5.5 ADDITIONAL APPLICATIONS

This section **introduces** three additional applications of integrals and once again illustrates the process of going from a problem to a Riemann sum and on to a definite integral. A fourth application is included which does not follow this process. It uses the idea of "area" to try to model an election and to qualitatively understand why certain election outcomes occur.

The main point of this section is to show the power of definite integrals to solve a wide variety of applied problems. Each of these new applications is treated more briefly than those in the previous sections.

1. Liquid Pressures and Forces

The hydrostatic pressure on an immersed object is the density of the fluid times the depth of the object: **pressure = (density)(depth)**. The total hydrostatic force against an immersed object is the sum of the hydrostatic forces against each part of the object. If our entire object is at the same depth, we can determine the total hydrostatic force simply by multiplying the density times the depth times the area. If the unit of density is "pounds per cubic foot" and the depth is given in "feet," then the unit of pressure is "pounds per square foot," a measure of **force per area**. If a pressure, with the units "pounds per square foot," is multiplied by an area with the units "square feet," the result is a force, "pounds."

Example 1: Find the total hydrostatic force against the bottom of the aquarium shown in Fig. 1.

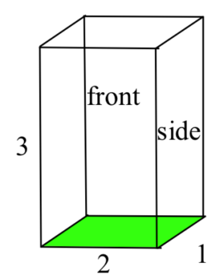

Fig. 1

Solution: The density of water is 62.5 pounds/ft^3, so

total hydrostatic force = (density)•(depth)•(area)

= (62.5 pounds/ft^3)•(3 feet)•(2 square feet) = 375 pounds.

Finding the total hydrostatic force against the front of the aquarium is a very different problem because different parts of the front are at different depths and are subject to different pressures. To find the force against the front of the aquarium, we can partition it into thin horizontal slices (Fig. 2) and focus on one of them. Since the slice is very thin, every part of the i^{th} slice is at almost the same depth so every part of the slice is subject to almost the same pressure. We can approximate the total hydrostatic force against the slice at the depth x_i as

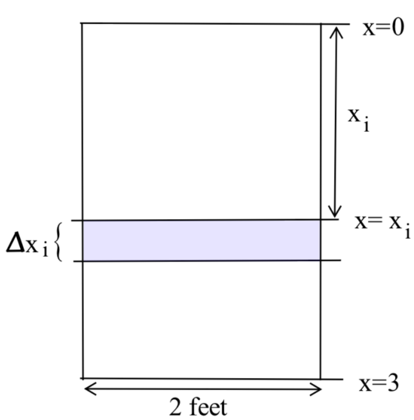

Fig. 2

$$\text{(density)•(depth)•(area)} = (62.5 \text{ pounds/ft}^3)(x_i \text{ feet})(2 \text{ feet})(\Delta x_i \text{ feet})$$

$$= 125 x_i \cdot \Delta x_i \text{ pounds.}$$

The total hydrostatic force against the front is the sum of the forces against each slice,

total hydrostatic force $\approx \sum 125x_i \, \Delta x_i$, a Riemann sum.

The limit of the Riemann sum as the slices get thinner ($\Delta x \to 0$) is a definite integral:

$$\text{total hydrostatic force} \approx \sum 125x_i \, \Delta x_i \longrightarrow \int_{x=0}^{3} 125x \, dx = 62.5\,x^2 \Big|_{x=0}^{3} = 562.5 \text{ pounds.}$$

Practice 1: Find the total hydrostatic force against one side of the aquarium.

Hydrostatic Force

If the width of a slice of a horizontal object at depth x is $w(x)$ (Fig. 3)

then the total hydrostatic force against the object between depths a and b is

$$\text{total hydrostatic force} \approx \sum (\text{density})\cdot(\text{depth})\cdot w(x_i)\, \Delta x_i$$

$$\longrightarrow \int_{x=a}^{b} (\text{density})\cdot x \cdot w(x)\, dx \;.$$

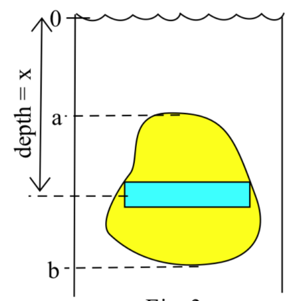

Fig. 3

Example 2: Find the total hydrostatic force against windows A and B in Fig. 4.

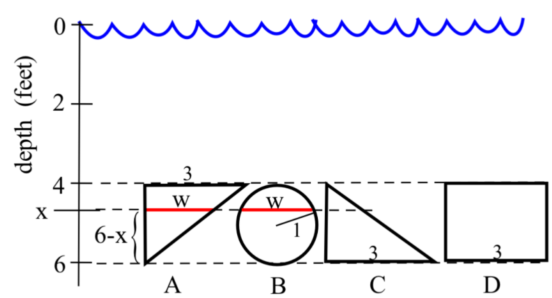

Fig. 4

Solution: Window A: Using similar triangles,

$$\frac{w}{6-x} = \frac{3}{2} \quad \text{so} \quad w(x) = \frac{3}{2}(6-x) \;.$$

Then total hydrostatic force

$$= \int_{x=4}^{6} (\text{density})\cdot x \cdot w(x)\, dx$$

$$= \int_{4}^{6} (60)\cdot x \cdot \tfrac{3}{2}(6-x)\, dx = 90\left(3x^2 - \frac{x^3}{3}\right)\Big|_{4}^{6} = 840 \text{ pounds.}$$

Window B: The equation of the circle is $(5-x)^2 + (w/2)^2 = 1$ so $w(x) = 2\sqrt{1-(5-x)^2}$. Then

$$\text{total hydrostatic force} = \int_{4}^{6} (60)\cdot x \cdot 2\sqrt{1-(5-x)^2}\, dx \approx 938.1 \text{ pounds (using a calculator).}$$

Practice 2: Find the total hydrostatic force against windows C and D in Fig. 4.

Because the total force at even moderate depths is so large, the underwater windows at aquariums are made of thick glass or plastic and strongly secured to their frames. Similarly, the bottom of a dam is much thicker than the top in order to withstand the greater force against the bottom.

