12.1 Selected Answers

The graphs for the odd problems 1 to 15 are given.

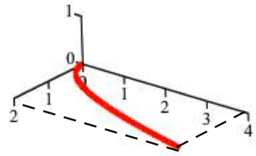


Fig. Problem 1

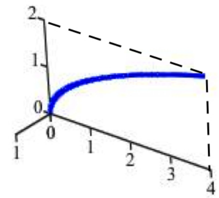


Fig. Problem 3

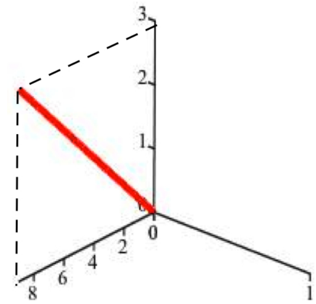


Fig. Problem 5

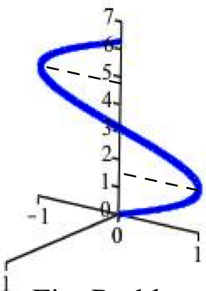


Fig. Problem 7

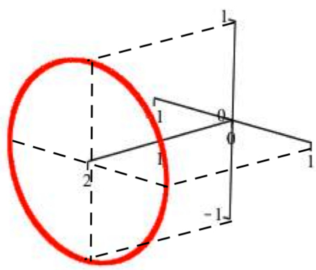


Fig. Problem 9

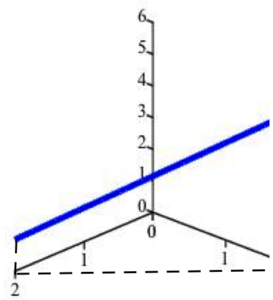


Fig. Problem 11

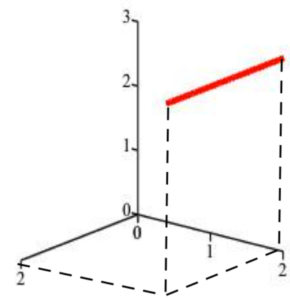


Fig. Problem 13

The graphs for problems 17 to 20 are given.

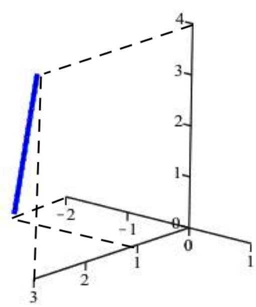


Fig. Problem 15

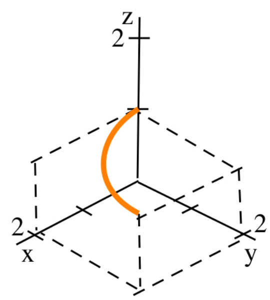


Fig. Problem 17

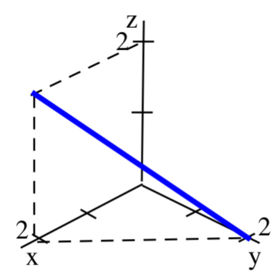


Fig. Problem 18

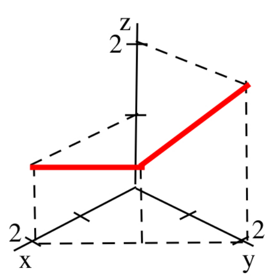


Fig. Problem 19

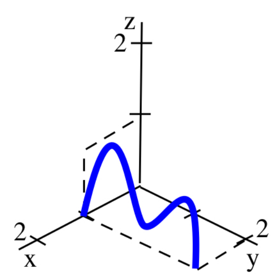


Fig. Problem 20

21. $\langle 3, 9/2, 1/2 \rangle$

22. $\langle 1, 0, 0 \rangle$

23. does not exist

24. $\langle 1, 8, 1 \rangle$

25. $\langle 0, 3, 0 \rangle$

26. does not exist

27. $\langle \pi/2, 0, 0 \rangle$

28. $\langle 1, 1, 1 \rangle$

29. continuous for all $t \neq 1$

30. continuous for all $t \neq 3$

31. continuous for all $t \geq -7$ and $t \neq 2$

32. continuous for all $t \geq 1$

33. continuous for all $t \neq -1, 0, 2$

34. continuous for all $t \neq -3, 2$

35. continuous for all $t > 0$

36. continuous for all $t \neq 0$

37. $x(t) = (1-t)^3(0) + 3(1-t)^2 \cdot t(1) + 3(1-t) \cdot t^2(1) + t^3(0) = -3t^2 + 3t$

$y(t) = (1-t)^3(0) + 3(1-t)^2 \cdot t(0) + 3(1-t) \cdot t^2(2) + t^3(2) = -4t^3 + 6t^2$

$z(t) = (1-t)^3(1) + 3(1-t)^2 \cdot t(1) + 3(1-t) \cdot t^2(0) + t^3(0) = 2t^3 - 3t^2 + 1$

38. $x(t) = (1-t)^3(0) + 3(1-t)^2 \cdot t(1) + 3(1-t) \cdot t^2(0) + t^3(1) = 4t^3 - 6t^2 + 3t$

$y(t) = (1-t)^3(0) + 3(1-t)^2 \cdot t(0) + 3(1-t) \cdot t^2(2) + t^3(3) = -3t^3 + 6t^2$

