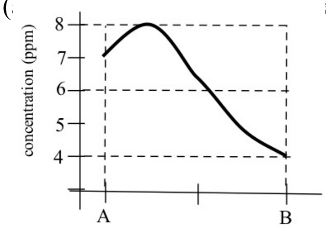
13.1 Selected Answers

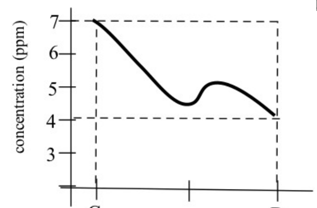
1. (a) minimum = 2, maximum = 30 (b) down 2, down 2 more, up 9 and up 5 more
   (c) up 5, up 5, up 5 up 5, then no change

3. (a) maximum = 10 m (b) depth increases by 3, decreases by 4, increase 1, decreases 1
   (c) depth decreases 2, decreases 1, decreases 1

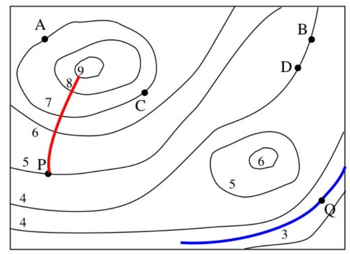
5. (el ires



Problem 5(b)

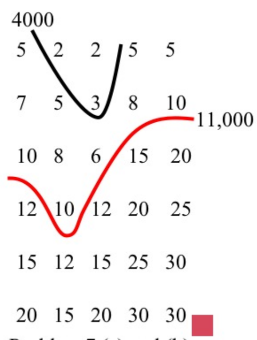


Problem 5(c)

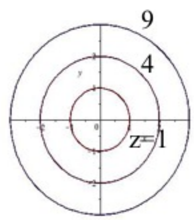


Problem 5(d) and (e)

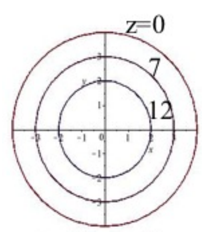
7 to 15. See figures below



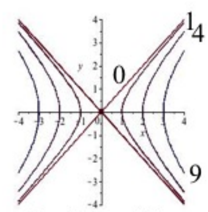
Problem 7 (a) and (b)



Problem 9

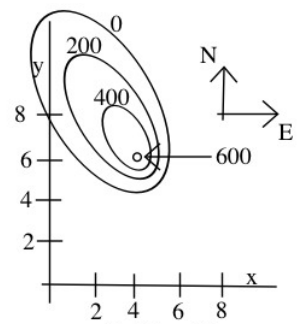


Problem 11



Problem 13

17. level curves D
18. level curves C
19. level curves A
20. level curves G
21. level curves B
22. level curves F
23. level curves E



Problem 15

13.2 Selected Answers

1. (a) 3 (b) 2 (c) 4 (d) about 2.4

3. (a) 3 (b) 2 (c) dne (d) dne

5. -927

7. -5/2

9. Π

11. 1

13. dne Paths y=0 and y=x give different limit values.

15. dne Paths y=0 and y=x give different limit values.

17. dne Paths y=0 and y=x give different limit values.

19. dne Paths y=0 and y=x give different limit values.

21. 1

23. dne Paths y=0 and y= x^2 give different limit values.

25. 2 Try rationalizing the denominator.

27. 0 The function equals $\dfrac{x(y-1)}{(x-1)^2+(y-1)^2}$ which is easier to analyze.

29. -3/5

31. dne If x=y=0 and z->0 then limit = -1. If y=z=0 and x->0 then limit = 1.

33. dne If y=z=0 and x->0 then limit = 0. If z=0 and y=x -> 0 then limit = ½..