2. Kinetic Energy of a Rotating Object

The **Kinetic Energy** (the energy of motion) of an object is defined to be half the mass of the object multiplied by the square of the velocity of the object: $\mathbf{KE = \frac{1}{2}\, m{\cdot}v^2}$.

The larger an object is or the faster it is moving, the greater its kinetic energy. If every part of the object has the same velocity, then it is easy to compute its kinetic energy. Sometimes, however, different parts of the object move with different velocities. For example, if an ice skater is spinning with an angular velocity of 2 revolutions per second, then her arms travel further in one second (have a greater linear velocity) when they are extended than when they are drawn in close to her body (Fig. 5). So the ice skater, spinning at 2 revolutions per second, has greater kinetic energy when her arms are extended. Similarly, the tip of a rotating propeller or of a swinging baseball bat has a greater has a greater linear velocity than other parts of the propeller or bat. If the units ofmass are "grams" and the units of velocity are "centimeters per second," then the units of kinetic energy $KE = \frac{1}{2}\, m{\cdot}v^2$ are "ergs."

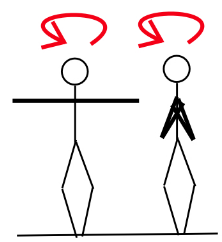

Fig. 5

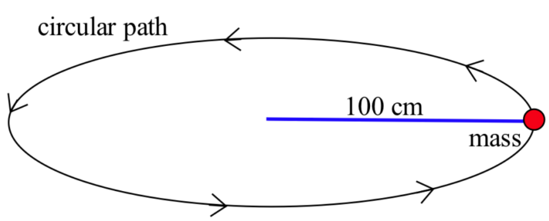

Fig. 6

Example 3: A point–mass of 1 gram at the end of a (massless) 100 centimeter long string is rotated at a rate of 2 revolutions per second (Fig. 6). (a) Find the kinetic energy of the point–mass. (b) Find its kinetic energy if the string is 200 centimeters long.

Solution: (a) In one second, the mass travels twice around a circle with radius 100 centimeters so it travels $2{\cdot}(2\pi{\cdot}\text{radius}) = 400\pi$ centimeters. The velocity is $v = 400\pi$ cm/s , and

$$KE = \frac{1}{2}\, m{\cdot}v^2 = \frac{1}{2}(1\ \text{g}){\cdot}(400\pi\ \text{cm/s})^2 = 80{,}000\,\pi^2\ \text{ergs}.$$

(b) If the string is 200 centimeters long, then the velocity is $2{\cdot}(2\pi{\cdot}\text{radius})/\text{second} = 800\pi$ cm/s ,

$$\text{and}\quad KE = \frac{1}{2}\, m{\cdot}v^2 = \frac{1}{2}(1\ \text{g}){\cdot}(800\pi\ \text{cm/sec})^2 = 320{,}000\,\pi^2\ \text{ergs}.$$

When the length of the string doubles, the velocity is twice as large and the kinetic energy is 4 times as large.

Practice 3: A 1 gram point–mass at the end of a 2 meter (massless) string is rotated at a rate of 4 revolutions per second. Find the kinetic energy of the point mass.

If different parts of a rotating object are different distances from the axis of rotation, then those parts have different linear velocities, and it is more difficult to calculate the total kinetic energy of the object. By now the method should seem very familiar: partition the object into small pieces, approximate the kinetic energy of each piece, and add the kinetic energies of the small pieces (a Riemann sum) to approximate the total kinetic energy of the object. The limit of the Riemann sum as the pieces get smaller is a definite integral.

Example 4: The density of a narrow bar (Fig. 7) is 5 grams per meter of length. Find the kinetic energy of the 3 meter long bar when it is rotated at a rate of 2 revolutions per second.

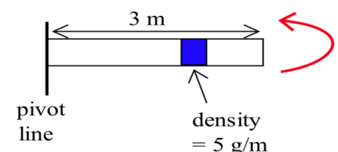

Fig. 7

Solution: If the length of the bar is partitioned (Fig. 8), the mass of the i^{th} piece is

$$m_i \approx (\text{length})\cdot(\text{density}) = (\Delta x_i \text{ meters})(5 \text{ grams/meter}) = 5\ \Delta x_i \text{ grams}.$$

In one second the i^{th} piece will make two revolutions and will travel approximately

$$2\cdot(2\pi\cdot\text{radius}) = 400\pi\cdot x_i \text{ centimeters so } v_i \approx 400\pi\, x_i \text{ cm/sec}.$$

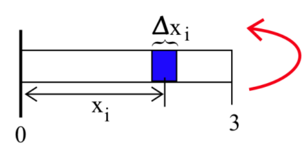

Fig. 8

The kinetic energy of the i^{th} piece is

$$ke_i = \frac{1}{2} m_i\cdot v_i^2 = \frac{1}{2}(5\ \Delta x_i \text{ grams})\cdot(400\pi\, x_i \text{ cm/sec})^2$$

$$= 400{,}000\pi^2 (x_i)^2\ \Delta x_i \text{ ergs},$$

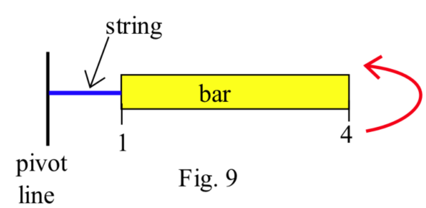

Fig. 9

and the total kinetic energy of the rotating bar is

$$KE = \sum ke_i = \sum 400{,}000\pi^2 (x_i)^2\ \Delta x_i \longrightarrow \int_0^3 400{,}000\pi^2\cdot x^2\ dx$$

$$= 400{,}000\ \pi^2 \frac{x^3}{3}\Big|_0^3 = 3{,}600{,}000\ \pi^2 \text{ ergs}.$$

Practice 4: Find the kinetic energy of the bar in the previous example if it is rotated at 2 revolutions per second at the end of a 100 centimeter (massless) string (Fig. 9).

