $r = 7$ 34. (a) $r^2 = 8r \cdot \cos(\theta) + 3$ (b) $r = 1$

35. (a) $r = 5 \cdot \cos(\theta)$ (b) $z = r^2$ 36. (a) $r = 5 \cdot \sec(\theta)$ (b) $z = r^2 \sin(\theta) \cos(\theta)$

In problems 37 – 40, convert the rectangular coordinate equations to equations in cylindrical coordinates.

37. (a) $z = x^2 + y^2 - 3x + 2y$ (b) $x = 3$

38. (a) $z = 3x^2 + 3y^2$ (b) $y = 2$

39. (a) $z = x^2 + 5y^2$ (b) $x + y + z = 5$

40. (a) $y = x^2$ (b) $x + 5y = z$

Spherical coordinates

In problems 41 – 44, plot the points whose spherical coordinates are given.

41. $A(3, \pi/2, \pi/4)$, $B(2, 0, \pi/6)$, $C(1, \pi, 90^o)$

42. $D(3, \pi/6, 0)$, $E(3, \pi/2, \pi/6)$, $F(2, 60^o, 20^o)$

43. $P(2, 2\pi/3, 2\pi/3)$, $Q(3, 1, 1)$, $R(0, 71^o, 7\pi/13)$

44. $S(3, \pi/3, \pi)$, $T(0, \pi/3, \pi/7)$, $U(2, 2, 1)$

In problems 45 – 50, values are given for two of the ρ, θ, φ variables. Plot the graph as the third variable takes all possible values.

45. $\rho = 3, \theta = 0$

46. $\rho = 1, \theta = \pi/6$

47. $\rho = 1, \varphi = \pi/4$

48. $\rho = 3, \varphi = \pi/2$

49. $\theta = \pi/4, \varphi = \pi/2$

50. $\theta = \pi, \varphi = \pi/6$

In problems 51 – 56, a value is given for one of the ρ, θ, φ variables. Plot the graph as the other two variables take all possible values.

51. $\rho = 1$

52. $\rho = 3$

53. $\theta = \pi/2$

54. $\theta = \pi/6$

55. $\varphi = 0$

56. $\varphi = \pi/2$

In problems 57 – 24 sketch the set of points that satisfy the given conditions.

57. $1 \le \rho \le 3$, $\theta = 90^o$, and $10^o \le \varphi \le 40^o$

58. $\rho = 3$, $0 \le \theta \le 90^o$, and $10^o \le \varphi \le 40^o$

59. $\rho = 3$, $0 \le \theta \le 30^o$, and $0 \le \varphi \le 70^o$

60. $0 \le \rho \le 3$, $0 \le \theta \le 30^o$, and $60^o \le \varphi \le 90^o$

In problems 61 – 64, convert the spherical coordinates of the given point to rectangular coordinates.

61. $(5, \pi/2, \pi/3)$

62. $(3, \pi/3, \pi/6)$

63. $(4, 45^o, 30^o)$

64. $(7, 90^o, 45^o)$

In problems 65 – 68, convert the rectangular coordinates of the given point to spherical coordinates.

65. $(1, 2, 3)$

66. $(5, 2, 7)$

67. $(-5, 3, 2)$

68. $(3, -4, -2)$

In problems 69 – 72, convert the spherical coordinate equations to equations in rectangular coordinates.

69. (a) $\rho = 5$ (b) $\theta = \pi/2$

70. (a) $\rho = 3$ (b) $\varphi = \pi/2$

71. (a) $\rho = 5\cdot\sin(\varphi)\cdot\cos(\theta)$ (b) $\rho = 3\cdot\sec(\varphi)$

72. $\rho\cdot\sin(\varphi) = 5\cos(\theta)$

In problems 73 – 76, convert the rectangular coordinate equation to an equation in spherical coordinates.

