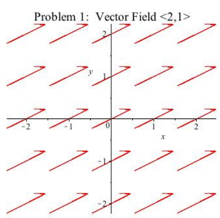
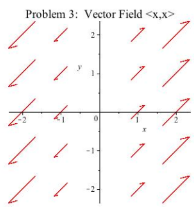
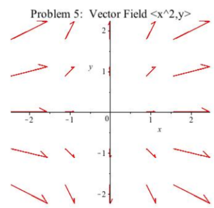
15.0 Odd Answers

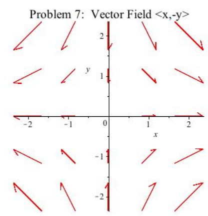
1. A small eddy or whirlpool.
3. Easier from D to San Juan Island.
5. Easier from A to B.
7. Easier from E to F.

15.1 Odd Answers



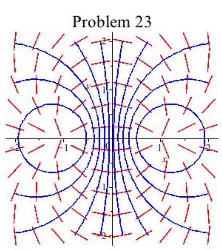




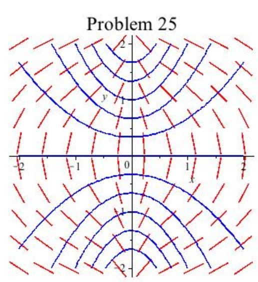


9. a-C, b-A, c-B
11. a-B, b-A, c-C
13. $\mathbf{F}(x,y) = \langle 2x, -2y \rangle$
15. $\mathbf{F}(x,y) = \langle y^2 - 2x \cdot y, 2x \cdot y - x^2 \rangle$
17. $\mathbf{F}(x,y,z) = \langle 3, 2, -1 \rangle$
19. $\mathbf{F}(x,y,z) = \langle x, y, z \rangle / \sqrt{x^2 + y^2 + z^2}$
21. $\mathbf{F}(x,y,z) = \langle y \cdot z \cdot \cos(x \cdot y \cdot z), x \cdot z \cdot \cos(x \cdot y \cdot z), x \cdot y \cdot \cos(x \cdot y \cdot z) \rangle$

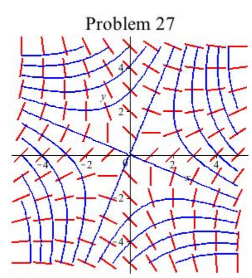
23.



25.



27.



15.2 Odd Answers

1. At A div $\mathbf{F} > 0$, at B div $\mathbf{F} < 0$, at C div $\mathbf{F} = 0$.
3. div $\mathbf{F}$ = $2x + 2$. At A div $\mathbf{F} = 4$, at B div $\mathbf{F} = 6$ and at C div $\mathbf{F} = 0$.
5. div $\mathbf{F}$ = $5 + 2$. At A div $\mathbf{F} = 7$, at B div $\mathbf{F} = 7$ and at C div $\mathbf{F} = 7$.
7. See Fig. 12. More going out than coming in.

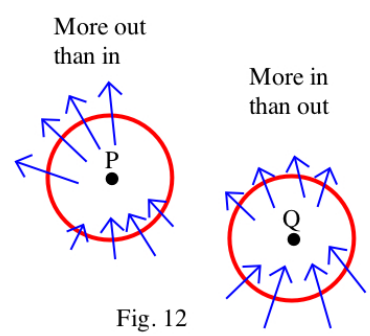


Fig. 12

9. At A curl $\mathbf{F} > 0$, at B curl $\mathbf{F} < 0$, at C curl $\mathbf{F} = 0$.
11. curl $\mathbf{F}$ = $1-3$. At A curl $\mathbf{F} = -2$, at B curl $\mathbf{F} = -2$, at C curl $\mathbf{F} = -2$.
13. curl $\mathbf{F}$ = $1-(-3)$. At A curl $\mathbf{F} = 4$, at B curl $\mathbf{F} = 4$, at C curl $\mathbf{F} = 4$.
15. See Fig. 13. Enough to rotate counterclockwise.
17. All of the partial derivatives are 0 so both the div and the curl are 0
19. No. The curl could be anything. See Fig. 14.

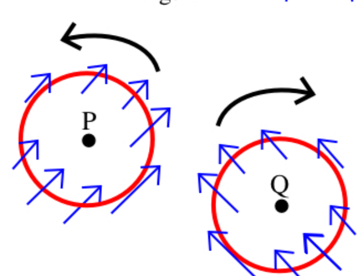
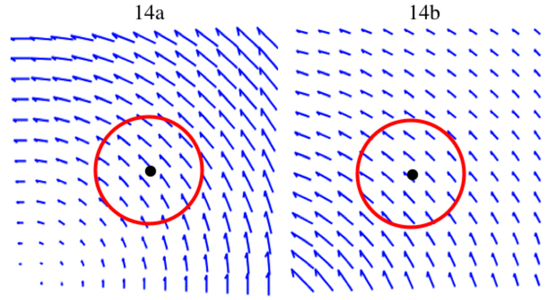


Fig. 13



In 14a the curl is positive. In 14b the curl is negative.

