14.0 Odd Answers

1. v=(24)(54/6)=216 (actual value is 368 – find out how in section 14.1)
3. (a) v=(32)(64/8)=256 (b) v=(32)(112/8)=448 (actual = 1184/3 = 393.667)
5. sum=39 inches, avg=39/16=2.4375 inches= 0.203125 ft, total = (avg)(area)=125.075 ft^3
7. v=(16/4)(4)(5)=80 ft^3
9. v=(19/5)(PI*2^2)=(76/5)*Pi=47.752 f^3
11. v=(26.7/15)*90=160.2 m^3
13. crude avg ht= 2, so vol=(2)(6)(10)(Pi)/4=30Pi = 94.25 m^3
15. avg =30/6=5, v=5*6*4= 120

14.1 Odd Answers

1. $\frac{8}{3}y^3, \frac{1}{4}x^2$ 3. $e^{2+y} + e^y = e^y(e^2+1)$, $xe^{x+1} - xe^x = xe^x(e-1)$ 5. 60

7. $3+e^4+4\sin(1) \approx 60.96$ 9. 26 11. 32/3 13. $\frac{4}{15}(31-9\sqrt{3})$ 15. 6

17. Easier $\int y\cdot\sqrt{x^2+y^2}\ dy = \frac{1}{3}(x^2+y^2)^{3/2} + C(x)$

19. $\int \sin(x)\cdot e^{x+y}\ dy = \int \sin(x)\cdot e^x\cdot e^y\ dy = \sin(x)\cdot e^x\cdot e^y + C(x)$

21. $\int \sqrt{x+y^2}\ dx = \frac{2}{3}(x+y^2)^{3/2} + C(y)$ 23. $\int e^{(y^2)}\ dx = x\cdot e^{(y^2)} + C(y)$

25. $-\frac{585}{8}$ 27. $\frac{\pi}{12} - \frac{3}{2} + \sqrt{3} - \frac{\pi}{12}\sqrt{3} \approx 0.0404$ 29 $\frac{1}{6}(60) = 10$ 31. $\frac{1}{6}(26) = \frac{13}{3}$

33. One reasonable approximation is 33 m^3. Your approximation should be close to this number, and you should be able to justify why your method is reasonable.

35. Using midpoints, one reasonable estimate is 88 m^3.

14.2 Odd Answers

1. 9 3. 63 5. $\frac{1}{2} - \frac{1}{2}\cos(1)$

7. $\int_{x=-2}^{x=2}\int_{y=0}^{y=4-x^2} f\ dy\ dx$, $\int_{y=0}^{y=4}\int_{x=-\sqrt{4-y}}^{x=\sqrt{4-y}} f\ dx\ dy$ 9. $\int_1^4\int_x^4 f\ dy\ dx$, $\int_1^4\int_1^y f\ dx\ dy$

11. $\int_0^2\int_{x^2}^{6-x} f\ dy\ dx$, $\int_0^4\int_0^{\sqrt{y}} f\ dx\ dy + \int_4^6\int_0^{6-y} f\ dx\ dy$

13. $\frac{1}{12}$ 15. $\frac{3}{4}$ 17. $\frac{32}{3}$ 19. $-\frac{72}{5}$

21. avg. value = volume/area = $\frac{44}{4} = 11$ 23. volume= $\frac{45}{4}$. area = $\frac{9}{2}$, avg. value = $\frac{5}{2}$

25. $\int_0^1 \int_y^1 f(x,y)\ dx\ dy$ 27. $\int_0^3 \int_0^{6-2x} f(x,y)\ dy\ dx$ 29. $\int_0^{\ln(2)} \int_{e^y}^2 f(x,y)\ dx\ dy$

31. $\int_0^4 \int_0^{2-y/2} f\ dx\ dy$ 33. $\int_0^2 \int_0^{y^2} f\ dx\ dy$ 35. $\int_1^e \int_{\ln(y)}^1 f\ dx\ dy$