35. Define f(2,1) = 4

37. No value of f(1,2) or f(3,2) will make f continuous at those points.
f(1,2) = 3 makes f continuous at (1,2).

39. Not continuous (not defined) on the circle where $x^2+y^2=1$.

41. Not continuous (not defined) if $2x+3y \le 0$ or $y \le -2x/3$.

43. In order for T to be continuous at (x,y) we need both $x+y \ge 0$ (so $y \ge -x$) and $x-y \ge 0$ (so $x \ge y$). That requires $x \ge 0$ and $-x \le y \le x$.

45. F is continuous at (x,y,z) if yz >0.

13.3 Selected Answers

1. $f_x(1,2) = -8,\ f_y(1,2) = -4$

2. $f_x(1,0) = -1/\sqrt{3}$, $f_y(1,0) = 0$

3. $f_x(3,-1) = -27$

4. 2

5. $\dfrac{\partial z}{\partial x} = (x^4 + 3x^2y^2 - 2xy^3)/(x^2+y^2)^2,\ \dfrac{\partial z}{\partial y} = (y^4 + 3x^2y^2 - 2x^3y)/(x^2+y^2)^2$

6. $(1/y) - (y/x^2)$

7. $(y-z)/(x-y),\ (x+z)/(x-y)$

8. $\dfrac{\cos(x)}{1-y\cdot e^z},\ \dfrac{e^z}{1-y\cdot e^z}$

9. $\dfrac{\partial z}{\partial x} = \dfrac{2xz}{2yz-x^2}$ $\dfrac{\partial z}{\partial y} = \dfrac{2y+z^2}{x^2-2yz}$

11. $y\sec(xy) + xy^2\sec(xy)\tan(xy)$

12. 0

13. y + z, x + z, x + y

14. $f_x(x,y) = 3x^2y^5 - 4xy + 1,\ f_y(x,y) = 5x^3y^4 - 2x^2$

15. $f_x(x,y) = 4x^3 + 2xy^2,\ f_y(x,y) = 2x^2y + 4y^3$

16. $f_x(x,y) = 2y/(x+y)^2,\ f_y(x,y) = -2x/(x+y)^2$

17. $f_x = e^x\{\tan(x-y) + \sec^2(x-y)\},\ f_y = -e^x\sec^2(x-y)$

18. $f_s = -3s/\sqrt{2-3s^2-5t^2}$, $f_t = -5t/\sqrt{2-3s^2-5t^2}$

19. $f_u = v/(u^2+v^2)$, $f_v = -u/(u^2+v^2)$

20. $g_x = 2xy^4 \sec^2(x^2y^3)$, $g_y = \tan(x^2y^3) + 3x^2y^3 \sec^2(x^2y^3)$

21. $\frac{\partial z}{\partial x} = 1/\sqrt{x^2+y^2}$, $\frac{\partial z}{\partial y} = y/(x^2+y^2+x\sqrt{x^2+y^2})$

22. $\frac{\partial z}{\partial x} = \frac{3}{2}\cosh(\sqrt{3x+4y})/\sqrt{3x+4y}$, $\frac{\partial z}{\partial y} = 2\cosh(\sqrt{3x+4y})/\sqrt{3x+4y}$

23. $f_x = -e^{(x^2)}$, $f_y = e^{(y^2)}$

24. $f_x = 2xyz^3 + y$, $f_y = x^2z^3 + x$, $f_z = 3x^2yz^2 - 1$

25. $f_x = yz\,x^{yz-1}$, $f_y = z\,x^{yz}\ln(x)$, $f_z = y\,x^{yz}\ln(x)$

26. $u_x = -yz\cos(y/(x+z))/(x+z)^2$, $u_y = z\cos(y/(x+z))/(x+z)$,
$u_z = \sin(y/(x+z)) - yz\cos(y/(x+z))/(x+z)^2$

27. $f_x = 1/(z-t)$, $f_y = -1/(z-t)$, $f_z = -(x-y)/(z-t)^2$, $f_t = (x-y)/(z-t)^2$

31. $\frac{\partial z}{\partial x} = f'(x)$, $\frac{\partial z}{\partial y} = g'(y)$