$z(t) = (1-t)^3(2) + 3(1-t)^2 \cdot t(1) + 3(1-t) \cdot t^2(0) + t^3(0) = t^3 - 3t + 2$

39. $x(t) = (1-t)^3(0) + 3(1-t)^2 \cdot t(0) + 3(1-t) \cdot t^2(2) + t^3(2) = -4t^3 + 6t^2$

$y(t) = (1-t)^3(1) + 3(1-t)^2 \cdot t(0) + 3(1-t) \cdot t^2(1) + t^3(0) = -4t^3 + 6t^2 - 3t + 1$

$z(t) = (1-t)^3(2) + 3(1-t)^2 \cdot t(2) + 3(1-t) \cdot t^2(0) + t^3(0) = 4t^3 - 6t^2 + 2$

40. $x(t) = (1-t)^3(0) + 3(1-t)^2 \cdot t(0) + 3(1-t) \cdot t^2(2) + t^3(1) = -5t^3 + 6t^2$

$y(t) = (1-t)^3(1) + 3(1-t)^2 \cdot t(0) + 3(1-t) \cdot t^2(0) + t^3(1) = 3t^2 - 3t + 1$

$z(t) = (1-t)^3(2) + 3(1-t)^2 \cdot t(2) + 3(1-t) \cdot t^2(0) + t^3(0) = 4t^3 - 6t^2 + 2$

The graphs for problems 41 to 44 are given.

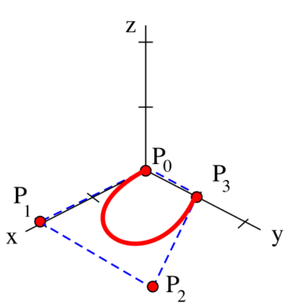


Fig. Problem 41

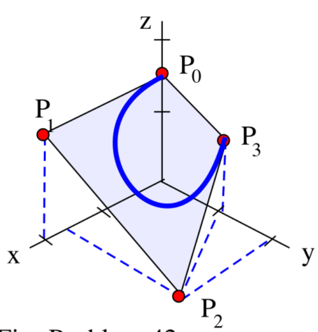


Fig. Problem 42

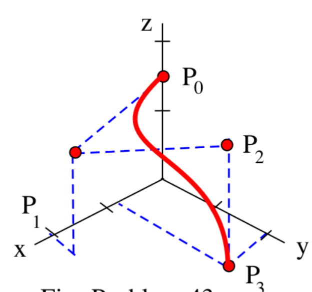


Fig. Problem 43

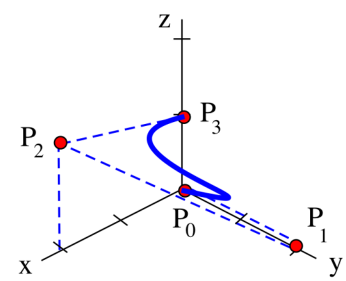


Fig. Problem 44

45. See Fig. 54

46. See Fig. 55

47. See Fig. 56

	A	B	C
x	D	D	D
y	I	I	I
z	D	D	D

Fig. 54

	A	B	C
x	D	D	D
y	I	I	I
z	D	I	D

Fig. 55

	A	B	C
x	D	I	D
y	I	I	I
z	D	D	D

Fig. 56

12.2 Selected Answers

1. $\mathbf{r}'(1) = \langle -, +, - \rangle$, $\mathbf{r}'(2) = \langle -, +, 0 \rangle$, and $\mathbf{r}'(3) = \langle -, 0, + \rangle$.

3. $\mathbf{r}'(1) = \langle +, -, + \rangle$, $\mathbf{r}'(2) = \langle +, -, + \rangle$, and $\mathbf{r}'(3) = \langle +, -, + \rangle$.

5. $\mathbf{r}'(t) = \langle 3t^2, 2, 2t \rangle$, $\mathbf{r}''(t) = \langle 6t, 0, 2 \rangle$

$\mathbf{v}(1) = \mathbf{r}'(1) = \langle 3, 2, 2 \rangle$, speed(1) $= |\mathbf{v}(1)| = \sqrt{17}$, direction(1) $= \dfrac{\mathbf{r}'(1)}{|\mathbf{r}'(1)|} = \dfrac{1}{\sqrt{17}} \langle 3, 2, 2 \rangle$

$\mathbf{v}(2) = \mathbf{r}'(2) = \langle 12, 2, 4 \rangle$, speed(2) $= |\mathbf{v}(2)| = \sqrt{164}$, direction(2) $= \dfrac{\mathbf{r}'(2)}{|\mathbf{r}'(2)|} = \dfrac{1}{\sqrt{164}} \langle 12, 2, 4 \rangle$.

$\mathbf{a}(1) = \mathbf{r}''(1) = \langle 6, 0, 2 \rangle$, $\mathbf{a}(2) = \mathbf{r}''(2) = \langle 12, 0, 2 \rangle$.