21. $\text{curl } \mathbf{F} = -\frac{\partial M}{\partial y}$ might be any value.

15.3 Odd Answers

1. $\int_0^3 \left((2t+1)^2 - 4(3+t^2)+11\right)\sqrt{(2)^2+(2t)^2}\,dt = \int_0^3 (4t)\sqrt{4+4t^2}\,dt = \frac{8}{3}\left(t^2+1\right)^{3/2}\Big|_0^3 = \frac{8}{3}\left(10\sqrt{10}-1\right)$

3. $\int_{t=a}^{t=b} y(t)\cdot\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2}\cdot dt$

4. $\int_{t=a}^{t=b} x(t)\cdot\sqrt{\left(\frac{dy}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2}\cdot dt$

5. $\text{mass} = \int_0^{\pi/2} (1+3\cos(t)+6\sin(t))(3)\,dt = 27+\frac{3}{2}\pi$

7. $\text{mass} = \int_0^2 (2+4t)(\sqrt{11})\,dt = 12\sqrt{11}$

9. $\int_0^2 (2t+9)(5)\,dt = 110$

11. $\int_0^3 (10t^2+10)(2t)\,dt = 495$

13. $\int_0^{\pi} (3-\sin(t)+\cos(t))(1)\,dt = -2+3\pi$

15. Along A work > 0. Along B work < 0.

17. Along A flow > 0. Along B flow > 0.

19. $\text{work} = \int_0^3 (14+37t)\,dt = 417/2$

21. $\text{work} = \int_0^{\pi} (-\sin(t)+2+3\cdot\cos(t))\,dt = 2\pi-2$

23. $\text{flow} = \int_1^2 (3t^2+32t)\,dt = 55$

25. $\text{flux} = \int_0^{2\pi} (2\cdot\cos(t))(2\cdot\cos(t)) - (1+4\cdot\sin(t))(-\sin(t))\,dt = 8\pi$

27. All of them are possible for the flux.

For 29 to 34 consider the integral formula used for each calculation.

29. Area is unchanged.

31. Sign of work is changed.

33. Sign of circulation is changed.

15.4 Odd Answers

1. $\mathbf{F} = \langle 2x, 6y\rangle$

3. $\mathbf{F} = \langle 3\cos(3x+2y), 2\cos(3x+2y)\rangle$

5. $\mathbf{F} = \left\langle \frac{2}{2x+5y}, \frac{5}{2x+5y}+e^y\right\rangle$

7. $f(x,y) = x^2+y^2 \quad f(5,1)-f(1,2) = 21$

9. $M_y = 0$, $N_x = 1$ so F is not conservative. $\mathbf{r}(t) = \langle 3t, 2+4t\rangle$ (t from 0 to 1).

$F(r(t))\bullet r'(t) = \langle 3t, 3t\rangle \bullet \langle 3, 4\rangle = 21t$ so $\text{work} = \int_{t=0}^{1} \mathbf{F}(\mathbf{r}(t))\bullet\mathbf{r}'(t)\,dt = \int_{t=0}^{1} 21t\,dt = \frac{21}{2}$

11. $f(x,y,z) = xyz \quad f(4,2,1)-f(1,0,0) = 8$

13. A to B: 5 C to D: 2 E to F: –3

15. A to B: 6 C to D: 0 E to F: -2

17. $f(x,y) = x^3+4x+6y$

19. $f(x,y) = \sin(xy) + x^3 y$

21. $f(x,y) = x \cdot \sin(y)$

23. $N_z \neq P_y$

15.5 Odd Answers

1. $M = x^2 y,\ N = 3y,\ \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = -x^2$ so $\iint_R -x^2\, dA = \int_0^1 \int_0^2 -x^2\, dx\, dy = -\frac{8}{3}$

3. $M = 3xy,\ N = 2x^2,\ \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = 4x - 3x$ so $\iint_R x\, dA = \int_0^1 \int_0^{2x} x\, dx\, dy = \frac{2}{3}$

5. $M = ax,\ N = by,\ \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = 0$ so $\iint_R 0\, dA = 0$