Example 5: Find the kinetic energy of the object in Fig. 10 when it rotates at 2 revolutions per second.

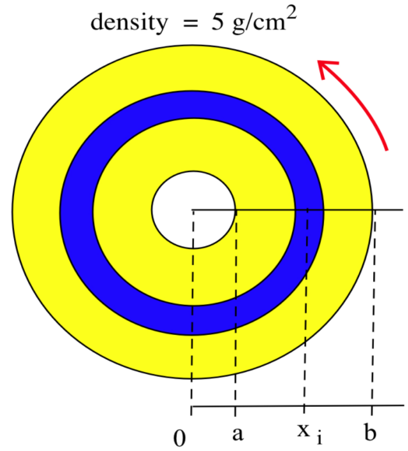

Fig. 10

Solution: We can partition along one radial line, and form circular "slices." Then the "slice" between x_i and $x_i+\Delta x$ is a thin circular band with mass $m_i = (\text{volume})\cdot(\text{density}) \approx 2\pi\, x_i\, \Delta x_i \cdot h \cdot d$ Newtons. Each part of the "slice" is approximately x_i centimeters from the axis of rotation so each part has approximately the same velocity:

$v_i \approx (2 \text{ rev/sec})\cdot(2\pi\, x_i \text{ centimeters/rev}) = 4\pi\cdot x_i$ centimeters/sec.

The kinetic energy of the i^{th} piece is

$$ke_i \approx = \frac{1}{2}\, m_i \cdot v_i^{\,2} = \frac{1}{2}(2\pi\cdot x_i\, \Delta x_i \cdot h \cdot d)\cdot(4\pi\cdot x_i\,)^2 = 16\,\pi^3 \cdot h \cdot d \cdot (x_i)^3\, \Delta x_i\,,$$

$$\text{so } KE = \sum ke_i = \sum 16\,\pi^3 \cdot h \cdot d \cdot (x_i)^3\, \Delta x$$

$$\longrightarrow \int_a^b 16\,\pi^3 \cdot h \cdot d \cdot x^3\, dx = 16\,\pi^3 hd \int_a^b x^3\, dx = 4\,\pi^3 hd\cdot(\, b^4 - a^4\,)\,.$$

Since b is raised to the 4^{th} power, a small increase in the value of b leads to a large increase in the kinetic energy.

3. Volumes of Revolution Using "Tubes" (Shells)

In Section 5.2 the "disk" method was used for finding the volume swept out when a region is revolved about a line (Fig. 11). To find the volume swept out when a region was revolved about the x–axis, we made cuts perpendicular to the x–axis so each slice was a "disk" with volume $\pi(\text{radius})^2\cdot(\text{thickness})$.

After adding the volumes of the slices together (a Riemann sum) and taking a limit as the thicknesses approached 0, we obtained a definite integral representation for the exact volume:

{ volume of revolution about the x–axis }

$$= \int_a^b \pi\, f^2(x)\, dx\,.$$

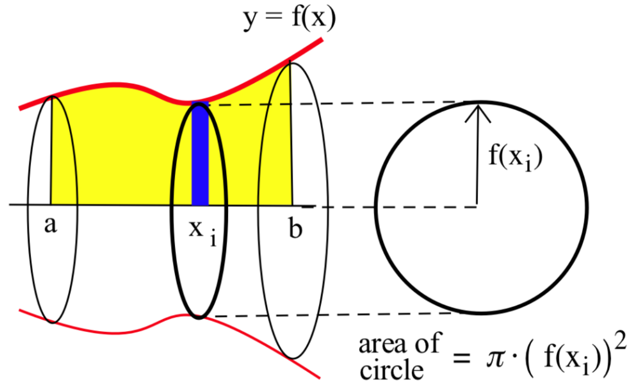

Fig. 11

However, the disk method can be cumbersome if we want the volume when the region in the figure is revolved about the y–axis or some other vertical line. To revolve the region about the y–axis, the disk method requires that we represent the original equation $y = f(x)$ as a function of y: $x = g(y)$.

Sometimes that is easy: if $y = 3x$ then $x = y/3$. But sometimes it is not easy at all: if $y = x + e^x$, then we can not solve for y as an elementary function of x. The "tube" method lets us use the original equation $y = f(x)$ to find the volume when the region is revolved about a vertical line.

We partition the x–axis to cut the region into thin, almost rectangular "slices." When the thin "slice" at x_i is revolved about a vertical line (Fig. 12a), the volume of the resulting "tube" can be approximated by cutting the wall of the tube and laying it out flat (Fig. 12b) to get a thin, solid rectangular box. The volume of the tube is approximately the same as the volume of the solid box:

$$V_{tube} \approx V_{box} = (\text{length})\cdot(\text{height})\cdot(\text{thickness}) = \mathbf{(2\pi\cdot radius)\cdot(\ height\ )\cdot(\ \Delta x_i\ )} = (2\pi\, x_i\,)\cdot(\,f(x_i)\,)\,\Delta x_i\ .$$

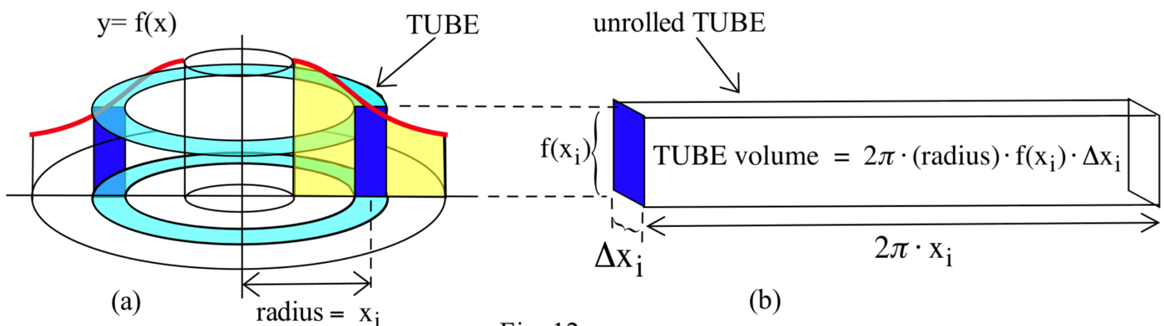

Fig. 12

The volume swept out when the whole region is revolved is the sum of the volumes of these "tubes", a Riemann sum. The limit of the Riemann sum is

$$\{\text{ volume of rotation about a vertical line }\} = \int_{x=a}^{b} 2\pi\cdot(\text{radius})\cdot(\text{height})\ dx\ .$$

The "tube" pattern for the volume of a region defined by a single function extends easily to regions between two functions.