7. $\mathbf{r}'(t) = \langle -1, -4/t^2, 0 \rangle$, $\mathbf{r}''(t) = \langle 0, 8/t^3, 0 \rangle$

$\mathbf{v}(1) = \mathbf{r}'(1) = \langle -1, -4, 0 \rangle$, speed(1) $= |\mathbf{v}(1)| = \sqrt{17}$, direction(1) $= \dfrac{\mathbf{r}'(1)}{|\mathbf{r}'(1)|} = \dfrac{1}{\sqrt{17}} \langle -1, -4, 0 \rangle$

$\mathbf{v}(2) = \mathbf{r}'(2) = \langle -1, -1, 0 \rangle$, speed(2) $= |\mathbf{v}(2)| = \sqrt{2}$, direction(2) $= \dfrac{\mathbf{r}'(2)}{|\mathbf{r}'(2)|} = \dfrac{1}{\sqrt{2}} \langle -1, -1, 0 \rangle$.

$\mathbf{a}(1) = \mathbf{r}''(1) = \langle 0, 8, 0 \rangle$, $\mathbf{a}(2) = \mathbf{r}''(2) = \langle 0, 1, 0 \rangle$.

9. $\mathbf{r}(t) = \langle t^3, 7, 1 + 5t \rangle$, $\mathbf{r}'(t) = \langle 3t^2, 0, 5 \rangle$.

$\dfrac{d}{dt}\mathbf{r}(2t) = \mathbf{r}'(2t)\dfrac{d}{dt}(2t) = \langle 3(2t)^2, 0, 5 \rangle (2) = \langle 24t^2, 0, 10 \rangle$.

Or, $\mathbf{r}(2t) = \langle (2t)^3, 7, 1 + 5(2t) \rangle = \langle 8t^3, 7, 1 + 10t \rangle$ so $\dfrac{d}{dt}\mathbf{r}(2t) = \langle 24t^2, 0, 10 \rangle$.

11. $\mathbf{r}(t) = \langle t, 2t^2, 3t^3 \rangle$, $\mathbf{r}'(t) = \langle 1, 4t, 9t^2 \rangle$.

$\dfrac{d}{dt}\{\sin(t)\,\mathbf{r}(t)\} = \sin(t)\,\mathbf{r}'(t) + \cos(t)\,\mathbf{r}(t) = \sin(t)\langle 1, 4t, 9t^2 \rangle + \cos(t)\langle t, 2t^2, 3t^3 \rangle$.

$= \langle \sin(t) + t\cos(t), 4t\sin(t) + 2t^2\cos(t), 9t^2\sin(t) + 3t^3\cos(t) \rangle$.

13. $\mathbf{r}(t) = (2 - 5t^3)\mathbf{i} + (7t)\mathbf{j} + (1 + t)\mathbf{k} = \langle 2 - 5t^3, 7t, 1 + t \rangle$, $\mathbf{r}'(t) = \langle -15t^2, 7, 1 \rangle$.

$\dfrac{d}{dt}\mathbf{r}(3t) = \mathbf{r}'(3t)\dfrac{d}{dt}(3t) = \langle -15(3t)^2, 7, 1 \rangle (3) = \langle -405t^2, 21, 3 \rangle$.

Or, $\dfrac{d}{dt}\mathbf{r}(3t) = \dfrac{d}{dt}\langle 2 - 5(3t)^3, 7(3t), 1 + (3t) \rangle$

$= \dfrac{d}{dt}\langle 2 - 135t^3, 21t, 1 + 3t \rangle = \langle -405t^2, 21, 3 \rangle$.

15. $\dfrac{d}{dt}\{\mathbf{u} + 2\mathbf{v}\} = \dfrac{d}{dt}\langle 2 + 10t, 8 - t, 6 + t^3 \rangle = \langle 10, -1, 3t^2 \rangle$

$\dfrac{d}{dt}\{\mathbf{u}\cdot\mathbf{v}\} = \dfrac{d}{dt}\{4t - t^2 + 3t^3\} = 4 - 2t + 9t^2$

$\dfrac{d}{dt}\{\mathbf{u} \times \mathbf{v}\} = \dfrac{d}{dt}\langle 3t - 4t^3 + t^4, t^3 + 5t^4, -t - 5t^2 \rangle = \langle 3 - 12t^2 + 4t^3, 3t^2 + 20t^3, -1 - 10t \rangle$.

17. $\frac{d}{dt}\{\mathbf{u}+2\mathbf{v}\} = \frac{d}{dt}\langle 5t^3-4t+2, 2-t, t+10\rangle = \langle 15t^2-4, -1, 1\rangle$

$\frac{d}{dt}\{\mathbf{u}\cdot\mathbf{v}\} = \frac{d}{dt}\{-10t^4+5t^3-21t^2+10t+8\} = -40t^3+15t^2-42t+10$

$\frac{d}{dt}\{\mathbf{u}\times\mathbf{v}\} = \frac{d}{dt}\langle -3t^2-34t+8, -20t^3-2t^2-3t+2, 15t^4-14t^2+11t-2\rangle$

$= \langle -6t-34, -60t^2-4t-3, 60t^3-28t+11\rangle$.

19. The curves intersect at the point (0,3,9) when t = 3: $\mathbf{u}(3) = \langle 0, 3, 9\rangle = \mathbf{v}(3)$.

$\mathbf{u}'(3) = \langle -1, 1, 6\rangle$ and $\mathbf{v}'(3) = \langle 0, 1, 0\rangle$.