37. One reasonable approximation of the volume is 24 m^3. Your approximation should be close to this.

14.3 Odd Answers

1. A: rectangular B: polar C: polar

3. A: rectangular B: rectangular (since the circle is not centered at the origin) C: polar

5. 63π 7. $A\cdot D^2\cdot \pi$

9. V= $\iint_R \sqrt{9-x^2-y^2}\ dA$ where $R=\{(x,y):\ x^2+y^2\le 9\}$.

$$\text{V}=\int_{\theta=0}^{2\pi}\int_{r=0}^{3}\sqrt{9-\text{r}^2}\ \text{r}\cdot\text{dr}\cdot\text{d}\theta=\int_{\theta=0}^{2\pi} 9\ \text{d}\theta=18\pi\ .$$

11. V= $\iint_R 2+\text{xy dA}$ where R is the region in inside the circle $x^2+y^2=4$ and above the x-axis..

$$\text{V}=\int_{\theta=0}^{\pi}\int_{r=0}^{2}\{2+r\cdot\cos(\theta)\cdot r\cdot\sin(\theta)\}\cdot r\cdot dr\cdot d\theta=\int_{\theta=0}^{\pi}\{4+4\cos(\theta)\sin(\theta)\}\ d\theta=4\pi\ .$$

13. Under the plane $z=5+2x+3y$ and above the disk $x^2+y^2\le 16$.

$$\text{V}=\int_{\theta=0}^{2\pi}\int_{r=0}^{4}\{5+2\cos(\theta)+3\sin(\theta)\}\ r\cdot dr\cdot d\theta=\int_{\theta=0}^{2\pi}\{40+16\cos(\theta)+24\sin(\theta)\}\ d\theta=80\pi$$

15. Between the surfaces $z=1+x+y$ and $z2=8+2x+3y$ for $x^2+y^2\le 1$ and $0\le$ y.

$$V=\iint_R\{\text{z2}-\text{z1}\}\ \text{dA}=\int_{\theta=0}^{\pi}\int_{r=0}^{1}\{z2-z1\}\ dA=\int_{\theta=0}^{\pi}\int_{r=0}^{1}\{7+x+2y\}\ r\cdot dr\cdot d\theta$$

$$=\int_{\theta=0}^{\pi}\int_{r=0}^{1}\{7+r\cdot\cos(\theta)+2r\cdot\sin(\theta)\}\ r\cdot dr\cdot d\theta=\int_{\theta=0}^{\pi}\left\{\frac{7}{2}+\frac{1}{3}\cos(\theta)+\frac{2}{3}\sin(\theta)\right\}\ d\theta=\frac{4}{3}+\frac{7}{2}\pi$$

17. Between the surface $z=f(x,y)=\dfrac{1}{1+x^2+y^2}$ and the xy-plane for (x,y) in the first quadrant and $1\le x^2+y^2\le 4$. The domain is $R=\{(x,y):1\le x^2+y^2\le 4\}=\left\{(r,\theta):1\le r\le 2\ \ 0\le\theta\le\frac{\pi}{2}\right\}$.

$$\text{Volume}=\iint_R \text{f}\cdot\text{dA}=\int_{\theta=0}^{\pi/2}\int_{r=1}^{2}\frac{1}{1+r^2}\cdot r\cdot dr\cdot d\theta$$

$$\int_{r=1}^{2} \frac{1}{1+r^2} \cdot r \cdot dr = \frac{1}{2}\ln(1+r^2)\Big|_{r=1}^{r=2} = \frac{1}{2}\ln\left(\frac{5}{2}\right) \text{ and } \int_{\theta=0}^{\pi/2} \frac{1}{2}\ln\left(\frac{5}{2}\right)\, d\theta = \frac{\pi}{2}\cdot\frac{1}{2}\ln\left(\frac{5}{2}\right) \approx 0.72 .$$

19. $\int_0^1 \int_0^{\pi} r \; d\theta \; dr = \frac{\pi}{2}$

21. $\int_0^1 \int_0^{\pi/2} r\cdot \sin(\theta)\cdot r \; d\theta \; dr = \int_0^1 r^2 \; dr = \frac{1}{3}$

23. $\int_0^{\pi/2} \int_0^1 e^{-r^2}\cdot r \; dr \; d\theta = \int_0^{\pi/2} \frac{1}{2}\left(1-\frac{1}{e}\right) d\theta = \frac{\pi}{4}\left(1-\frac{1}{e}\right)$

25. $f(x,y) = 7+3x+2y$ with $R = \left\{(x,y):\ x^2+y^2 \le 9\right\}$. (from Problem 5)

Area of R = 9π and $\iint_R$ f dA = 63π (from Problem 5) so the average value of f on R is 7.