> **Volume of Revolution Using "Tubes" (Shells)**
>
> If region R is bounded between the functions $f(x) \ge g(x)$ for $0 \le a \le b$ (Fig. 13),
>
> then { volume obtained when R is revolved about a vertical line } = $\displaystyle\int_{x=a}^{b} 2\pi\cdot(\text{radius})\cdot\{ f(x) - g(x) \}\, dx$.

Example 6: Find the volume when the region R in Fig. 14 is revolved about a vertical line.

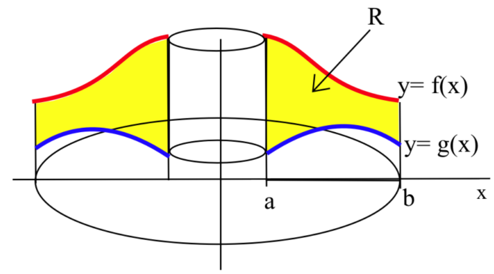

Fig. 13

Solution: We can partition the interval [2, 4] on the x–axis to get thin slices of R. When the slice at x_i is revolved around the y–axis, a tube is swept out, and the volume if this i^{th} tube is

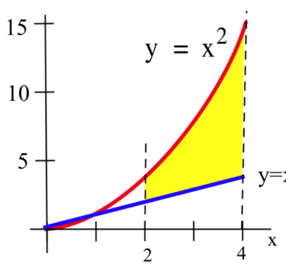

Fig. 14

$v_i \approx (2\pi\cdot\text{radius})\cdot(\text{height})\cdot(\text{thickness}) \approx 2\pi\,(x_i)(x_i^2 - x_i)(\Delta x_i) = 2\pi(x_i^3 - x_i^2)\,\Delta x_i$

The total volume is the sum of the volumes of the tubes:

$V = \sum v_i = \sum 2\pi(x_i^3 - x_i^2)\,\Delta x_i$ (a Riemann sum)

$$\longrightarrow \int_2^4 2\pi(x^3 - x^2)\, dx = 2\pi\left(\frac{x^4}{4} - \frac{x^3}{3}\right)\Bigg|_2^4 = \frac{248\,\pi}{3} \approx 259.7 .$$

Example 7: Write a definite integral to represent the volume swept out when the region in Fig. 15 is revolved about the vertical line $x = 4$.

Solution: $V = \displaystyle\int_a^b \mathbf{2\pi\cdot(radius)\cdot(height)}\, dx$

$= \displaystyle\int_0^{\pi} 2\pi\cdot(4 - x)\cdot\sin(x)\; dx$

≈ 30.526 (using calculator integrate command))

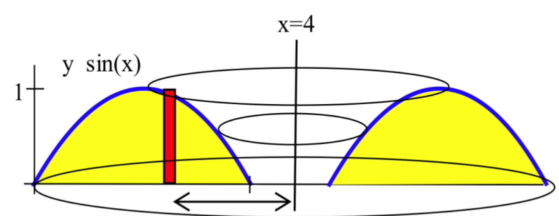

Fig. 15

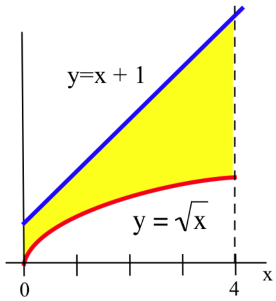

Fig. 16

Practice 5: Represent the volume when the region in Fig. 16 is revolved about the y–axis.

In theory, each method works for each volume of revolution problem. In practice, however, for any particular problem one of the methods is often easier to use than the other.

4. Areas & Elections

The previous applications used definite integrals to determine areas, volumes, pressures, and energies **exactly**. But exactness and numerical precision are not the same as "understanding," and sometimes we can gain insight and understanding simply by determining which of two areas or integrals is larger. One situation of this type involves elections.

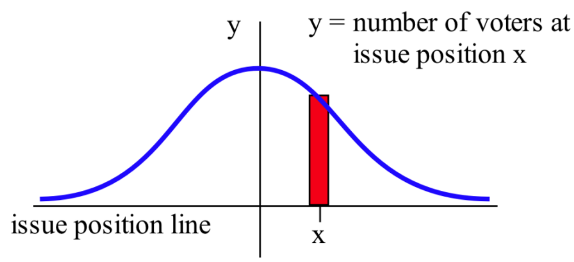

Fig. 17

Suppose the voters of a state have been surveyed about their positions on a **single** issue, and the distribution of voters who place themselves at each position on this issue is shown in Fig. 17. Also suppose that each voter votes for the candidate closest to that voter. If two candidates have taken the positions labeled A and B in Fig. 18, then a voter at position c votes for the candidate at A since A is closer to c than B is to c.

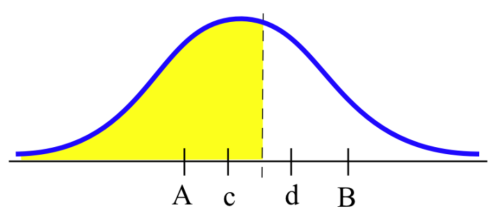

Fig. 18

Similarly, a voter at position d votes for the candidate at B. The total votes for the candidate at A in this election is the shaded area under the curve, and the candidate with the larger number of votes, the larger area, is the winner. In this illustration, the candidate at A wins.