$\cos(\theta) = \frac{\mathbf{u}'(3)\cdot\mathbf{v}'(3)}{|\mathbf{u}'(3)|\,|\mathbf{v}'(3)|} = \frac{1}{\sqrt{38}} \approx 0.162$ so $\theta \approx 1.408$ ($\approx 80.7^o$)

21. The curves intersect at the point (5,9,3) when t=–1 and s=3: $\mathbf{u}(-1) = \langle 5, 9, 3\rangle = \mathbf{v}(3)$.

$\mathbf{u}'(-1) = \langle -10, 0, -1\rangle$ and $\mathbf{v}'(3) = \langle 1, 3, -1\rangle$.

$\cos(\theta) = \frac{\mathbf{u}'(-1)\cdot\mathbf{v}'(3)}{|\mathbf{u}'(-1)|\,|\mathbf{v}'(3)|} = \frac{-9}{\sqrt{101}\sqrt{11}} \approx -0.270$ so $\theta \approx 1.844$ ($\approx 105.7^o$)

23. Area of the parallelogram $= |\mathbf{u}\times\mathbf{v}| = |\langle -2t^3, t^3, -t^2\rangle| = \sqrt{5t^6+t^4}$

Rate of change of area $= \frac{d}{dt}\sqrt{5t^6+t^4} = \frac{15t^5+2t^3}{\sqrt{5t^6+t^4}} = \frac{17}{\sqrt{6}}$ when t = 1 and $\frac{496}{\sqrt{336}}$ when t = 2.

25. Area of the triangle $= \frac{1}{2}|\mathbf{u}\times\mathbf{v}| = \frac{1}{2}|\langle 0, 6t, -2t^2\rangle| = \frac{1}{2}\sqrt{36t^2+4t^4} = \sqrt{9t^2+t^4}$

Rate of change of area $= \frac{d}{dt}\sqrt{9t^2+t^4} = \frac{9t+2t^3}{\sqrt{9t^2+t^4}} = \frac{11}{\sqrt{10}}$ when t = 1 and $\frac{34}{\sqrt{52}}$ when t = 2.

27. Volume of the tetrahedron $= \frac{1}{6}|\mathbf{u}\cdot(\mathbf{v}\times\mathbf{s})| = \frac{1}{6}|3t^2| = \frac{1}{2}t^2$.

Rate of change of volume $= \frac{d}{dt}\frac{1}{2}t^2 = t = 1$ when t = 1 and 2 when t = 2.

29. $\mathbf{r}(t) = \langle 6t^2+1, 4t^3+2, 6e^t-3\rangle$

30. $\mathbf{r}(t) = \langle 3t+2t^2+7, \sin(t)+2, t-3t^2+5\rangle$

31. $\mathbf{r}(t) = \langle 2t^3+4, 4t-2, 4t^2-5t-2\rangle$

33 and 34. See Fig. 18

35 and 36. See Fig. 19

t	$\mathbf{r}''(t)$	$\mathbf{r}'(t)$	$\mathbf{r}(t)$
0	⟨ 0, 2, 5 ⟩	⟨ 1 , 2 , 3 ⟩	⟨ 0 , 3 , 1 ⟩
1	⟨ 4, 1, 3 ⟩	⟨ 5 , 3 , 6 ⟩	⟨ 5 , 6 , 7 ⟩
2	⟨ 6, 0, 1 ⟩	⟨ 11 , 3 , 7 ⟩	⟨ 16 , 9 , 14 ⟩
3	⟨ 4, –2, 0 ⟩	⟨ 15 , 1 , 7 ⟩	⟨ 31 , 10 , 21 ⟩
4	⟨ 2, 0, 2 ⟩	⟨ 17 , 1 , 9 ⟩	⟨ 48 , 11 , 30 ⟩
5	⟨ 8, 3, 4 ⟩	⟨ 25 , 4 , 13 ⟩	⟨ 73 , 15 , 43 ⟩

Fig. 18

t	$\mathbf{r}''(t)$	$\mathbf{r}'(t)$	$\mathbf{r}(t)$
0	⟨ 1, 2, 3 ⟩	⟨ 1 , 6 , 4 ⟩	⟨ 17, 3 , 2 ⟩
1	⟨ 4, 2, 2 ⟩	⟨ 5 , 8 , 6 ⟩	⟨ 22, 11, 8 ⟩
2	⟨ 3, 1, 0 ⟩	⟨ 8 , 9 , 6 ⟩	⟨ 30, 20, 14 ⟩
3	⟨ 2, 3, 1 ⟩	⟨ 10, 12 , 7 ⟩	⟨ 40, 32, 21 ⟩
4	⟨ 1, 4, 0 ⟩	⟨ 11, 16 , 7 ⟩	⟨ 51, 48, 28 ⟩
5	⟨ 0, 1, 3 ⟩	⟨ 11, 17 , 10 ⟩	⟨ 62, 65, 38 ⟩

Fig. 19

12.3 Selected Answers

1. $\mathbf{L} = \int_{t=a}^{t=b} |\mathbf{r'}(t)|\,dt = \int_{t=0}^{t=2\pi} \sqrt{4\sin^2(t)+4\cos^2(t)+1}\;dt = \int_{t=0}^{t=2\pi} \sqrt{5}\;dt = 2\pi\cdot\sqrt{5} \approx 14.05$