27. $f(x,y) = 5+2x+3y$ with $R = \left\{(x,y):\ x^2+y^2 \le 16\right\}$. (from Problem 13)

Area of R = 16π and $\iint_R$ f dA = 80π (from Problem 5) so the average value of f on R is 5 .

29. A sprinkler (located at the origin) sprays water so after one hour the depth at location (x,y) feet is

$f(x,y) = K\cdot e^{-(x^2+y^2)}$ feet.

(a) How much water reaches the annulus $2 \le r \le 4$ and the annulus $8 \le r \le 10$ in one hour?

From Practice 2 we know that the total amount of the water in a circle of radius A is

$K\cdot\left(1-e^{-A^2}\right)\pi$. So the amount in the annulus $2 \le r \le 4$ is

$K\cdot\left(1-e^{-16}\right)\pi - K\cdot\left(1-e^{-4}\right)\pi = K\cdot\left(e^{-4}-e^{-16}\right)\pi$ and the amount in the annulus

$8 \le r \le 10$ is $K\cdot\left(e^{-64}-e^{-100}\right)\pi$.

(b) The area of the $2 \le r \le 4$ annulus is $(4^2-2^2)\pi = 12\pi$ square feet, and the area of the $8 \le r \le 10$

annulus is 36π square feet. The average depth for the first annulus is $\frac{K\cdot\left(e^{-4}-e^{-16}\right)\pi}{12\pi} \approx 0.00153K$

and it is $\frac{K\cdot\left(e^{-64}-e^{-100}\right)\pi}{36\pi} \approx 4.5\cdot 10^{-30}\cdot K$ (almost no water) for the second annulus.

14.4 Odd Answers

Author confession: Many of these integrals are very messy and take a long time. I used Maple to evaluate them.

1. Area = $\iint_R 1 \; dA = \int_{x=0}^{2}\int_{y=0}^{4} 1 \; dy \; dx = \int_{x=0}^{2} 4 \; dx = 8$ = M

$$M_x = \iint_R y\cdot\delta \; dA = \int_{x=0}^{2}\int_{y=0}^{4} y\cdot(1) \; dy \; dx = \int_{x=0}^{2} 8 \; dx = 16$$

$$M_y = \iint_R x\cdot\delta \; dA = \int_{x=0}^{2}\int_{y=0}^{4} x\cdot(1)\; dy\; dx = \int_{x=0}^{2} 4x\; dx = 4, \text{ and } \bar{x}=1, \quad \bar{y}=2$$

3. $$\text{Area} = \iint_R 1\; dA = \int_{x=0}^{2}\int_{y=0}^{x^2} 1\; dy\; dx = \int_{x=0}^{2} x^2\; dx = \frac{8}{3}$$

$$\text{M} = \iint_R \delta\; dA = \int_{x=0}^{2}\int_{y=0}^{x^2} (1+x)\; dy\; dx = \int_{x=0}^{2} x^3 + x^2\; dx = \frac{20}{3}$$

$$M_x = \iint_R y\cdot\delta\; dA = \int_{x=0}^{2}\int_{y=0}^{x^2} y\cdot(1+x)\; dy\; dx = \int_{x=0}^{2} \frac{1}{2}(x^4+x^5)\; dx = \frac{128}{15}$$

$$M_y = \iint_R x\cdot\delta\; dA = \int_{x=0}^{2}\int_{y=0}^{x^2} x\cdot(1+x)\; dy\; dx = \int_{x=0}^{2} (x^3+x^4)\; dx = \frac{52}{5}$$

$$\bar{x} = \frac{M_y}{\text{M}} = \frac{39}{25} \text{ and } \bar{y} = \frac{M_x}{\text{M}} = \frac{32}{25}$$

5. $\text{Area} = \frac{15}{2}$, $M = 24$, $M_x = \frac{341}{8}$, $M_y = \frac{405}{8}$, $\bar{x} = \frac{135}{64}$, $\bar{y} = \frac{341}{192}$