32. $\frac{\partial z}{\partial x} = f'(x+y)$, $\frac{\partial z}{\partial y} = f'(x+y)$

33. $\frac{\partial z}{\partial x} = f'(x/y)(1/y)$, $\frac{\partial z}{\partial y} = f'(x/y)(-x/y^2)$

34. $f_{xx} = 2y$, $f_{xy} = 2x + 1/(2\sqrt{y}) = f_{yx}$, $f_{yy} = -x/(4y\sqrt{y})$

35. $z_{xx} = 3(2x^2+y^2)/\sqrt{x^2+y^2}$, $z_{xy} = 3xy/\sqrt{x^2+y^2} = z_{yx}$, $z_{yy} = 3(x^2+2y^2)/\sqrt{x^2+y^2}$

13.4 Selected Answers

1. $4x + 8y - z = 8$

3. $2x + 4y - z + 6 = 0$

5. $2x + y - z = 1$

7. $2x - y - z + 2 = 0$

9. $dz = 2xy^3\,dx + 3x^2y^2\,dy$

11. $dz = -2x(x^2+y^2)^{-2}\,dx - 2y(x^2+y^2)^{-2}\,dy$

13. $du = e^x \cdot (\cos(xy) - y\cdot\sin(xy))\,dx - x\cdot e^x\cdot\sin(xy)\,dy$

15. $dw = 2xy\,dx + (x^2+2yz)\,dy + y^2\,dz$

17. $dw = (x^2+y^2+z^2)^{-1}(x\,dx + y\,dy + z\,dz)$

19. $\Delta z = 0.9225$ and $dz = 0.9$

21. 2.9923.

23. –0.28

25. 5.4 cm^2

27. 16 cm^3

29. 150

13.5 Selected Answers

1. $-4+\sqrt{3}$ 3. 1

5. (a) $\nabla f(x,y) = \langle 3x^2 - 8xy, -4x^2 + 2y \rangle$ (b) $\langle 0, -2 \rangle$ (c) $-8/5$

7. (a) $\nabla f(x,y,z) = \langle y^2z^3, 2xyz^3, 3xy^2z^2 \rangle$ (b) $\langle 4, -4, 12 \rangle$ (c) $20/\sqrt{3}$

9. 7/52 11. $29/\sqrt{13}$

13. 1/6 15. $\sqrt{5}$, $\langle 1/\sqrt{5}, 2/\sqrt{5} \rangle$

17. $\sqrt{17}$ / 6 , $\langle 4/\sqrt{17}, 1/\sqrt{17} \rangle$

19. $\sqrt{(13/2)}$, $\langle -3/\sqrt{13}, -2/\sqrt{13} \rangle$

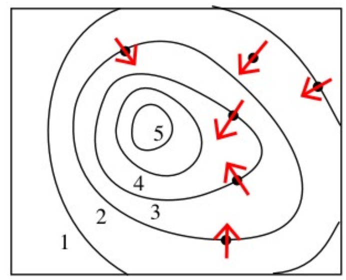


Fig. 10

21, See Fig. 10. Note that each gradient vector is perpendicular to the level curve and points uphill.

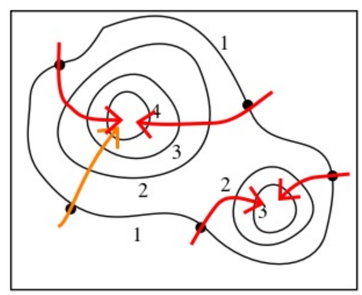


Fig. 11

23. See Fig. 11. Note that "uphill gradient" path is always perpendicular to the level curves.

27. (a) $-40/(3\sqrt{3})$

29. (a) $32/\sqrt{3}$ (b) $u = \langle 38, 6, 12 \rangle/(2\sqrt{406})$ (c) $2\sqrt{406}$