73. (a) $x^2 + y^2 + z^2 = 9$ (b) $x + z = 5$

74. (a) $x^2 + y^2 = 9$ (b) $y + z = 2$

75. (a) $z = 2x^2 + 2y^2$ (b) $z^2 = 25 - x^2$

76. (a) $z = 3$ (b) $z^2 = 25 - x^2 - y^2$

PRACTICE ANSWERS

Practice 1: The points $P(3, \pi/6, -1)$, $Q(3, \pi/2, 2))$, and $R(0, \pi, 3)$ are shown on Fig. 21.

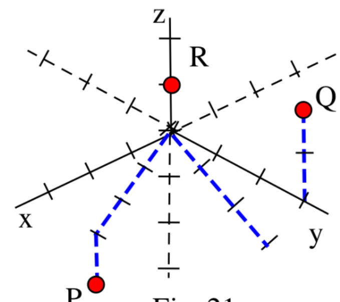


Fig. 21

Practice 2: Fig. 22 illustrates cylindrical coordinates as "rectangular coordinates on a door" by treating r and z as the rectangular coordinates (r along the bottom edge of the door and z as the amount above the bottom edge) and θ as the angular opening of the door.

Sometimes this is an excellent way to think of cylindrical coordinates.

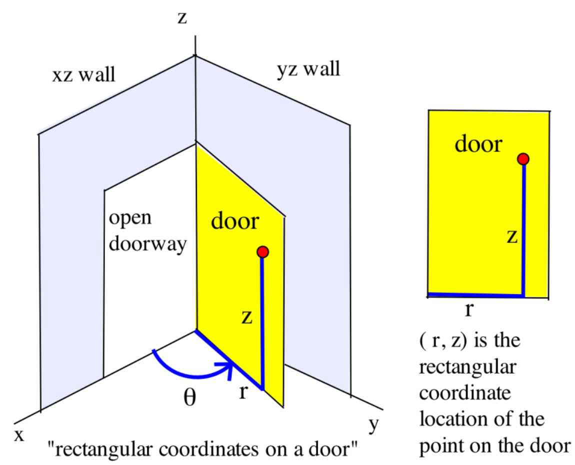


Fig. 22

Practice 3: The (approximate) graph of the path is shown in Fig. 23.

Practice 4: (a) $(x,y,z) = (-5,12,1)$:

$r^2 = x^2 + y^2 = (-5)^2+12^2 = 13^2$. $\tan(\theta) = 12/(-5)$

so $\theta = \arctan(-12/5) \approx -1.176$ (-67.4^o)

$\theta = \arctan(-12/5) \approx -1.176$ (-67.4^o) but the point $(-5,12)$ is in the second quadrant so the angle with the positive x–axis is $-1.176 + \pi \approx 1.966$ (112.6^o). $z = 1$.

The cylindrical coordinates of P are approximately $(13, 1.966, 1)$.

(b) $x = r\cdot\cos(3\pi/4) = 4(-\frac{\sqrt{2}}{2}) = -2\sqrt{2}$. $y = r\cdot\sin(3\pi/4) = 4(\frac{\sqrt{2}}{2}) = 2\sqrt{2}$.

The rectangular coordinates of Q are approximately $(-2\sqrt{2}, 2\sqrt{2}, 3)$.

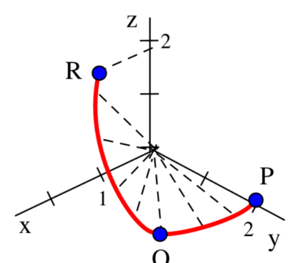


Fig. 23

Practice 5: The points $P(3, 3\pi/4, \pi/6)$, $Q(3, 0, \pi/4)$, and $R(2, \pi/6, 0)$ are shown on Fig. 24.

Practice 6: (a) $(x,y,z) = (2,9,6)$. $\rho^2 = x^2 + y^2 + z^2 = 2^2 + 9^2 + 6^2 = 11^2$.

$\tan(\theta) = 9/2 = 4.5$ so $\theta = \arctan(4.5) \approx 1.352$ (77.5^o).

$\cos(\varphi) = z/\rho = 6/11$ so $\varphi = \arccos(6/11) \approx 0.994$ (56.9^o).

The spherical coordinates of A are approximately $(11, 1.352, 0.994)$.