7. $$\text{Area} = \iint_R 1\; dA = \int_{\theta=0}^{\pi/3}\int_{r=0}^{\sin(3\theta)} 1\; r\; dr\; d\theta = \frac{\pi}{12} = M = \text{mass} ,$$

$$M_x = \iint_R y\cdot\delta\; dA = \int_{\theta=0}^{\pi/3}\int_{r=0}^{\sin(3\theta)} r\cdot\sin(\theta)\; r\; dr\; d\theta = \frac{27}{640}\sqrt{3} \approx 0.07307$$

$$M_y = \iint_R x\cdot\delta\; dA = \int_{\theta=0}^{\pi/3}\int_{r=0}^{\sin(3\theta)} r\cdot\cos(\theta)\; r\; dr\; d\theta = \frac{81}{640} \approx 0.1266$$

$$\bar{x} = \frac{M_y}{M} = \frac{243}{160\pi} \approx 0.4834, \quad \bar{y} = \frac{M_x}{M} = \frac{81\sqrt{3}}{160\pi} \approx 0.2791$$

9. $Area = \frac{3}{4}\pi$, $M = \frac{5}{6}\pi$, $M_x = \frac{8}{5}$, $M_y = \frac{7}{8}\pi$, $\bar{x} = \frac{21}{20} \approx 1.05$, $\bar{y} = \frac{48}{25\pi} \approx 0.611$

11. $\text{M} = 8, \quad \text{I}_\text{x} = \frac{128}{3}, \quad \text{R}_\text{x} = \frac{4}{3}\sqrt{3}$

13. $\text{M} = 16, \quad \text{I}_\text{x} = 128, \quad \text{R}_\text{x} = 2\sqrt{2}$

15. The density of the bar is 4 kg/m. The force between the point mass and a small Δx piece of the bar at location x is $\text{f}_\text{i} = \text{GM}\frac{4\cdot\Delta x}{x^2}$. Forming the usual Riemann sum of the little forces and taking the limit,

total force = $\text{G}\cdot 10\cdot\int_2^4 \frac{4}{x^2}\; \text{dx}$,

17. The density is $k = 5$ kg/m for the first bar and $K = 3$ kg/m for the second bar. The force between a small Δx piece of the first bar at location x and a small Δy piece of the second bar at location y is $f_{ij} = G \cdot \frac{(5\Delta x)(3\Delta y)}{(y-x)^2}$.

Total force $= G \cdot \int_4^7 \int_0^2 \frac{15}{(y-x)^2}\, dx\, dy$

14.5 Odd Answers

1. $SA = \int_0^2 \int_0^{2x} \sqrt{1+(2x)^2+1}\; dy\; dx = \int_0^2 2x\sqrt{4x^2+2}\; dx = \frac{26}{3}\sqrt{2}$

3. $SA = \int_0^4 \int_0^{y/2} \sqrt{1+(4)^2+(2y)^2}\; dx\; dy = \int_0^4 \frac{y}{2}\sqrt{4y^2+17}\; dx = \frac{243}{8} - \frac{17}{24}\sqrt{17}$

5. $SA = \int_{-2}^{2} \int_0^{4-x^2} \sqrt{26}\; dy\; dx = \int_{-2}^{2} (4-x^2)\sqrt{26}\; dx = \frac{32}{3}\sqrt{26}$

7. $SA = \iint_R \sqrt{26}\; dA = 9\pi \cdot \sqrt{26}$ since R is a circle of radius 3

or $SA = \int_{\theta=0}^{2\pi} \int_{r=0}^{3} \sqrt{26} \cdot r\, dr\, d\theta = \int_{\theta=0}^{2\pi} \frac{9}{2}\sqrt{26}\; d\theta = 9\sqrt{26} \cdot \pi$

9. $8\pi\sqrt{3}$

11. R is an annulus ($1 \le r \le 3$). $SA = \int_{\theta=0}^{2\pi} \int_{r=1}^{3} \sqrt{2} \cdot r\, dr\, d\theta = 9\pi\sqrt{2} - 1\pi\sqrt{2} = 8\pi\sqrt{2}$

