# CHAPTER 8: IMPROPER INTEGRALS and INTEGRATION TECHNIQUES

## Introduction

In previous sections we examined a variety of applications which require integrals, and we found antiderivatives of many important groups of functions: polynomials, some rational functions, the trigonometric functions, the logarithm functions and some exponential functions. With this information, it is easy to find antiderivatives of their sums and differences, but finding antiderivatives of their products, quotients, and compositions can still be quite difficult. This chapter introduces several techniques of integration which greatly expand the number and variety of functions you can integrate. The first section, however, does not discuss a technique for finding antiderivatives.

- Section 8.1 describes how to evaluate an **Improper Integral**, the integral of a function over an infinitely long interval or over a finite interval when the function is not bounded at one endpoint of the interval.

The rest of the chapter is devoted to finding antiderivatives, and the overall theme **transformation**, how to transform a new type of integrand into one we can integrate immediately or into one we can find in the tables at the end of the book. Our goal is to change the pattern of some new function or combination of functions into a pattern we recognize or can find in the tables.

- Section 8.2 **reviews** some of the most common patterns we have already encountered, and it emphasizes the powerful technique of **substitution**. When the substitution technique works, it is among the easiest and quickest to use.

The remaining four sections present additional techniques for finding antiderivatives. Each of the new techniques is very useful for finding antiderivatives of particular patterns and combinations of functions. There are more integration techniques besides the four presented here, but these four are the ones most commonly needed.

- Section 8.3 introduces a technique called **Integration By Parts** which is particularly useful for finding antiderivatives of products of functions. Integration by parts is the technique used to derive many of the integration formulas in the tables.
- Section 8.4 introduces an algebraic technique called **Partial Fraction Decomposition** for transforming difficult rational functions into sums of easier rational functions which can then be integrated using previous integration techniques.
- Section 8.5 introduces a technique called **Trigonometric Substitution** which is particularly useful for integrands containing sums and differences of squares of the forms $x^2 + a^2$, $x^2 - a^2$, and $a^2 - x^2$.
- Section 8.6 considers a variety of ways in which trigonometric identities and transformations can be used to find antiderivatives of some combinations of trigonometric functions.

Unfortunately, some functions simply do not have antiderivatives which are elementary combinations (sums, differences, products, quotients, roots, and compositions) of polynomials, rational functions, trigonometric, logarithmic and exponential functions, and none of the integration methods of this chapter will find their antiderivatives.

Historically, the integration techniques in this chapter and tables of antiderivatives were very important for people who needed to apply calculus and solve differential equations. Recently we have gained additional tools, computers and even calculators that can calculate the antiderivatives of many (but not all) functions. These electronic aides, like earlier tables of antiderivatives, can remove some computational difficulties on the way to an answer, but it is still up to you to understand and set up the problems and to interpret and use the answers. Now, perhaps more than ever, it is important that you master the concepts of calculus and understand how these concepts are related and are used. **The computer may help you get an answer once the problem has been understood and formulated in mathematical terms, but it is your understanding of the concepts that will enable you to formulate the problems so computers can help.**

Although the computational techniques in this chapter are less important than they were several years ago, they still contribute to understanding calculus and to recognizing patterns of functions and their derivatives and antiderivatives.