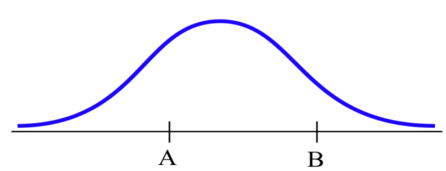

Fig. 19

Example 8: The distribution of voters on an issue is shown in Fig. 19. If these voters decide between candidates on the basis of that single issue, which candidate will win the election?

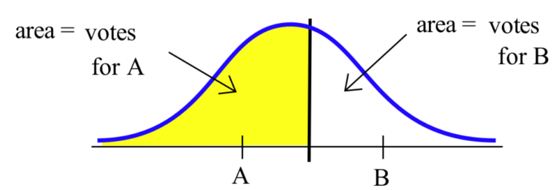

Fig. 20

Solution: Fig. 20 illustrates that A has a larger area (more votes) than B. A will win.

Practice 6: In an election between candidates with positions A and B in Fig. 21, who will win?

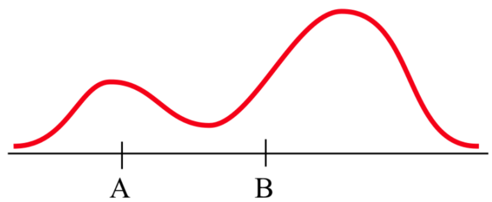

Fig. 21

If voters behave as described and if the election is between **2 candidates**, then we can give the candidates some advice.

The best position for a candidate is at the "median point," the location that divides the voters into two equal sized (equal area) groups so half of the voters are on one side of the median point and half are on the other side (Fig. 22). A candidate at the median point gets more votes than a candidate at any other point. (Why?)

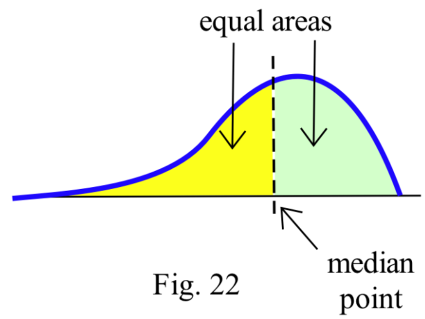

Fig. 22

If two candidates have positions on opposite sides of the median point (Fig. 23), then a candidate can get more votes by moving a bit toward the median point. This "move toward the middle ground commonly occurs in elections as candidates try to sell themselves as "moderates" and their opponents as "extremists."

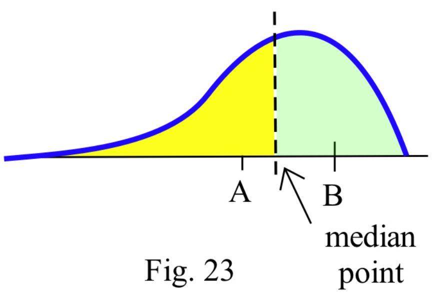

Fig. 23

If there are more than two candidates running in an election, then the situation changes dramatically, and a candidate at the median position, the unbeatable place in a 2–candidate election, can even get the fewest votes. If Fig. 24 is the distribution of voters on the single issue in the election, then candidate A would beat B in an election just between A and B (Fig. 25a); and A would beat C in an election just between A and C (Fig. 25b). But in an election among all 3 candidates, A would get the fewest votes of the 3 candidates (Fig. 25c). This type of situation really does occur. It leads to the political saying about a primary election with lots of candidates and a general election between the final nominees of the two parties:

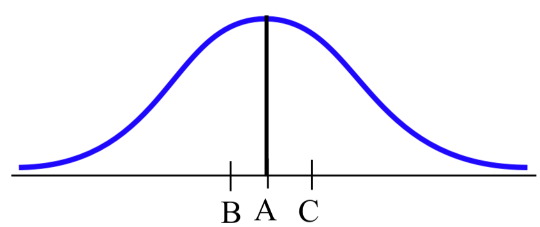

Fig. 24

"extremists can win primaries, but moderates are elected to office."

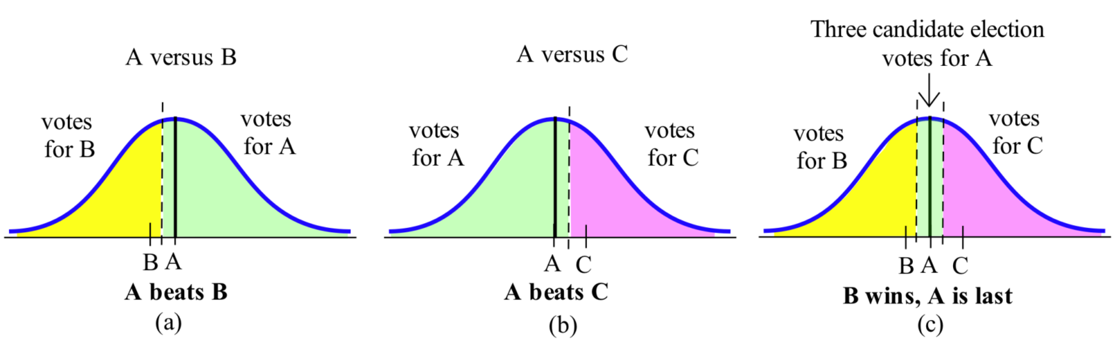

Fig. 25

The previous discussion of elections and areas is greatly oversimplified. Most elections involve several issues of different importance to different voters, and the views of the voters are seldom completely known before the election. Many candidates take "fuzzy" positions on issues. And it is not even certain that real voters vote for the "closest" candidate: perhaps they don't vote at all unless some candidate is "close enough" to their position. But the very simple model of elections can still help us understand how and why some things happen in elections. It is also a starting place for building more sophisticated models to help understand more complicated election situations and to test assumptions about how voters really do make voting decisions.

PROBLEMS

Liquid Pressure

For problems 1–5, assume that the liquid has density d.