13. $SA = \int_{-2}^{2} \int_{x^2}^{4} \sqrt{1+(y^2)^2+(2xy)^2}\; dy\; dx$

15. $SA = \int_0^2 \int_0^3 \sqrt{1+\cos^2(x)+\sin^2(y)}\; dy\; dx$

14.6 Odd Answers

1. $\int_{x=0}^{4} \int_{y=0}^{2} \int_{z=0}^{4-2y} f\, dz\, dy\, dx$

3. $\int_{x=-2}^{2} \int_{y=-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{z=\sqrt{x^2+y^2}}^{4} f\, dz\, dy\, dx$

5. $\int_{x=0}^{2} \int_{y=0}^{4-x^2} \int_{z=0}^{16-4x^2-y^2} f\, dz\, dy\, dx$

7. 7/6

9. -5

11. 24

13. $(e^3 - 1)/6$ 15. 16/3 17. 144

19. int(2*x+3, y=0..2*x, x=0..2, z=0..3); 21. int(x*y*z, z=0..8-2*y, y=x..4, x=0..4);

14.7 Odd Answers

1. $\frac{\pi}{3}\left(2\sqrt{2}-1\right)$ 3. $\frac{23}{24}\pi$ 5. $\frac{1}{54}\pi^5$ 7. 8π

9. $\int_0^4 \int_{\pi/4}^{\pi/2} \int_0^1 r \cdot e^{-r^2} \; dr \cdot d\theta \cdot dz = \frac{\pi}{2}\left(1-\frac{1}{e}\right)$ 11. $\int_0^1 \int_0^{\pi/2} \int_0^{4-r\cdot\sin(\theta)} r \; dz \cdot d\theta \cdot dr = \pi - \frac{1}{3}$

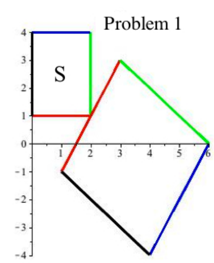
13. $\iiint_R \text{f dV} = \int_0^{2\pi} \int_0^3 \int_3^5 r \cdot r \; dz \cdot dr \cdot d\theta = 36\pi$

15. $\iiint_R \text{f dV} = \int_0^{2\pi} \int_0^{\sqrt{7}} \int_0^{1+r^2} \left(e^z\right) \cdot r \; dz \cdot dr \cdot d\theta = 2\pi\left(\frac{1}{2}e^8 - \frac{1}{2}e - \frac{7}{2}\right)$

17. $\iiint_R \text{f dV} = \int_0^{\pi/2} \int_0^3 \int_{r^2}^{36-3r^2} r \; dz \cdot dr \cdot d\theta = \frac{81}{2}\pi$

19. 0 21. $\frac{5}{3}\pi$ 23. $\frac{7}{3}\pi$ 25. $\frac{31}{160}\pi$

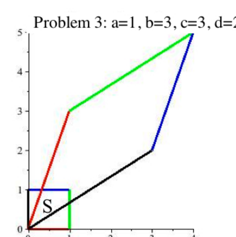
14.8 Odd Answers



1. $S = \{(u,v):\; 0 \le u \le 2,\; 1 \le v \le 4\}$ under $x = u + v$ and $y = 2u - v$.

$J(u,v) = \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = -3, \quad J(x,y) = -\frac{1}{3}$

$\iint_R \text{f(x,y) dx dy} = \int_1^4 \int_0^2 f(u+v,\, 2u-v)(3) \; du\, dv$



3. $S = \{(u,v):\; 0 \le u \le 1,\; 0 \le v \le 1\}$ under

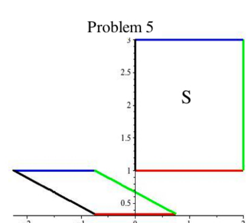
$x = au + bv$ and $y = cu + dv$.

$J(u,v) = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc, \quad J(x.y) = \frac{1}{ad - bc}$

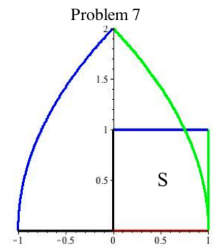
$\iint_R \text{f(x,y) dx dy} = \int_0^1 \int_0^1 f(au+bv,\, cu+dv)\, |\, ad - bc \,|\; du\, dv$

5. $S = \{(x,y):\; 0 \le x \le 2,\; 1 \le y \le 3\}$ under $u = \frac{3x - 3y}{4}$ and $v = \frac{y}{3}$.