7. circulation $= \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA = \iint_R (-1-2)\, dA = \int_0^2 \int_0^2 -3\, dx\, dy = -12$

flux $= \iint_R \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right) dA = \iint_R (1+1)\, dA = \int_0^2 \int_0^2 2\, dx\, dy = 8$

9. circulation $= \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA = \iint_R (2x-2y)\, dA = \int_0^2 \int_0^x 2x - 2y\, dy\, dx = \frac{8}{3}$

flux $= \iint_R \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right) dA = \iint_R (2x-2y)\, dA = \int_0^2 \int_0^x 2x - 2y\, dy\, dx = \frac{8}{3}$

11. C_1 is from –2 to 2: $\mathbf{r}(t) = \langle t, t^2 \rangle$ so $\frac{1}{2}\int_{-2}^{2} (t)(2t) - (t^2)(1)\, dt = \frac{1}{2}\int_{-2}^{2} t^2\, dt = \frac{8}{3}$

C_2 is from 2 to –2: $\mathbf{r}(t) = \langle t, 8 - t^2 \rangle$ so $\frac{1}{2}\int_{2}^{-2} (t)(-2t) - (8-t^2)(1)\, dt = \frac{1}{2}\int_{-2}^{2} (-t^2 - 8)\, dt = \frac{56}{3}$

So the total area is $\frac{64}{3}$.

Note: $\int_{-2}^{2} ((8-x^2) - (x^2)\, dx$ is much easier. Green's Theorem does not make everything easier.

13. Call D the boundary of R so $\int_D \mathbf{F} \bullet d\mathbf{r} = \int_{C_1} \mathbf{F} \bullet d\mathbf{r} - \int_{C_2} \mathbf{F} \bullet d\mathbf{r} = \int_{C_1} \mathbf{F} \bullet d\mathbf{r} - 20$ ·

But $\int_D \mathbf{F} \bullet d\mathbf{r} = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA = \iint_R 5\, dA = 500$ so $\int_{C_1} \mathbf{F} \bullet d\mathbf{r} = 520$·

15. Call D the boundary of R so $\int_D \mathbf{F} \bullet d\mathbf{r} = \int_{C_1} \mathbf{F} \bullet d\mathbf{r} - \int_{C_2} \mathbf{F} \bullet d\mathbf{r} - \int_{C_3} \mathbf{F} \bullet d\mathbf{r} = \int_{C_1} \mathbf{F} \bullet d\mathbf{r} - 3\pi - 4\pi$ ∴

Area of R = $= 42 - 2\pi$.

But $\int_D \mathbf{F}\bullet d\mathbf{r} = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA = \iint_R 9\ dA = 9(42 - 2\pi) = 378 - 18\pi$ so $\int_{C_1} \mathbf{F}\bullet d\mathbf{r} = 378 - 11\pi$.

17. All of the partial derivatives are 0 so the double integrals for circulation and flux are 0.

19. $\text{circulation} = \oint_C M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA = \iint_R (c-b)\ dA = (c-b)\cdot(\text{area of R})$

15.6 Odd Answers

1. div $\mathbf{F} = 3xz^2 + 2xy + xz$, curl $\mathbf{F} = \langle -xy, -z^3, -x^2 + yz \rangle$

3. div $\mathbf{F} = e^z + z\cdot e^y$, curl $\mathbf{F} = \langle e^x - e^y,\ xe^z - ye^x,\ 0 \rangle$

5. div $\mathbf{F} = 3$, curl $\mathbf{F} = \langle -1,\ -1,\ -1 \rangle$

In problems 7 to 10, X = not meaningful.

7. div f =X , div F =scalar, div (curl F)=scalar , gradient (curl F)=X, curl F=vector

9. gradient f =vector, curl (curl F)=vector , curl (div (gradient f))=X

11. Not conservative.

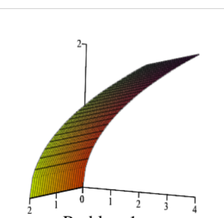
13. Not conservative.

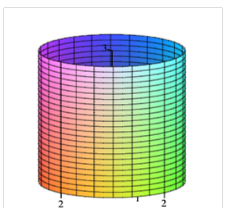
15. Conservative. $f(x,y,z) = x\cdot\sin(yz) + 2z$

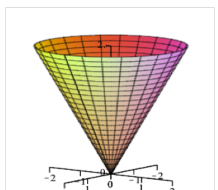
17. Your location = (5, 10, 5)= (1, 2, 3) + (**2**)(2, 4, 1) so you have moved in the direction of the curl. Looking back, you will see a Counterclockwise rotation.