(b) $(\rho,\theta,\varphi) = (4, 3\pi/4, \pi/2)$. $x = 4\cdot\sin(\pi/2)\cdot\cos(3\pi/4) = 4(1)(-\frac{\sqrt{2}}{2}) = -2\sqrt{2}$.

$y = 4\cdot\sin(\pi/2)\cdot\sin(3\pi/4) = 4(1)(\frac{\sqrt{2}}{2}) = 2\sqrt{2}$. $z = 4\cdot\cos(3\pi/4) = 4(0) = 0$.

The rectangular coordinates of B are approximately

$(-2\sqrt{2}, 2\sqrt{2}, 0) \approx (-1.414, 1.414, 0)$.

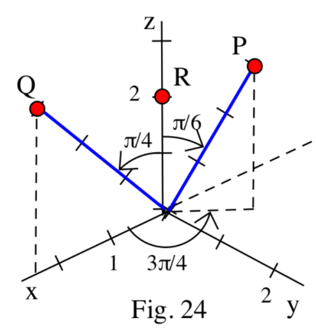


Fig. 24

Appendix: MAPLE Commands for Graphs in Cylindrical & Spherical Coordinates

The computer language Maple has the ability to plot points whose locations are specified in cylindrical or spherical coordinates as well as paths and surfaces in cylindrical or spherical coordinates.

The following commands illustrate how to begin using Maple to graph paths and surfaces in cylindrical and spherical coordinates.

Begin your session with the following command to load the special commands we need:

with(plots); then press the *enter* key.

Now try the following, and some of your own too.

Cylindrical coordinate paths given parametrically:

cylinderplot([3, theta, cos(theta)], theta=0..2*Pi, z = –1..1, axes = NORMAL);

cylinderplot([3, theta, cos(theta)], theta=0..2*Pi, z = –1..1, grid = [30,30], axes = NORMAL);

Cylindrical coordinate surfaces:

cylinderplot(1, theta = 0..2*Pi, z = –1..1, axes = NORMAL); a cylinder

cylinderplot((z+3*cos(2*theta), theta=0..Pi, z=0..3, axes = NORMAL); strange, but nice

Spherical coordinate surfaces:

sphereplot(1, theta=0..2*Pi, phi=0..Pi, grid = [30,30], axes = NORMAL); a sphere

sphereplot([t, 0.3, r], t=4..5, r=0..Pi/2, axes = NORMAL);

sphereplot([r=0..5, t=0..2*Pi, Pi/4], axes = NORMAL);