Then x=4u/3+3v and y=3v. $J(x,y) = \begin{vmatrix} \frac{3}{4} & -\frac{3}{4} \\ 0 & \frac{1}{3} \end{vmatrix} = \frac{1}{4}, \quad J(u,v) = 4$



7. $S = \{(x,y):\ 0 \le x \le 1,\ 0 \le y \le 1\}$ under $u = x^2 - y^2$ and $v = 2xy$.

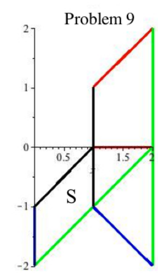


$J(x,y) = \begin{vmatrix} 2x & -2y \\ 2y & 2x \end{vmatrix} = 4x^2 + 4y^2,\quad J(u,v) = \dfrac{1}{4x^2 + 4y^2}$

(It is difficult/impossible to solve for x and y.)

9. R = {trapezoid with vertices (x,y) = (1,0), (2,0), (0,-2) and (0,-1)}

under u=x-y and v=x+y . Then $x = \dfrac{u+v}{2}$ and $y = \dfrac{-u+v}{2}$.



$J(u,v) = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 2,\quad J(x,y) = \dfrac{1}{2}$

11. $\iint\limits_R 1\ dV = \iint\limits_S 1 \cdot |J(u,v)|\,du\ dv = \iint\limits_S 7\ du\ dv = 7(14) = 98.$

13. $J(x,y) = 3$ so $J(u,v) = \dfrac{1}{3}$. Then

$\iint\limits_R 1\ dV = \iint\limits_S 1 \cdot |J(u,v)|\,dV \iint\limits_S \dfrac{1}{3}\ dV = \dfrac{1}{3}(15) = 5.$

15. The new uv domain is a triangle with vertices (0,), (1,0) and (0,1). $J(u,v) = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 3$.

$\int_0^1 \int_0^{1-u} [(2u+v) + 3(u+2v)] \cdot 3\ dv\ du = 6$.

17. Using the substitution from Fig. 4, u=x+y, v=x-2y, so $J(x,y) = \begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} = -3$ and $J(u,v) = -\dfrac{1}{3}$.

$\int_3^6 \int_0^6 \left[\dfrac{u}{3}\right] \cdot \dfrac{1}{3}\ du\ dv = \int_3^6 2\ dv\ du = 6$ (Note: The substitution u=(x+y)/3 and v=(x–2y) from example

leads to the same integral value since with this substitution u goes from 0 to 2 and v from 1 to 2.)

19. u=y–x v=2x+y so $J(x,y) = \begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} = -3$ and $J(u,v) = -\dfrac{1}{3}$.

x=(v–u)/3 y=(2u+v)/3 so 3x+6y=3u+3v. $\int_6^9 \int_0^2 [3u + 3v] \cdot \dfrac{1}{3}\ du\ dv = \int_6^9 (2 + 2v)\ dv = 51$

21. This is from Problem 8: $x = u/v$ and $y = uv$

$J(u,v) = \begin{vmatrix} \dfrac{1}{v} & -\dfrac{u}{v^2} \\ v & u \end{vmatrix} = \dfrac{2u}{v}$ Then $u = \sqrt{xy},\ v = \sqrt{\dfrac{y}{x}}$ with u=1 to 2 and v=1 to 2.

$\int_1^2 \int_1^2 [v + u] \cdot \left(\dfrac{2u}{v}\right) du\ dv = \int_1^2 \int_1^2 \left[2u + \dfrac{2u^2}{v}\right] du\ dv = \int_1^2 3 + \dfrac{14}{3}\dfrac{1}{v}\ dv = 3 + \dfrac{14}{3}\ln(2)$.

(Note: See the Two Transformations, Same Result page at the end of the problem answers.)

23. R is the region bounded by the ellipse. If x=au and y=bv then the uv shape is $u^2 + v^2 \le 1$, a circle

of radius 1 and area π. $J(u,v) = \begin{vmatrix} a & 0 \\ 0 & b \end{vmatrix} = ab$ so $\iint\limits_{\text{ellipse}} 1\ dx\ dy = \iint\limits_{\text{circle}} ab\ du\ dv = ab \cdot \iint\limits_{\text{circle}} 1\ du\ dv = ab\pi$