1. Calculate the total force against windows A and B in Fig. 26

2. Calculate the total force against windows C and D in Fig. 26.

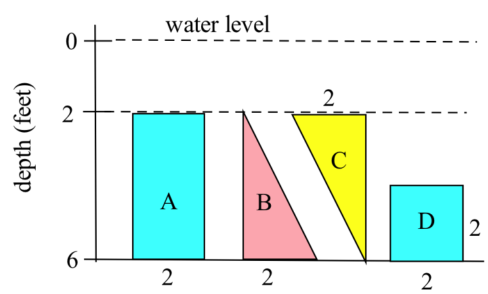

Fig. 26

3. Calculate the total force against each end of the tank in Fig. 27. How does the total force against the ends of the tank change if the length of the tank is doubled?

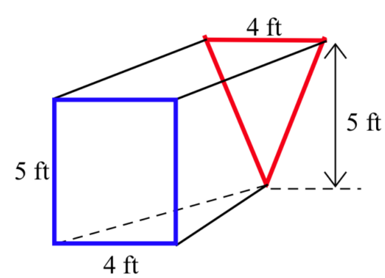

Fig. 27

4. Calculate the total force against the ends of the tank in Fig. 28.

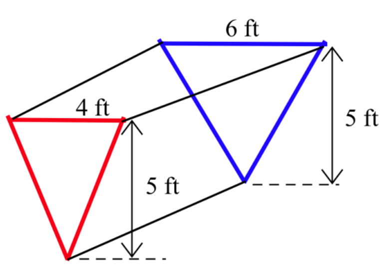

Fig. 28

5. Calculate the total force against the end of the tank in Fig. 29.

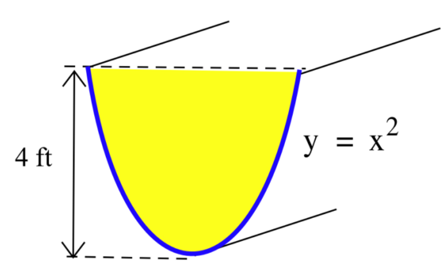

Fig. 29

6. The 3 tanks in Fig. 30 are all 6 feet tall and the top perimeter of each tank is 10 feet. Which tank has the greatest total force against its sides?

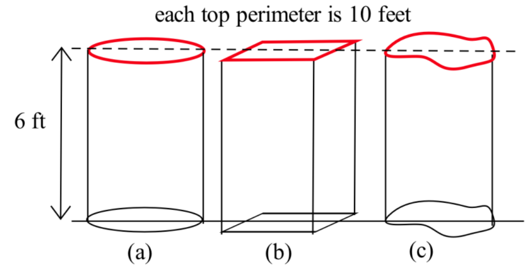

Fig. 30

7. The 3 tanks in Fig. 31 are all 6 feet tall and the cross sectional area of each tank is 16 square feet. Which tank has the greatest total force against its sides?

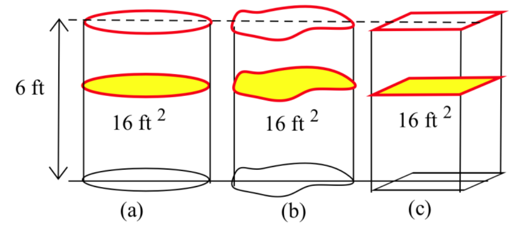

Fig. 31

8. Calculate the total force against the bottom 2 feet of the sides of a 40 foot by 40 foot tank which is filled (a) to a depth of 30 feet with water and (b) to a depth of 35 feet.

9. Calculate the total force against the bottom 2 feet of the sides of a cylindrical tank with a radius of 20 feet which is filled (a) to a depth of 30 feet with water and (b) to a depth of 35 feet.

10. Calculate the total force against the sides and bottom of a 12 ounce can of cola (height = 12 cm, radius = 3 cm). (The density of cola is approximately the same as water.) Why is the total force more than 12 ounces?

Kinetic energy

11. Calculate the kinetic energy of a 20 gram object rotating at 3 revolutions per second at the end of (a) a 15 cm (massless) string, and (b) a 20 cm string.

12. One centimeter of a metal bar weighs 3 grams. Calculate the kinetic energy of a 50 centimeter bar which is rotating at a rate of 2 revolutions per second about one end.

13. One centimeter of a metal bar weighs 3 grams. Calculate the kinetic energy of a 50 centimeter bar which is rotating at a rate of 2 revolutions per second at the end of a 10 cm piece of string.

14. Calculate the kinetic energy of a 20 gram meter stick which is rotating at a rate of 1 revolution per second about one end.

15. Calculate the kinetic energy of a 20 gram meter stick if it is rotating at a rate of 1 revolution per second about its middle point.

16. A flat, circular plate is made from material which weighs 2 grams per cubic centimeter. The plate is 5 centimeters thick, has a radius of 30 centimeters and is rotating about its center at a rate of 2 revolutions per second. (a) Calculate its kinetic energy. (b) Find the radius of the plate that would have twice the kinetic energy of this plate? (Assume the density, thickness, and rotation rate are the same.)

17. Each "washer" in Fig. 32 is made from material weighing 1 gram per cubic centimeter, and each is rotating about its center at a rate of 3 revolutions per second. Calculate the kinetic energy of each washer.

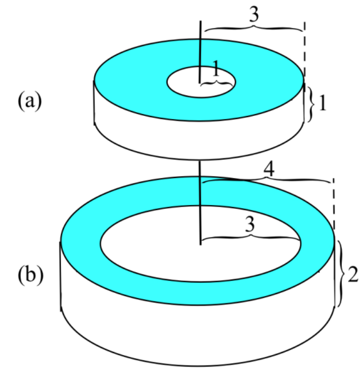

Fig. 32

18. The rectangular plate is 1 cm thick, 10 cm long and 6 cm wide and is made of material which weighs 3 grams per cubic centimeter. Calculate the kinetic energy of of the plate if it is rotated at a rate of 2 revolutions per second
(a) about its 10 cm side (Fig. 33) and
(b) about its 6 cm side.