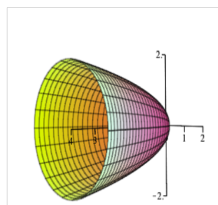
19. Your location = (4, 4, 3)= (6, 3, 2) + (**–1**)(2, –1, 1) so you have moved in the opposite direction of the curl. Looking toward (6, 3, 2), you will see a Clockwise rotation.

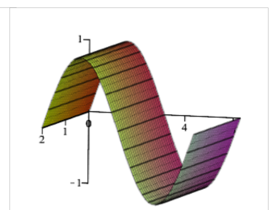
15.7 Odd Answers


Problem 1


Problem 3


Problem 5


Problem 7


Problem 9

11. $\mathbf{r}(u,v) = \langle u,\ v,\ \sqrt{v} \rangle$, $\mathbf{r}_u = \langle 1, 0, 0 \rangle$, $\mathbf{r}_v = \left\langle 0,\ 1,\ \frac{1}{2\sqrt{v}} \right\rangle$, $\mathbf{r}_u\mathbf{x}\mathbf{r}_v = \left\langle 0,\ \frac{1}{2\sqrt{v}},\ 1 \right\rangle$

Surface area = $\iint_R |\mathbf{r}_u\mathbf{x}\mathbf{r}_v|\ dA = \int_{u=0}^{2} \int_{v=0}^{4} \sqrt{\frac{1}{4v} + 1}\ dv\ du$.

13. $\mathbf{r}(u,v) = \langle 2\cdot\cos(u),\ 2\cdot\sin(u),\ v \rangle$, $\mathbf{r}_u = \langle -2\cdot\sin(u),\ 2\cdot\cos(u),\ 0 \rangle$, $\mathbf{r}_v = \langle 0, 0, 1 \rangle$,

$\mathbf{r}_u\mathbf{x}\mathbf{r}_v = \langle 2\cdot\cos(u),\ 2\cdot\sin(u),\ 0 \rangle$. Surface area = $\iint_R |\mathbf{r}_u\mathbf{x}\mathbf{r}_v|\ dA = \int_{u=0}^{2\pi} \int_{v=0}^{3} 2\ dv\ du$.

15. $r(u,v) = \langle u \cdot \cos(v),\ u \cdot \sin(v),\ u \rangle$, $\mathbf{r}_u = \langle \cos(v),\ \sin(v),\ 1 \rangle$, $\mathbf{r}_v = \langle -u \cdot \sin(v),\ u \cdot \cos(v),\ 0 \rangle$

$r_u x r_v = \langle -u\cos(v),\ u \cdot \sin(v),\ u \rangle$. Surface area = $\iint_R |\mathbf{r}_u \mathbf{x} \mathbf{r}_v|\ dA = \int_{u=0}^{2} \int_{v=0}^{2\pi} u\sqrt{2}\ dv\ du$.

17. Surface area = $\int_{u=0}^{4} \int_{v=0}^{2\pi} \sqrt{u + \frac{1}{4}}\ dv\ du$.

19. Surface area = $\int_{u=0}^{2} \int_{v=0}^{2\pi} \sqrt{1 + \cos^2(v)}\ dv\ du$.

15.8 Odd Answers

1. $r_u = \langle 1,2,3 \rangle$, $r_v = \langle 3,-1,1 \rangle$, $\mathbf{r}_u \mathbf{x} \mathbf{r}_v = \langle 5,8,-7 \rangle$ and $|\mathbf{r}_u \mathbf{x} \mathbf{r}_v| = \sqrt{138}$. $f(\mathbf{r}(1,2)) = f(7,0,5) = 12$,

$\Delta A = \Delta u \cdot \Delta v = 0.03$ so $f(\mathbf{r}(u,v)) \cdot |\mathbf{r}_u \mathbf{x} \mathbf{r}_v|\ \Delta A = (12)(\sqrt{138})(0.03) \approx 4.23$.

3. When u=2 and v=1, then $r_u = \langle 2u,\ 3,\ 0 \rangle = \langle 4,3,0 \rangle$, $r_v = \langle 0,\ 1,\ 2v \rangle = \langle 0,1,2 \rangle$,

$\mathbf{r}_u \mathbf{x} \mathbf{r}_v = \langle 6v,\ -4uv,\ 2u \rangle = \langle 6,-8,4 \rangle$ and $|r_u x r_v| = \sqrt{116}$. $f(\mathbf{r}(2,1)) = f(4,7,1) = 56$,

$\Delta A = \Delta u \cdot \Delta v = 0.06$ so $f(\mathbf{r}(u,v)) \cdot |\mathbf{r}_u \mathbf{x} \mathbf{r}_v|\ \Delta A = (56)(\sqrt{116})(0.06) \approx 36.19$.