2. $\mathbf{L} = \int_{t=a}^{t=b} |\mathbf{r'}(t)|\,dt = \int_{t=0}^{t=2\pi} \sqrt{9\sin^2(t)+9\cos^2(t)+1}\;dt = \int_{t=0}^{t=2\pi} \sqrt{10}\;dt = 2\pi\cdot\sqrt{10} \approx 19.87$

3. $\mathbf{L} = 2\pi\sqrt{17} \approx 25.91$

4. $\mathbf{L} = 2\pi\sqrt{R^2+1}$

5. $\mathbf{L} = \int_{t=a}^{t=b} |\mathbf{r'}(t)|\,dt = \int_{t=0}^{t=2\pi} \sqrt{4\sin^2(t)+9\cos^2(t)+1}\;dt \approx 17.08$

6. $\mathbf{L} = \int_{t=a}^{t=b} |\mathbf{r'}(t)|\,dt = \int_{t=0}^{t=2\pi} \sqrt{4\sin^2(t)+25\cos^2(t)+1}\;dt \approx 23.93$

7. $\mathbf{L} = \int_{t=a}^{t=b} |\mathbf{r'}(t)|\,dt = \int_{t=0}^{t=2\pi} \sqrt{A^2\sin^2(t)+B^2\cos^2(t)+1}\;dt$

8. $\mathbf{L} = \int_{t=a}^{t=b} |\mathbf{r'}(t)|\,dt = \int_{t=0}^{t=2\pi} \sqrt{4\sin^2(t)+4\cos^2(t)+1}\;dt = \int_{t=0}^{t=2\pi} \sqrt{5}\;dt = 2\pi\cdot\sqrt{5} \approx 14.05$

9. $\mathbf{L} = \int_{t=a}^{t=b} |\mathbf{r'}(t)|\,dt = \int_{t=0}^{t=2\pi} \sqrt{t^2+2}\;dt \approx 22.43$

11. $\mathbf{L} = \int_{t=a}^{t=b} |\mathbf{r'}(t)|\,dt = \int_{t=0}^{t=2\pi} \sqrt{\left(-2t\sin(t)+2\cos(t)\right)^2+\left(t\cos(t)+\sin(t)\right)^2+1}\;dt \approx 34.02$

13. $x'(t) = 3 - 6t$, $y'(t) = 18t - 21t^2$, $z'(t) = 3 - 18t + 5t^2$.

$$\mathbf{L} = \int_{t=a}^{t=b} |\mathbf{r'}(t)|\,dt = \int_{t=0}^{t=1} \sqrt{(x'(t))^2+(y'(t))^2+(z'(t))^2}\;dt$$

15. $x'(t) = -6 + 24t - 24t^2$, $y'(t) = 3 - 6t + 9t^2$, $z'(t) = 3 - 6t$.

$$\mathbf{L} = \int_{t=a}^{t=b} |\mathbf{r'}(t)|\,dt = \int_{t=0}^{t=1} \sqrt{(x'(t))^2+(y'(t))^2+(z'(t))^2}\;dt$$

17. $\mathbf{r}'(t) = \langle -\sin(t), \cos(t), 1 \rangle$, $\mathbf{r}''(t) = \langle -\cos(t), -\sin(t), 0 \rangle$, $\mathbf{r}'(t) \times \mathbf{r}''(t) = \langle \sin(t), -\cos(t), 1 \rangle$.

Then $|\mathbf{r}'(t)| = \sqrt{2}$ and $|\mathbf{r}'(t) \times \mathbf{r}''(t)| = \sqrt{2}$, so

$$\kappa = \frac{|\mathbf{r}' \times \mathbf{r}''|}{|\mathbf{r}'|^3} = \frac{\sqrt{2}}{(\sqrt{2})^3} = \frac{1}{2} \text{ for all values of } t.$$

19. $\mathbf{r}'(t) = \langle -R\sin(t), R\cos(t), 1 \rangle$, $\mathbf{r}''(t) = \langle -R\cos(t), -R\sin(t), 0 \rangle$, $\mathbf{r}'(t) \times \mathbf{r}''(t) = \langle R\sin(t), -R\cos(t), R^2 \rangle$.

Then $|\mathbf{r}'(t)| = \sqrt{R^2+1}$ and $|\mathbf{r}'(t) \times \mathbf{r}''(t)| = |R|\sqrt{R^2+1}$, so

$$\kappa = \frac{|\mathbf{r}' \times \mathbf{r}''|}{|\mathbf{r}'|^3} = \frac{|R|\sqrt{R^2+1}}{(\sqrt{R^2+1})^3} = \frac{|R|}{R^2+1} \text{ for all values of } t.$$

21. $x'(t) = -6 - 18t + 9t^2$, $y'(t) = 18t - 21t^2$, $z'(t) = 3 - 18t + 15t^2$.

$x''(t) = -18 + 18t$, $y''(t) = 18 - 42t$, $z''(t) = -18 + 30t$.