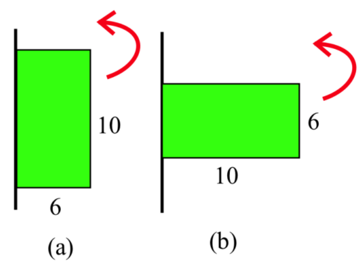

Fig. 33

19. Calculate the kinetic energy of of the plate in Problem 18 if it is rotated at a rate of 2 revolutions per second about a vertical line
(a) through the center of the plate (Fig. 34a) and
(b) through the center of the plate (Fig. 34b).

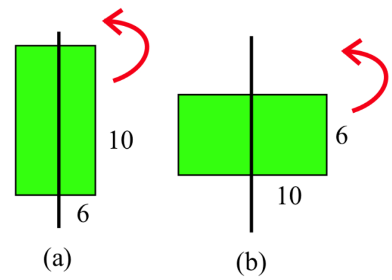

Fig. 34

Volumes by "Tubes"

In problems 20 – 25, sketch each region and calculate the volume swept out when the region is revolved about the specified vertical line.

20. The region between $y = 2x - x^2$ and the x–axis for $0 \le x \le 2$ is rotated about the y-axis.

21. The region between $y = \sqrt{1 - x^2}$ and the x–axis for $0 \le x \le 1$ is rotated about the y-axis.

22. The region between $y = \dfrac{1}{1 + x^2}$ and the x–axis of $0 \le x \le 3$ is rotated about the y-axis.

23. The region between $y = 2x$ and $y = x^2$ for $0 \le x \le 3$ is rotated about the $x = 4$ line.

24. The region between $y = x$ and $y = 2x$ for $1 \le x \le 3$ is rotated about the $x = 1$ line.

25. The region between $y = 1/x$ and $y = 1/3$ for $1 \le x \le 3$ is rotated about the $x = 5$ line.

In problems 26 – 30, write a definite integral representing the volume swept out when the region is revolved about the y–axis, and use a calculator to evaluate the integral.

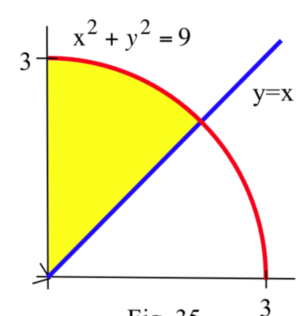

Fig. 35

26. The region between $y = e^x$ and $y = x$ for $0 \le x \le 2$.

27. The region between $y = \ln(x)$ and $y = x$ for $1 \le x \le 4$.

28. The region between $y = x^2$ and $y = 6 - x$ for $1 \le x \le 4$.

29. The shaded region in Fig. 35.

30. The shaded region in Fig. 36.

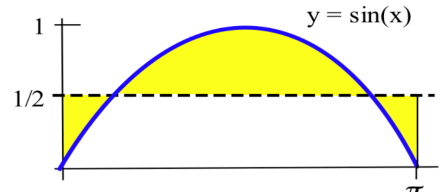

Fig. 36

Areas & Elections

31. For the voter distribution in Fig. 37, which candidates would the voters at positions a , b and c vote for?

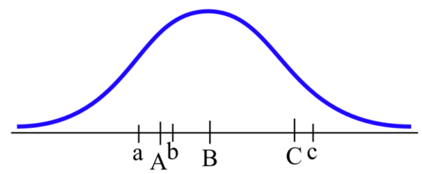

Fig. 37

32. For the voter distribution in Fig. 38, which candidates would the voters at positions a , b and c vote for?

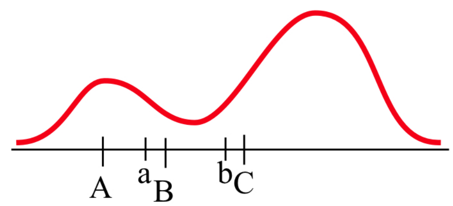

Fig. 38

33. Shade the region representing votes for candidate A in Fig. 39. Which candidate wins?

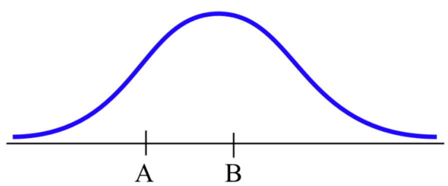

Fig. 39

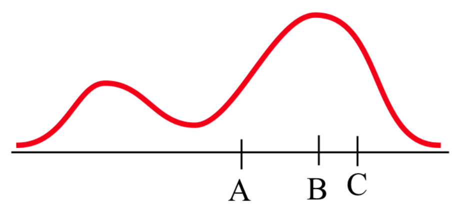

Fig. 40

34. Shade the region representing votes for candidate A in Fig. 40. Which candidate wins?

35. In Fig. 41, (a) which candidate wins?

(b) If candidate B withdraws before the election then which candidate will win?

(c) If candidate B stays in the election, but C withdraws, then who will win?

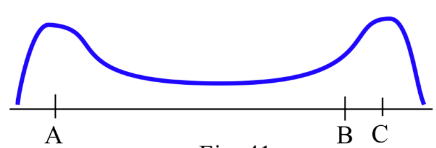

Fig. 41

36. In Fig. 42, (a) which candidate wins?

(b) If candidate A withdraws before the election, which candidate wins?

(c) If candidate B stays in the election, but C withdraws, then who wins?

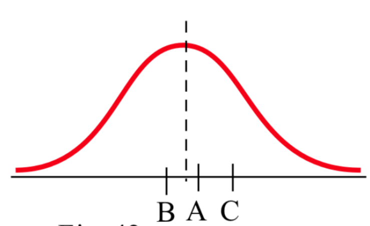

Fig. 42

37. In Fig. 43,

(a) if the election was only between A and B, who would win?

(b) If the election was only between A and C, who would win?

(c) If the election was among A, B, and C, who would win?

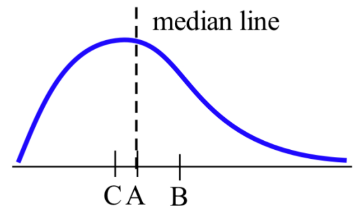

Fig. 43

38. In Fig. 44,

(a) if the election was only between A and B, who would win?