5. S can be parameterized by $r(u,v) = \langle u,v,4 \rangle$ with $0 \le u \le 3$ and $0 \le v \le 2$. Then $r_u = \langle 1,0,0 \rangle$,

$r_v = \langle 0,1,0 \rangle$, $\mathbf{r}_u \mathbf{x} \mathbf{r}_v = \langle 0,0,1 \rangle$ and $|\mathbf{r}_u \mathbf{x} \mathbf{r}_v| = 1$. $f(r(u,v)) = u^2 + 4v + 4$.

$\iint_S f(x,y,z)\ dS = \iint_R f(\mathbf{r}(u,v)) \cdot |\mathbf{r}_u \mathbf{x} \mathbf{r}_v|\ dA = \int_{v=0}^{2} \int_{u=0}^{3} u^2 + 4v + 4\ du\ dv = \int_{v=0}^{2} 9 + 12v + 12\ dv = 66$.

7. S can be parameterized by $r(u,v) = \langle u,\ v,\ 4 - u - v \rangle$ with $0 \le u \le 4$ and $0 \le v \le 4 - u$. Then $r_u = \langle 1,\ 0,\ -1 \rangle$,

$r_v = \langle 0,\ 1,\ -1 \rangle$,, $\mathbf{r}_u \mathbf{x} \mathbf{r}_v = \langle 1,\ 1,\ 1 \rangle$ and $|r_u x r_v| = \sqrt{3}$. $f(r(u,v)) = v^2$.

$\iint_S f(x,y,z)\ dS = \iint_R f(\mathbf{r}(u,v)) \cdot |\mathbf{r}_u \mathbf{x} \mathbf{r}_v|\ dA = \int_0^4 \int_0^{4-u} v^2\sqrt{3}\ dv\ du = -(4-u)^4 \frac{\sqrt{3}}{12} \Big|_0^4 = 64 \cdot \frac{\sqrt{3}}{3}$

9. S can be parameterized by $r(u,v) = \langle u,v,4 \rangle$ with $0 \le u \le 3$ and $0 \le v \le 2$. Then $r_u = \langle 1,0,0 \rangle$,
$r_v = \langle 0,1,0 \rangle$, and $\mathbf{r}_u \mathbf{x} \mathbf{r}_v = \langle 0,0,1 \rangle$. $\mathbf{F}(\mathbf{r}(u,v)) = \langle u,\ 2v,\ 3 \cdot 4 \rangle$.

Flux = $\iint_S \mathbf{F} \cdot \mathbf{n}\ d\mathbf{S} = \iint_{\mathbf{R}} \mathbf{F} \bullet (\mathbf{r}_u \mathbf{x} \mathbf{r}_v)\ dA = \iint_{\mathbf{R}} \langle u,\ 2v,\ 3 \cdot 4 \rangle \bullet \langle 0,\ 0,\ 1 \rangle\ dA = \int_0^2 \int_0^3 12\ du\ dv = 72$.
The flux units are (liters/second)(meter[2]).

11. S can be parameterized by $\mathbf{r}(u,v) = \langle 2 \cdot \cos(u),\ \sin(u),\ v \rangle$ with $0 \le u \le 2\pi$ and $0 \le v \le 3$. Then

$\mathbf{r}_u = \langle -2 \cdot \sin(u),\ \cos(u),\ 0 \rangle$, $\mathbf{r}_v = \langle 0,\ 0,\ 1 \rangle$ and $\mathbf{r}_u \mathbf{x} \mathbf{r}_v = \langle \cos(u),\ 2 \cdot \sin(u),\ 0 \rangle$. $\mathbf{F}(\mathbf{r}(u,v)) = \langle 2 \cdot \cos(u),\ \sin(u),\ v \rangle$.

Flux = $\iint_S \mathbf{F} \cdot \mathbf{n}\ d\mathbf{S} = \iint_{\mathbf{R}} \mathbf{F} \bullet (\mathbf{r}_u \mathbf{x} \mathbf{r}_v)\ dA = \iint_{\mathbf{R}} \langle 2 \cdot \cos(u),\ \sin(u),\ v \rangle \bullet \langle \cos(u),\ 2 \cdot \sin(u),\ 0 \rangle\ dA$

$= \int_0^3 \int_0^{2\pi} 2\ du\ dv = 12\pi$.