31. 327/13

13.6 Selected Answers

1. Minimum $f(-2,3) = -13$ 3. Minimum $f(0,-1) = -1$

5. Local maximum: f(1,0)=7. Saddle points at (-1,0), (2,3) and (2,-3)

7. Local minimum: $f(0,0) = 4$. Saddle points: $(\pm\sqrt{2}, -1)$

9. Local minimum: $f(1,1) = -1$. Saddle point $f(0,0) = 0$

11. Saddle point $f(1,2) = -2$

13. Local maximum $f(-1/2, 4) = -6$ 15. None

17. Local maximum $f(0,0) = 2$, local minimum $f(0,2) = -2$, saddle points $(\pm 1, 1)$

19. Saddle points $(0, n\pi)$, n and integer

21. Minimum $f(4,0) = -7$, maximum $f(4,5) = 13$

23. Maximum $f(-1,1) = f(1,1) = 7$, minimum $f(0,0) = 4$

25. Critical points: f1,1)=0, f(-2,4)=- -9 minimum, f(2,4)= 3 maximum, f(-1/3, 1/9) = 32/27

27. Critical points: f(0,0) = 30, **f(1,1)=f(–1,1)=31=maximum**, f(–2,4)=f(2,4)=22, **f(0,4) = 14 minimum**

29. Critical points: f(1,3)=17, f(–2,0)=11, f(0,4) = 19 maximum, f(–3/2, 1/2) = 43/4 minimum

31. Critical points: f(0,0)=0 minimum, $f(0, \pm 1) = 1$, $f(\frac{1}{2}, \pm\frac{\sqrt{3}}{2}) = \frac{85}{16}$ maximum

33. (2/7, 4/7, 6/7)

35. $\frac{100}{3}$, $\frac{100}{3}$, $\frac{100}{3}$

37. $16/\sqrt{3}$

39. 4/3

41. Cube, edge length c/12

13.7 Selected Answers

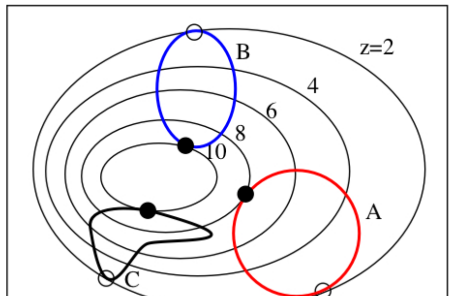


Fig. 8

1. See Fig. 8. The solid circles marks the locations of the maximum z values along each path, and the open circles mark the locations of the minimums, A: max z=8, min z=2. B: max z=10, min z=2, C: max z=10, min z=2.

3. maximum f(±1,0) = 1, minimum f(0,±1) = – 1

5. maximum $f(\sqrt{2}/3, \sqrt{2}) = f(-\sqrt{2}/3, -\sqrt{2}) = 2/3$, minimum $f(\sqrt{2}/3, -\sqrt{2}) = f(-\sqrt{2}/3, \sqrt{2}) = -2/3$

7. maximum $f(1/\sqrt{35}, 3/\sqrt{35}, 5/\sqrt{35}) = \sqrt{35}$, minimum $f(-1/\sqrt{35}, -3/\sqrt{35}, -5/\sqrt{35}) = -\sqrt{35}$

9. $x = \pm\sqrt{2}$, $y = \pm 1$, $z = \pm\sqrt{\frac{2}{3}}$. maximum f is $2/\sqrt{3}$ (when all are positive or one is positive and two are negative), minimum f is $-2/\sqrt{3}$.

11. maximum is $\sqrt{3} = f\left(\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}\right)$, minimum is $1 = f(\pm 1,0,0) = f(0,\pm 1,0) = f(0,0,\pm 1)$

13. V=xyz with xy+2xz+2yz=A. Maximum V is $\frac{1}{2}\left(\frac{A}{3}\right)^{3/2}$ and that occurs when $x = y = \sqrt{\frac{A}{3}}$ and $z = \frac{1}{2}\sqrt{\frac{A}{3}}$.