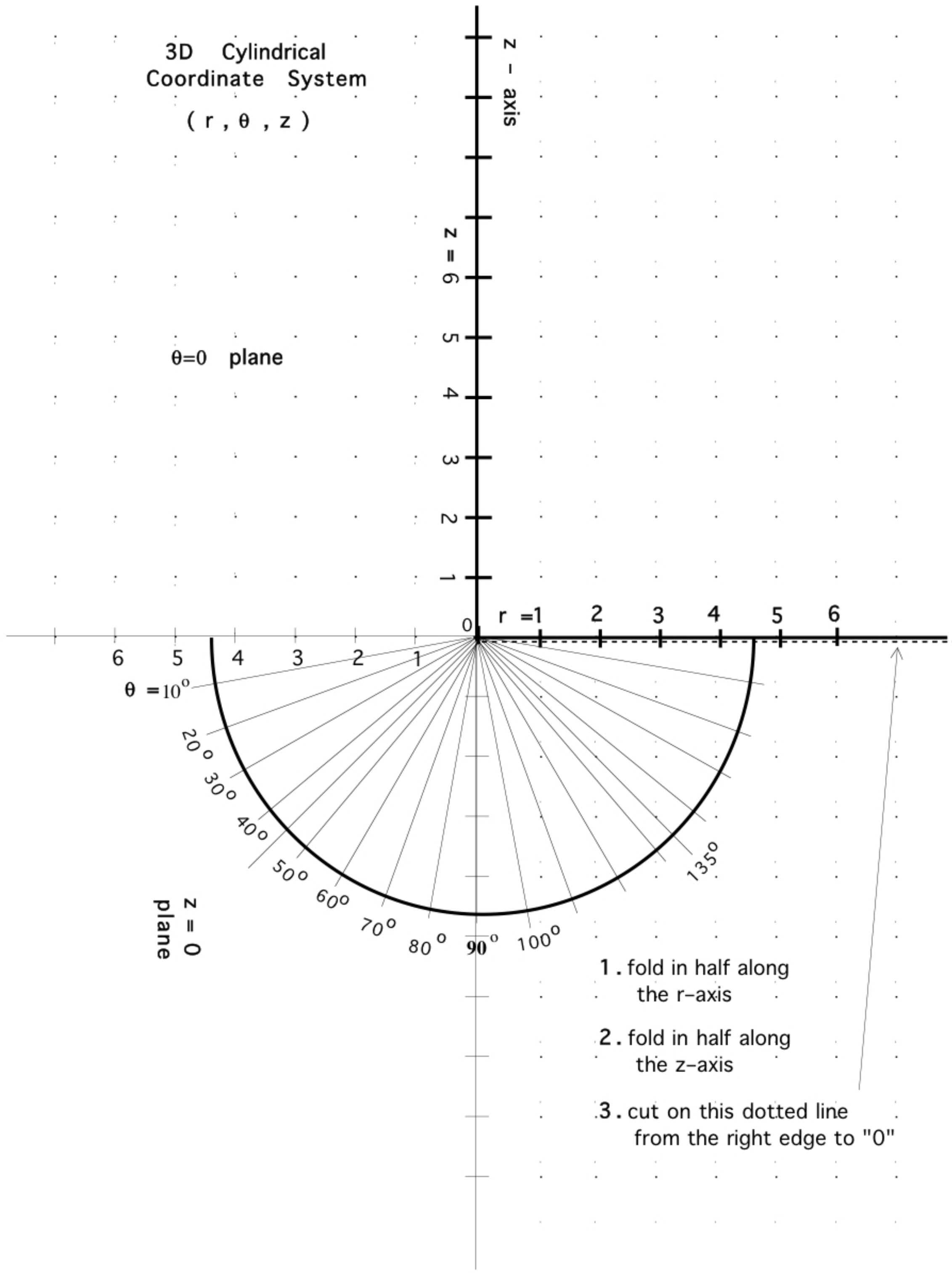
sphereplot([p*t, exp(t/10, p^2], t=0..Pi, p=–2..2, axes = NORMAL); strange