13. S can be parameterized by $\mathbf{r}(u,v) = \langle v\cdot\cos(u),\ v\cdot\sin(u),\ A^2 - v^2 \rangle$ with $0 \le u \le 2\pi$ and $0 \le v \le A$. Then

$\mathbf{r}_u = \langle -v\cdot\sin(u),\ v\cdot\cos(u),\ 0\rangle$, $\mathbf{r}_v = \langle \cos(u),\ \sin(u),\ -2Cv\rangle$ and $\mathbf{r}_u \mathbf{x} \mathbf{r}_v = \langle -2v^2C\cdot\cos(u),\ -2v^2C\cdot\sin(u),\ -v\rangle$.

$\mathbf{F}(\mathbf{r}(u,v)) = \langle 0, 0, K\rangle$.

$$\text{Flux} = \iint_S \mathbf{F}\cdot\mathbf{n}\, d\mathbf{S} = \iint_{\mathbf{R}} \mathbf{F}\bullet(\mathbf{r}_u \mathbf{x} \mathbf{r}_v)\ dA = \iint_{\mathbf{R}} \langle 0, 0, K\rangle \bullet \langle -2v^2C\cdot\cos(u),\ -2v^2C\cdot\sin(u),\ -v\rangle\ dA$$

$$= \int_0^A \int_0^{2\pi} -Kv\ du\ dv = -KA^2\pi .$$

This is the same flux as in Problem 12. In fact, the flux is the same even if C=0 so that S is just the disk $x^2 + y^2 \le A^2$ in the xy-plane.

15.9 Odd Answers

1. curl $\mathbf{F} = \nabla\mathbf{x}\mathbf{F} = \langle 0, 0, 1\rangle$, $R = \{(x,y):\ x^2 + 4y^2 \le 4\}$, area of R = $\pi(1)(2) = 2\pi$ and $\mathbf{n} = \langle 0, 0, 1\rangle$.

$$\text{circulation} = \oint_{\mathbf{C}} \mathbf{F}\bullet\mathbf{T}\ ds = \iint_R (\nabla\mathbf{x}\mathbf{F})\bullet\mathbf{n}\ dA = \iint_R \langle 0, 0, 1\rangle\bullet\langle 0, 0, 1\rangle\ dS = \text{area of R} = 2\pi .$$

3. curl $\mathbf{F} = \nabla\mathbf{x}\mathbf{F} = \langle x, 1-y, 0\rangle$, $\mathbf{n} = \frac{1}{\sqrt{12}}\langle 2, 2, 2\rangle$, $R = \{(x,y):\ 0 \le x \le 3,\ 0 \le y \le 3 - x\}$

$$\text{circulation} = \oint_{\mathbf{C}} \mathbf{F}\bullet\mathbf{T}\ ds = \iint_R (\nabla\mathbf{x}\mathbf{F})\bullet\mathbf{n}\ dA = \iint_R \langle x, 1-y, 0\rangle\bullet\frac{1}{\sqrt{12}}\langle 2, 2, 2\rangle\ dS$$

$$= \frac{1}{\sqrt{12}}\int_0^3 \int_0^{3-x} 2x + 2 - 2y\ dy\ dx = \frac{9}{\sqrt{12}} .$$

5. curl $\mathbf{F} = \nabla\mathbf{x}\mathbf{F} = \langle 0, 0, 2\rangle$, and $\mathbf{n} = \frac{1}{\sqrt{3}}\langle 1, 1, 1\rangle$.

$$\text{circulation} = \oint_{\mathbf{C}} \mathbf{F}\bullet\mathbf{T}\ ds = \iint_R (\nabla\mathbf{x}\mathbf{F})\bullet\mathbf{n}\ dA = \iint_R \langle 0, 0, 2\rangle\bullet\frac{1}{\sqrt{3}}\langle 1, 1, 1\rangle\ dA = \frac{2}{\sqrt{3}}\cdot\iint_R 1\ dA$$

$= \frac{2}{\sqrt{3}}$ (area of R) where R is the area enclosed by C.

7. curl $\mathbf{F} = \nabla\mathbf{x}\mathbf{F} = \langle 0, 0, 3-2y\rangle$ and $\mathbf{n} = \langle 0, 0, 1\rangle$.

$$\text{circulation} = \oint_{\mathbf{C}} \mathbf{F}\bullet\mathbf{T}\ ds = \iint_R (\nabla\mathbf{x}\mathbf{F})\bullet\mathbf{n}\ dA = \iint_R \langle 0, 0, 3-2y\rangle\bullet\langle 0, 0, 1\rangle\ dA$$

$$= \int_0^2 \int_0^{2\pi} (3 - 2r\cdot\sin(\theta))\cdot r\ d\theta\, dr = \int_0^2 6\pi r\ dr = 12\pi .$$