15. V=xyz with 5xy + 1(2xz + 2yz) = 1500 (working in cents). Maximum volume is 2500 in^3 when x = y = 10 inches and z = 25 inches. Note that the cost of the bottom is \$5.00, the total cost of the two ends is \$5.00, and the total costs of the other two sides is \$5.00 .

16. V = xyz with Bxy + S2xz + 2Syz = T, $x = y = \sqrt{\frac{T}{3B}}$, $z = \frac{1}{2S}\sqrt{\frac{BT}{3}}$ and maximum volume is $V = \frac{T}{3B} \cdot \frac{1}{2S} \cdot \sqrt{\frac{BT}{3}}$. The cost of the bottom is T/3.

17. maximum volume is $64/\sqrt{\pi}$ when $r = 4/\sqrt{\pi}$ and $h = 4/\sqrt{\pi}$.

13.8 Selected Answers

1. (a) When t = 1, x=2 and y=3 so $\frac{\partial f}{\partial x} = 11$ and $\frac{\partial f}{\partial y} = 6$. Also $\frac{dx}{dt} = -1$ and $\frac{dy}{dt} = -3$.

Then $\frac{df}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt} = (11) \cdot (-1) + (6) \cdot (-3) = -29$.

(b) When t = 3, x=1 and y=0 so $\frac{\partial f}{\partial x} = 1$ and $\frac{\partial f}{\partial y} = 6$. Also $\frac{dx}{dt} = -2$ and $\frac{dy}{dt} = -1$.

Then $\frac{df}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt} = (1) \cdot (-2) + (6) \cdot (-1) = -8$.

3. When t = π, then x = cos(π)= -1, y = sin(π)= 0, $\frac{dx}{dt} = -\sin(t) = -\sin(\pi) = 0$,

$\frac{dy}{dt} = \cos(t) = \cos(\pi) = -1$, $\frac{\partial f}{\partial x} = 2x = -2$, and $\frac{\partial f}{\partial y} = 2y = 0$ so

$\frac{df}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt} = (-2) \cdot (0) + (0) \cdot (-1) = 0$

5. When t = 2, x = 7, y = 4, z = 10, $\frac{dx}{dt} = 2$, $\frac{dy}{dt} = t^2 = 4$, $\frac{dz}{dt} = 5$,

$\frac{\partial f}{\partial x} = 2xy + z = 66$, $\frac{\partial f}{\partial y} = x^2 + z = 59$ and $\frac{\partial f}{\partial z} = y + x = 11$.

$\frac{df}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial f}{\partial z} \cdot \frac{dz}{dt}$

$= (66) \cdot (2) + (59) \cdot (4) + (11) \cdot (5) = 423$

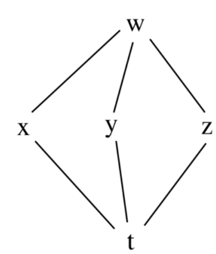


Fig. Problems 5 and 7

7. When t = 1, then $x = \ln(2)$, $y = \tan^{-1}(1) = \frac{\pi}{4}$, $z = e$, $\frac{dx}{dt} = \frac{2t}{1+t^2} = 1$, $\frac{dy}{dt} = \frac{1}{1+t^2} = \frac{1}{2}$,

$\frac{dz}{dt} = e^t = e$, $\frac{\partial f}{\partial x} = 2ye^x = 2 \cdot \frac{\pi}{4} \cdot e^{\ln(2)} = \pi$, $\frac{\partial f}{\partial y} = 2e^x = 2e^{\ln(2)} = 4$ and $\frac{\partial f}{\partial z} = -\frac{1}{z} = -\frac{1}{e}$.