When $t = 0.2$, $x' = 2.76$, $y' = 2.76$, $z' = 0$, $x'' = -17.28$, $y'' = -26.4$, $z'' = -12$ so

$\mathbf{r}' \times \mathbf{r}'' = -33.12\mathbf{i} + 33.12\mathbf{j} - 25.1712\mathbf{k}$, $|\mathbf{r}' \times \mathbf{r}''| \approx 53.17$, $|\mathbf{r}'| \approx 3.90$

and $\kappa = \dfrac{|\mathbf{r}' \times \mathbf{r}''|}{|\mathbf{r}'|^3} = \dfrac{53.17}{(3.90)^3} = 0.896$. The radius of curvature is $\dfrac{1}{\kappa} \approx 1.12$.

When $t = 0.5$, $x' = -0.75$, $y' = 3.75$, $z' = -2.25$, $x'' = -9$, $y'' = -3$, $z'' = -3$

$\mathbf{r}' \times \mathbf{r}'' = -18\mathbf{i} + 18\mathbf{j} + 36\mathbf{k}$, $|\mathbf{r}' \times \mathbf{r}''| \approx 44.09$, $|\mathbf{r}'| \approx 4.44$

and $\kappa = \dfrac{|\mathbf{r}' \times \mathbf{r}''|}{|\mathbf{r}'|^3} = \dfrac{44.09}{(4.44)^3} = 0.504$. The radius of curvature is $\dfrac{1}{\kappa} \approx 1.98$.

23. $x'(t) = -3\sin(t)$, $y'(t) = 5\cos(t)$, $x''(t) = -3\cos(t)$, $y''(t) = -5\sin(t)$. Then

$$\kappa = \frac{|x'y'' - x''y'|}{((x')^2 + (y')^2)^{3/2}} = \frac{|15\sin^2(t) - -15\cos^2(t)|}{(9\sin^2(t) + 25\cos^2(t))^{3/2}} .$$

When $t = 0$, $\kappa = \dfrac{15}{(9\sin^2(0) + 25\cos^2(0))^{3/2}} = \dfrac{15}{125} = 0.12$. Radius of curvature = $\dfrac{25}{3} \approx 8.33$.

When $t = \dfrac{\pi}{4}$, $\kappa = \dfrac{15}{(9\sin^2(\pi/4) + 25\cos^2(\pi/4))^{3/2}} = \dfrac{15}{17^{3/2}} = 0.214$. Radius of curvature ≈ 4.67 .

When $t = \dfrac{\pi}{2}$, $\kappa = \dfrac{15}{(9\sin^2(\pi/2) + 25\cos^2(\pi/2))^{3/2}} = \dfrac{15}{27} = 0.555$. Radius of curvature $= \dfrac{9}{5} = 1.8$.

25. $x'(t) = -A\sin(t)$, $y'(t) = B\cos(t)$, $x''(t) = -A\cos(t)$, $y''(t) = -B\sin(t)$. Then

$$\kappa = \frac{|x'y'' - x''y'|}{((x')^2 + (y')^2)^{3/2}} = \frac{|AB\sin^2(t) + AB\cos^2(t)|}{(A^2\sin^2(t) + B^2\cos^2(t))^{3/2}} .$$

When $t = 0$, $\kappa = \dfrac{|AB|}{(A^2\sin^2(0) + B^2\cos^2(0))^{3/2}} = \dfrac{|AB|}{|B|^3} = \dfrac{|A|}{|B|^2}$. Radius of curvature = $\dfrac{1}{\kappa} = \dfrac{|B|^2}{|A|}$.

When $t = \dfrac{\pi}{4}$, $\kappa = \dfrac{|AB|}{(A^2\sin^2(\pi/4) + B^2\cos^2(\pi/4))^{3/2}} = \dfrac{|AB|}{((A^2 + B^2)/2)^{3/2}} =$. Radius of curvature = $\dfrac{1}{\kappa}$.

When $t = \dfrac{\pi}{2}$, $\kappa = \dfrac{|AB|}{(A^2\sin^2(\pi/2) + B^2\cos^2(\pi/2))^{3/2}} = \dfrac{|AB|}{|A|^3} = \dfrac{|B|}{|A|^2}$. Radius of curvature = $\dfrac{1}{\kappa} = \dfrac{|A|^2}{|B|}$.

27. $y' = 3$ and $y'' = 0$ so $\kappa = \dfrac{|y''|}{(1 + (y')^2)^{3/2}} = \dfrac{0}{10^{3/2}} = 0$. (As we might expect, the curvature of the straight line $y = 3x + 5$ is 0.)

29. $y' = 2x$ and $y'' = 2$ so $\kappa = \dfrac{|y''|}{(1 + (y')^2)^{3/2}} = \dfrac{2}{(1 + 4x^2)^{3/2}}$.

When $x = 1$, $\kappa = \dfrac{2}{5^{3/2}} \approx 0.1789$. When $x = 2$, $\kappa = \dfrac{2}{17^{3/2}} \approx 0.0285$. When $x = 3$, $\kappa = \dfrac{2}{37^{3/2}} \approx 0.0089$.

12.4 Selected Answers

1 – 23 Odd: The answers are shown in the figures.