9. The boundary of S is the circle C parameterized by $\mathbf{r}(t) = \langle 4\cdot\cos(t),\ 4\cdot\sin(t),\ 0\rangle$.

$$\iint_S \text{curl}\ \mathbf{F}\bullet d\mathbf{S} = \int_C \mathbf{F}\bullet d\mathbf{r} = \int_{t=0}^{2\pi} \langle 4\cdot\sin(t),\ -4\cdot\cos(t),\ 0\rangle\bullet\langle -4\cdot\sin(t),\ 4\cdot\cos(t),\ 0\rangle\ dt$$

$$= \int_{t=0}^{2\pi} -16\sin^2(t) - 16\cos^2(t)\ dt = -32\pi .$$

11. The solid torus is a closed surface so it has no boundary curve so $\iint_S \text{curl } \mathbf{F} \bullet d\mathbf{S} = 0$.

13. $\nabla \mathrm{x} \mathbf{F} = \langle 0,0,0 \rangle$ so F is a conservative field.

15. $\nabla \mathrm{x} \mathbf{F} = \langle cx - bx,\ ay - cy,\ bx - az \rangle$ so F is not a conservative field.

17. $\nabla \mathrm{x} \mathbf{F} = \langle 0,0,0 \rangle$ so F is a conservative field.

19. Quick check: $\nabla \mathrm{x} \mathbf{F} = \langle 2, 2, 2 \rangle$.

The top and bottom boundaries of S can be parameterized by $\mathbf{r}_1 = \langle 4 \cdot \cos(t), 4 \cdot \sin(t), 0 \rangle$ and $\mathbf{r}_2 = \langle 4 \cdot \cos(t), 4 \cdot \sin(t), 3 \rangle$ for $0 \le t \le 2\pi$ (both going counterclockwise).

$$\int_{C_1} \mathbf{F} \bullet \mathbf{r}_1' \ dt = \int_0^{2\pi} \langle 0 - 4 \cdot \sin(t), 4 \cdot \cos(t) - 0, 4 \cdot \sin(t) - 4 \cdot \cos(t) \rangle \bullet \langle -4 \cdot \sin(t), 4 \cdot \cos(t), 0 \rangle \ dt$$

$$= \int_0^{2\pi} 16 \cdot \sin^2(t) + 16 \cdot \cos^2(t) \ dt = 32\pi .$$

$$\int_{C_2} \mathbf{F} \bullet \mathbf{r}_2' \ dt = \int_0^{2\pi} \langle 3 - 4 \cdot \sin(t), 4 \cdot \cos(t) - 3, 4 \cdot \sin(t) - 4 \cdot \cos(t) \rangle \bullet \langle -4 \cdot \sin(t), 4 \cdot \cos(t), 0 \rangle \ dt$$

$$= \int_0^{2\pi} -12 \cdot \sin(t) + 16 \cdot \sin^2(t) + 16 \cdot \cos^2(t) - 12 \cdot \cos(t) \ dt = 32\pi .$$

Finally, $\iint_S \text{curl } \mathbf{F} \bullet d\mathbf{S} = \int_{C_1} F \bullet \mathbf{r}_1' \ dt - \int_{C_2} F \bullet \mathbf{r}_1' \ dt = 0$.

21. Quick check $\nabla \mathrm{x} \mathbf{F} = \langle 0, 0, 0 \rangle$ so $\iint_S \text{curl } \mathbf{F} \bullet d\mathbf{S} = \iint_S \langle 0, 0, 0 \rangle \bullet d\mathbf{S} = 0$.

23. Quick check $\nabla \mathrm{x} \mathbf{F} = \langle -1, -1, 1 \rangle$.

The boundary curve of S can be parameterized by $\mathbf{r} = \langle 2 \cdot \cos(t), 0, -5 \cdot \sin(t) \rangle$.
(goes counterclockwise looking back along the y-axis).

$$\iint_S \text{curl } \mathbf{F} \bullet d\mathbf{S} = \int_C \mathbf{F} \bullet d\mathbf{r} = \int_{t=0}^{2\pi} \langle 0, -5 \cdot \sin(t), 2 \cdot \cos(t) \rangle \bullet \langle -2 \cdot \sin(t), 0, -5 \cdot \cos(t) \rangle \ dt$$

$$= \int_{t=0}^{2\pi} -10 \cdot \cos^2(t) \ dt = -10\pi .$$

25. Quick check $\nabla \mathbf{x} \mathbf{F} = \langle -1, 0, 1 \rangle$.

When z=0, $x^2 + y^2 = 25$ is a circle of radius 5: $\mathbf{r}_1 = \langle 5\cdot\cos(t), 5\cdot\sin(t), 0\rangle$.