$\frac{df}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial f}{\partial z} \cdot \frac{dz}{dt} = (\pi) \cdot (1) + (4) \cdot \left(\frac{1}{2}\right) + \left(-\frac{1}{e}\right) \cdot (e) = \pi + 1$

9. $w = xy + yz + xz$. When (u, v) = (–2, 0) then x = -2, y = -2, z = 0,

$$\frac{dx}{du} = 1,\ \frac{dx}{dv} = 1,\ \frac{dy}{du} = 1,\ \frac{dy}{dv} = -1,\ \frac{dz}{du} = v = 0,\ \frac{dz}{dv} = u = -2$$

$$\frac{\partial w}{\partial x} = y + z = -2,\ \frac{\partial w}{\partial y} = x + z = -2,\ \frac{\partial w}{\partial z} = y + x = -4$$

$$\frac{dw}{du} = \frac{\partial w}{\partial x}\cdot\frac{dx}{du} + \frac{\partial w}{\partial y}\cdot\frac{dy}{du} + \frac{\partial w}{\partial z}\cdot\frac{dz}{du} = (-2)(1) + (-2)(1) + (-4)(0) = -4$$

$$\frac{dw}{dv} = \frac{\partial w}{\partial x}\cdot\frac{dx}{dv} + \frac{\partial w}{\partial y}\cdot\frac{dy}{dv} + \frac{\partial w}{\partial z}\cdot\frac{dz}{dv} = (-2)(1) + (-2)(-1) + (-4)(-2) = 8$$

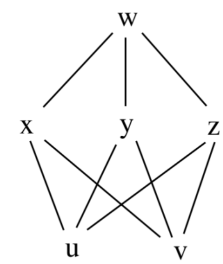


Fig. Problem 9

11. $z = \cos(xy) + x\cdot\sin(y),\ x = u + v + 2,\ y = uv$.

When u = 0 and v = 0 then x = 2, y = 0 and z = 1.

$$\frac{dx}{du} = 1,\ \frac{dx}{dv} = 1,\ \frac{dy}{du} = v = 0,\ \frac{dy}{dv} = u = 0$$

$$\frac{\partial z}{\partial x} = -y\cdot\sin(xy) + \sin(y) = 0,\ \frac{\partial z}{\partial y} = -x\cdot\sin(xy) + x\cdot\cos(y) = 2$$

$$\frac{dz}{du} = \frac{\partial z}{\partial x}\cdot\frac{dx}{du} + \frac{\partial z}{\partial y}\cdot\frac{dy}{du} = (0)(1) + (2)(0) = 0$$

$$\frac{dz}{dv} = \frac{\partial z}{\partial x}\cdot\frac{dx}{dv} + \frac{\partial z}{\partial y}\cdot\frac{dy}{dv} = (0)(1) + (2)(0) = 0$$

(That was a lot of work just to get a couple 0s.)

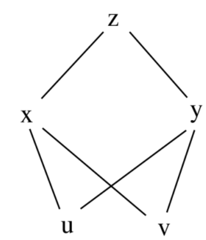


Fig. Problem 11

13. We know that T = 310 K, $\frac{dT}{dt} = -0.2\ \frac{K}{s}$, V = 80 L and $\frac{dV}{dt} = 0.1\ \frac{L}{s}$.

$P = 8.31\frac{T}{V}$ so $\frac{\partial P}{\partial T} = \frac{8.31}{V}$ and $\frac{\partial P}{\partial V} = -8.31\frac{T}{V^2}$. By the Chain Rule

$$\frac{dP}{dt} = \frac{\partial P}{\partial T}\cdot\frac{\partial T}{\partial t} + \frac{\partial P}{\partial V}\cdot\frac{\partial V}{\partial t} = \left(\frac{8.31}{V}\right)\cdot\left(\frac{\partial T}{\partial t}\right) + \left(-\frac{8.31T}{V^2}\right)\cdot\left(\frac{\partial V}{\partial t}\right)$$

$$= \left(\frac{8.31}{80}\right)\cdot(-0.2) + \left(-\frac{8.31\cdot 310}{80^2}\right)\cdot(0.1) = -0.061\ \frac{kPa}{sec}$$