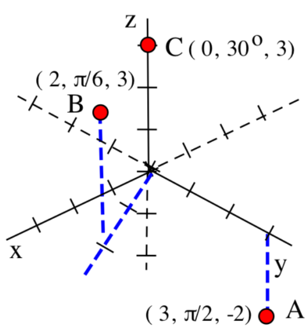


Fig. Problem 1

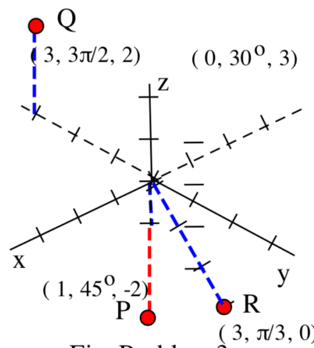
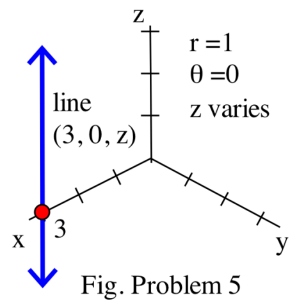


Fig. Problem 3



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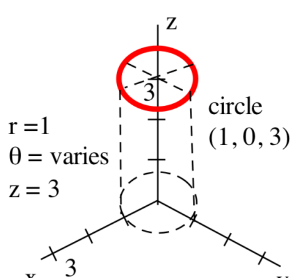


Fig. Problem 7

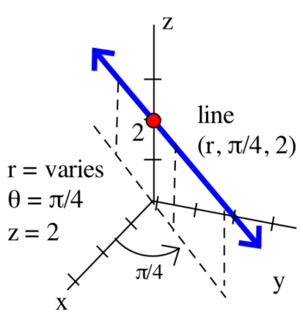


Fig. Problem 9

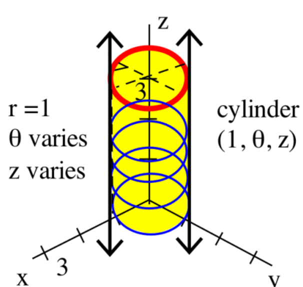
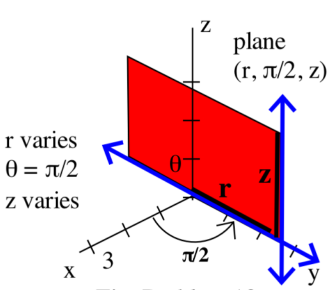


Fig. Problem 11



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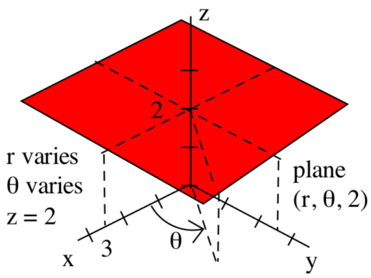


Fig. Problem 15

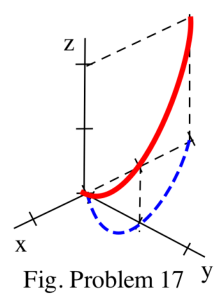


Fig. Problem 17

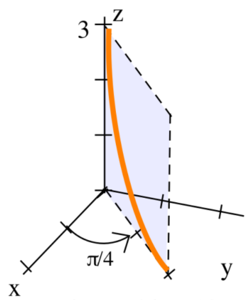


Fig. Problem 19

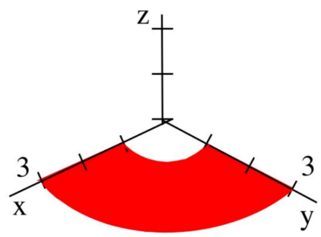


Fig. Problem 21

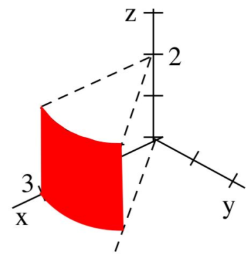


Fig. Problem 23

25. $x = 5\cos(\pi/6) \approx 4.33$, $y = 5\sin(\pi/6) = 2.5$, $z = 3$.

27. $x = 3\cos(35^o) \approx 2.46$, $y = 3\sin(35^o) \approx 1.72$, $z = -2$.

29. $r = \sqrt{x^2 + y^2} = \sqrt{1^2 + 2^2} = \sqrt{5}$,

$\theta = \arctan(y/x) = \arctan(2/1) \approx 1.107$, $z = 3$.

31. $r^2 = x^2 + y^2 = 5^2 = 25$,

$\theta = \arctan(y/x) = \arctan(-3/4) \approx -0.644$,

$z = -1$. To get the correct location, we need to use $r = -5$.

33. (a) $x^2 + y^2 = 4y - 1$ or $x^2 + (y-2)^2 = 3$. (b) $\sqrt{x^2 + y^2} = 7$ or $x^2 + y^2 = 49$

35. (a) $x^2 + y^2 = 5x$ or $(x - 5/2)^2 + y^2 = 25/4$. (b) $z = x^2 + y^2$

37. (a) $z = r^2 - 3r\cdot\cos(\theta) + 2r\cdot\sin(\theta)$ (b) $r\cdot\cos(\theta) = 3$

39. $z = r^2\cos^2(\theta) + 5r^2\sin^2(\theta)$ (b) $r\cdot\cos(\theta) + r\cdot\sin(\theta) + z = 5$

41 – 59 Odd: The answers are shown in the figures.