$$\int_{C_1} \mathbf{F}\bullet\mathbf{r}_1{}' \, dt = \int_0^{2\pi} \langle 5\cdot\sin(t), 0, 5\cdot\cos(t)\rangle \bullet \langle -5\cdot\sin(\theta), 5\cdot\cos(t), 0\rangle \, dt = \int_0^{2\pi} -25\cdot\sin^2(t) \, dt = -25\pi .$$

When z=16, $x^2 + y^2 = 9$ is a circle of radius : $\mathbf{r}_2 = \langle 3\cdot\cos(t), 3\cdot\sin(t), 16\rangle$.

$$\int_{C_2} \mathbf{F}\bullet\mathbf{r}_2{}' \, dt = \int_0^{2\pi} \langle 3\cdot\sin(t), 16, 3\cdot\cos(t)\rangle \bullet \langle -3\cdot\sin(t), 3\cdot\cos(t), 0\rangle \, dt$$

$$= \int_0^{2\pi} -9\cdot\sin^2(t) + 48\cdot\cos(t) \, dt = -9\pi . \text{ So } \iint_S \text{curl } \mathbf{F}\bullet d\mathbf{S} = \int_{C_1} F\bullet\mathbf{r}_1{}' \, dt - \int_{C_2} F\bullet\mathbf{r}_1{}' \, dt = -16\pi .$$

15.10 Odd Answers

1. Distance to the origin of P1 is $\sqrt{14} \approx 3.7$, of P2 is $\sqrt{6} \approx 2.4$, of P3 is $\sqrt{8} \approx 2.8$, of P4 is $\sqrt{2} \approx 1.4$, of P5 is $\sqrt{12} \approx 3.5$, and of P6 is $\sqrt{19} \approx 4.4$.

   (a) Only P4: net flux = 6 m^3/s. (b) P1, P2, P3, P4 and P5: net flux=(7)+(–3)+(2)+(6)+(–4)= 8 m^3/s.

3. Dist to center of P1 is $\sqrt{10} \approx 3.2$, of P2 is $\sqrt{18} \approx 4.2$, of P3 is 0 , of P4 is $\sqrt{6} \approx 2.4$, of P5 is 2, and of P6 is $\sqrt{11} \approx 3.3$.

   (a) Only P3: net flux= +2 m^3/s. (b) P3, P4, P5: net flux=(+2)+(+6)+(–4)= +4 m^3/s.

5. Outward flux sphere 1: 5 m^3/s. outward flux of sphere 2: 14 m^3/s. Net flux = (14)–(5)= 9 m^3/s.

7. div $\mathbf{F}$ = 3 so $\text{flux} = \iiint_E \text{div } \mathbf{F} \, dV = 3(\text{volume of E}) = 3\left(\frac{4}{3}\pi(2)^3\right) = 32\pi$.

9. div $\mathbf{F}$ = 2 so $\text{flux} = \iiint_E \text{div } \mathbf{F} \, dV = 2(\text{volume of E}) = 2\cdot\left(\frac{4}{3}\pi(3)^3\right) = 72\pi$.

11. div $\mathbf{F}$ = 4 so $\text{flux} = \iiint_E \text{div } \mathbf{F} \, dV = (4)\iiint_E \mathbf{1} \, dV = 4\cdot(\text{volume of E}) = (4)\cdot(2)(3(4) = 96$.

13. div $\mathbf{F}$ = 2x + 2y + 2z so $\text{flux} = \iiint_E \text{div } \mathbf{F} \, dV = \int_0^4 \int_0^3 \int_0^2 (2x + 2y + 2z) \, dx \, dy \, dz = 216$.

15. $0 = \iiint_D \text{div } \mathbf{F} \, dV = \iint_S \mathbf{F}\bullet\mathbf{n} \, dS = \iint_{S_1} \mathbf{F}\bullet\mathbf{n} \, dS - \iint_{S_2} \mathbf{F}\bullet\mathbf{n} \, dS$ so the outward flux from the outer sphere equals the outward flux from the inner sphere.

17. div $\mathbf{F}$ = 0 at every point so the Flux across every sphere is zero.

19. div $\mathbf{F}$ = 1 at every point so Flux across a sphere equals the volume of the sphere so the Flux across every sphere is always positive.

21. div $\mathbf{F}$ = 2x + 2y + 2z (a) Flux = 0 because of the symmetry of the triple integral

    (b) Flux is positive because x, y and z are all positive in this sphere so the integral is positive.

    (c) Flux is positive.