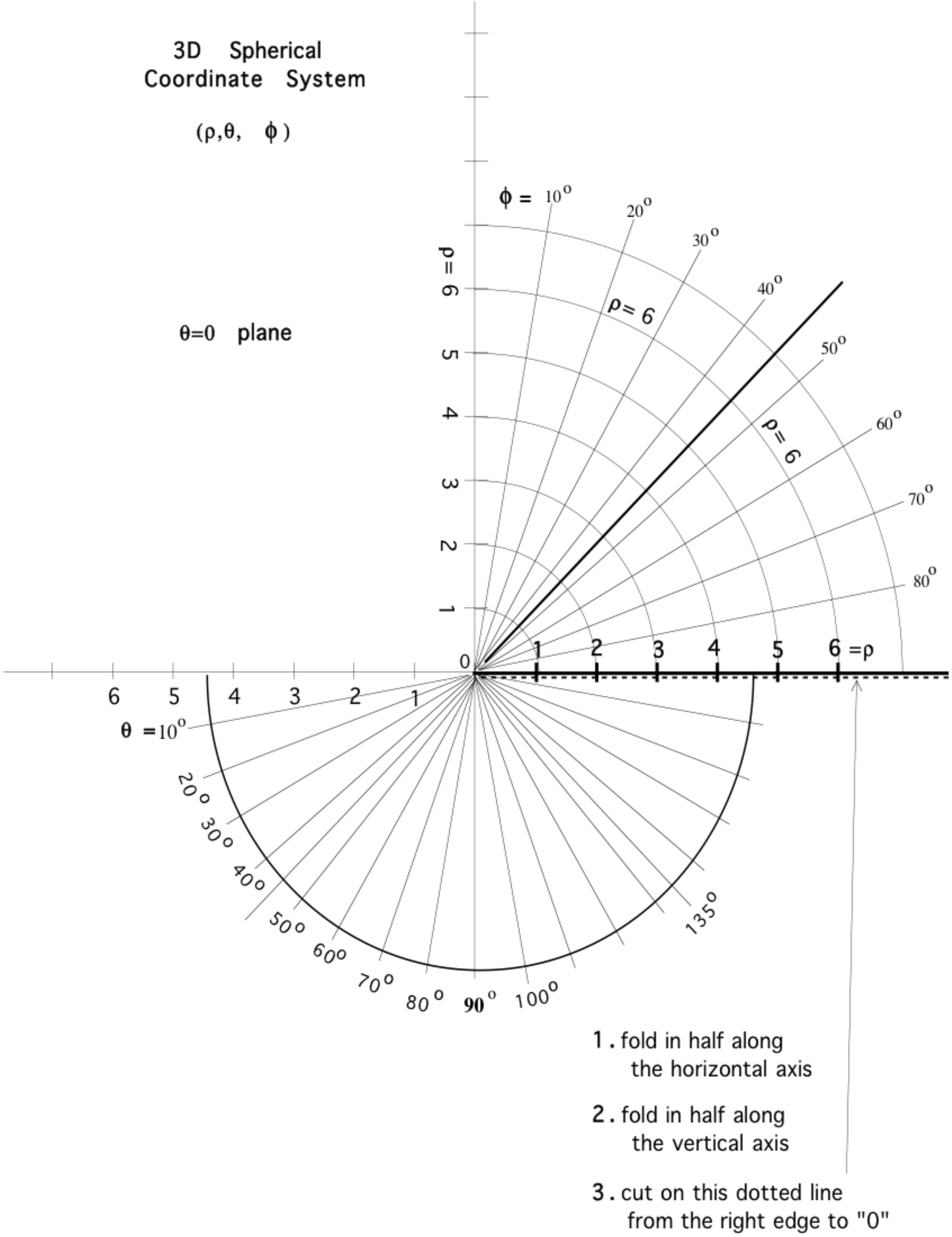
sphereplot((3*sin(x)^2–1)/2, x = –Pi..Pi, y = 0..Pi);

In Chapter 13 we will be working with surfaces z = f(x,y). The Maple option for how we view these surfaces is "orientation = [theta, phi]" which specifies our viewing angle along the ray whose spherical coordinates have angles theta and phi (given in degrees). Try viewing the paraboloid of revolution $z = x^2 + y^2$ with several different orientations:

plo3d(x^2 + y^2, x= –3..3, y= –3..3, orientation = [10,30]); (try this command with some other orientations)

Now position the cursor on the graph, press (and hold) down the mouse button, and slowly move the mouse. This will rotate a "box" containing the graph (and display the orientation coordinates). When the button is released, the graph will be redrawn with the new orientation.


3D Cylindrical
Coordinate System
(r , θ , z)
z – axis
z = 6
5
4
3
2
1
0
θ=0 plane
r =1
2
3
4
5
6
θ =10°
20°
30°
40°
50°
60°
70°
80°
90°
100°
135°
z = 0
plane
1. fold in half along
the r–axis
2. fold in half along
the z–axis
3. cut on this dotted line
from the right edge to "0"


3D Spherical
Coordinate System
(ρ,θ, ϕ)
θ=0 plane
ϕ = 10°
20°
30°
40°
50°
60°
70°
80°
ρ= 6
ρ= 6
ρ= 6
1 2 3 4 5 6 =ρ
0
6 5 4 3 2 1
θ =10°
20° 30° 40° 50° 60° 70° 80° 90° 100°
135°
1. fold in half along
the horizontal axis
2. fold in half along
the vertical axis
3. cut on this dotted line
from the right edge to "0"