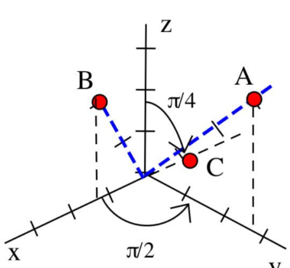


Fig. Problem 41

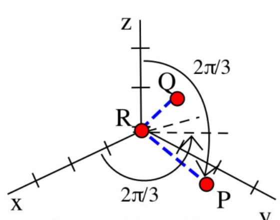
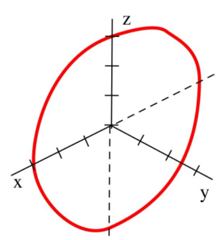


Fig. Problem 43



circle on the xz-plane
Fig. Problem 45

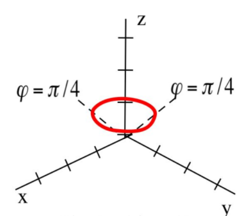


Fig. Problem 47

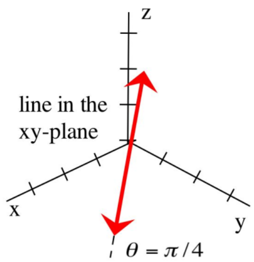


Fig. Problem 49

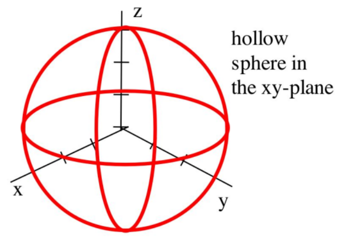


Fig. Problem 51

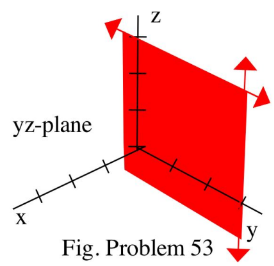


Fig. Problem 53

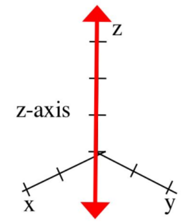


Fig. Problem 55

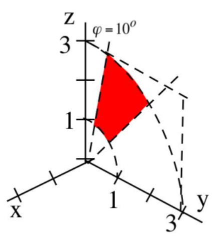


Fig. Problem 57

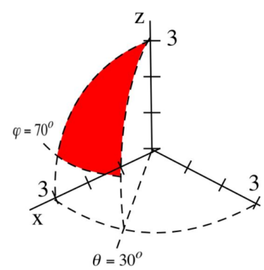


Fig. Problem 59

61. $x = 5\cdot\sin(\pi/3)\cdot\cos(\pi/2) = 0$, $y = 5\cdot\sin(\pi/3)\cdot\sin(\pi/2) = 5\sqrt{3}/2 \approx 4.330$, $z = 5\cdot\cos(\pi/3) = 5/2$

63. $x = 4\cdot\sin(30^o)\cdot\cos(45^o) = (4)(\frac{1}{2})(\frac{\sqrt{2}}{2}) = \sqrt{2} \approx 1.414$

$y = 4\cdot\sin(30^o)\cdot\sin(45^o) = (4)(\frac{1}{2})(\frac{\sqrt{2}}{2}) = \sqrt{2} \approx 1.414$, $z = 4\cdot\cos(30^o) = (4)(\frac{\sqrt{3}}{2}) = 2\sqrt{3} \approx 3.468$

65. $\rho = \sqrt{x^2 + y^2 + z^2} = \sqrt{14} \approx 3.742$, $\theta = \arctan(y/x) = \arctan(2/1) \approx 1.107$ $(\approx 63.4^o)$,

$\varphi = \arccos(z/) = \arccos(3/\sqrt{14}) \approx 0.641$ $(\approx 36.7^o)$

67. $\rho = \sqrt{x^2 + y^2 + z^2} = \sqrt{38} \approx 6.164$, $\theta = \arctan(y/x) = \arctan(-3/5) \approx -0.540$ $(\approx -30.9^o)$,

$\varphi = \arccos(z/\rho) = \arccos(2/\sqrt{38}) \approx 1.240$ $(\approx 71.0^o)$

69. (a) $5 = \rho = \sqrt{x^2 + y^2 + z^2}$ or $x^2 + y^2 + z^2 = 25$, (b) (Graphically) $x = 0$

71. (a) $\rho = 5\cdot\sin(\varphi)\cdot\cos(\theta)$

$\rho^2 = 5\rho\cdot\sin(\varphi)\cdot\cos(\theta)$

$x^2 + y^2 + z^2 = 5x$

(b) $\rho = 3\cdot\sec(\varphi) = 3\frac{1}{\cos(\varphi)}$

$\rho\cdot\cos(\varphi) = 3$

$z = 3$

73. (a) $\rho^2 = 9$ (b) $\rho\cdot\sin(\varphi)\cdot\cos(\theta) + \rho\cdot\cos(\varphi) = 5$

75. (a) $\rho\cdot\cos(\varphi) = 2\rho^2\cdot\sin^2(\varphi)$ (b) $\rho^2\cdot\cos^2(\varphi) = 25 - \rho^2\cdot\sin^2(\varphi)\cdot\cos^2(\theta)$