(b) If the election was only between A and C, who would win?

(c) If the election was among A, B, and C, who would win?

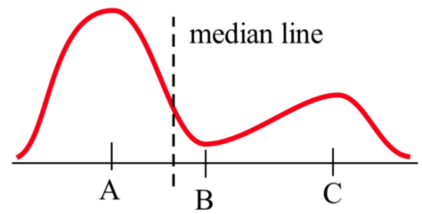

Fig. 44

39. Sketch what a voter distribution might look like for a 2 issue election.

Section 5.5 Practice Answers

Practice 1: The reasoning is the same as in Example 1 except that the width of the front is 2 feet but the width of the side is 1 foot. Then

(density)·(depth)·(area) = (62.5 pounds/ft^3)(x_i feet)(1 feet)(Δx_i feet) = 62.5 $x_i \cdot \Delta x_i$ pounds

and

hydrostatic force $\approx \sum 125 x_i \,\Delta x_i \longrightarrow \int_{x=0}^{3} 62.5x\,dx = 31.25\,x^2 \Big|_{x=0}^{3} = 281.25$ pounds.

Practice 2: C: $\frac{w}{x-4} = \frac{3}{2}$ in Fig. 45 so $w = \frac{3}{2}(x-4)$. Then

hydrostatic force $= \int_{x=a}^{b} (\text{density}) \cdot (\text{depth}) \cdot w(x)\,dx$

$= \int_{x=4}^{6} (60) \cdot (x) \cdot \frac{3}{2}(x-4)\,dx$

$= 90 \int_{x=4}^{6} (x^2 - 4x)\,dx = 90\left(\frac{x^3}{3} - 2x^2\right) \Big|_{4}^{6}$

= **960 pounds**.

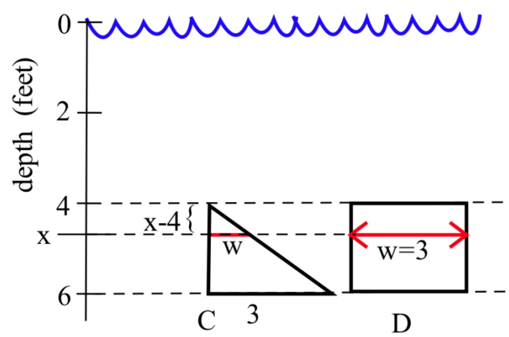

Fig. 45

D: w = 3 at all depths so

hydrostatic force $= \int_{x=a}^{b} (\text{density}) \cdot (\text{depth}) \cdot w(x)\,dx = \int_{x=4}^{6} (60) \cdot (x) \cdot (3)\,dx$

$= 180 \int_{x=4}^{6} x\,dx = 90\,x^2 \Big|_{4}^{6}$ = **1800 pounds**.

Windows A and C (with a flip) fit together to form window D, and it is encouraging that the sum of the total hydrostatic forces on A and C is the same as the total hydrostatic force on D.

Practice 3: The object travels $2\pi(\text{radius}) = 2\pi(2 \text{ meters}) = 4\pi$ meters in one revolution so in the 1 second it takes to make 4 revolutions it travels 16π meters: $v = 1600\pi$ cm/second.

$KE = \frac{1}{2} m \cdot v^2 = \frac{1}{2}(1 \text{ g}) \cdot (1600\pi \text{ cm/s})^2 = 1{,}280{,}000\pi^2$ ergs $\approx 12{,}633{,}094$ ergs.

Practice 4: Since the bar and the number of revolutions per second are the same as in Example 4,

$$ke_i = \frac{1}{2}\, m_i \cdot v_i^2 = \frac{1}{2}(5\,\Delta x_i \text{ grams}) \cdot (400\pi\, x_i \text{ cm/sec})^2 = 400{,}000\pi^2\,(x_i)^2\,\Delta x_i \text{ ergs.}$$

Then, since the bar is at the end of a 1 meter string, we integrate from $x = 1$ to $x = 1+3 = 4$:

$$KE = \sum ke_i = \sum 400{,}000\pi^2\,(x_i)^2\,\Delta x_i \text{ æææÆ } \int_{\mathbf{1}}^{\mathbf{4}} 400{,}000\pi^2 \cdot x^2\, dx$$

$$= 400{,}000\pi^2 \frac{x^3}{3}\Bigg|_{\mathbf{1}}^{\mathbf{4}} = 8{,}400{,}000\,\pi^2 \text{ ergs.}$$

Practice 5: $\{$ volume obtained when R is revolved about the y–axis $\} = \displaystyle\int_{x=a}^{b} 2\pi \cdot x \cdot \{ f(x) - g(x) \}\, dx$

$$\text{so volume} = \int_{x=0}^{4} 2\pi \cdot x \cdot \{ (x+1) - \sqrt{x} \}\, dx = 2\pi \int_{x=0}^{4} (x^2 + x - x^{3/2})\, dx$$

$$= 2\pi \left(\frac{x^3}{3} + \frac{x^2}{2} - \frac{2}{5}\, x^{5/2} \right)\Bigg|_{0}^{4} \approx 2\pi(16.53) \approx 103.9 \,.$$

Practice 6: The shaded regions in Fig. 46 show the total votes for each candidate: **B wins**.

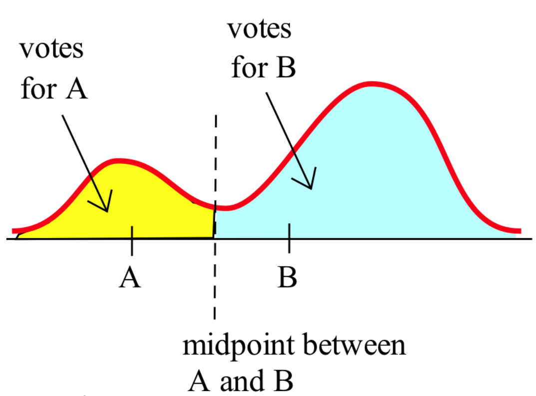

Fig. 46