**PROBLEM ANSWERS Chapter Eight**

**Section 8.1**

1. $\int = \lim_{A\to\infty} \{ -\frac{1}{2x^2} \Big|_{10}^{A} \} = \lim_{A\to\infty} \{ ( -\frac{1}{2A^2} ) - ( -\frac{1}{200} ) \} = \lim_{A\to\infty} \{ \frac{1}{200} - \frac{1}{2A^2} \} = \frac{1}{200}$ .

3. $\int = \lim_{A\to\infty} \{ 2\cdot\arctan(x) \Big|_{3}^{A} \} = \lim_{A\to\infty} \{ 2\cdot\arctan(A) - 2\cdot\arctan(3) \} = 2( \frac{\pi}{2} ) - 2\cdot\arctan(3) \approx 0.644$ .

5. Use $u = \ln(x)$ .

$\int = \lim_{A\to\infty} \{ 5\cdot\ln( \ln(x) ) \Big|_{e}^{A} \} = \lim_{A\to\infty} \{ 5\cdot\ln( \ln(A) ) - 5\cdot\ln( \ln(e) ) \} = \lim_{A\to\infty} \{ 5\cdot\ln( \ln(A) ) - 0 \} = \infty$ .

$\int$ DIVERGES.

7. $\int = \lim_{A\to\infty} \{ \ln(x-2) \Big|_{3}^{A} \} = \lim_{A\to\infty} \{ \ln(A-2) - \ln(3-2) \} = \lim_{A\to\infty} \{ \ln(A-2) - 0 \} = \infty$ . $\int$ DIVERGES.

9. $\int = \lim_{A\to\infty} \{ \frac{-1}{2(x-2)^2} \Big|_{3}^{A} \} = \lim_{A\to\infty} \{ \frac{-1}{2(A-2)^2} - \frac{-1}{2(3-2)^2} \} = \lim_{A\to\infty} \{ \frac{-1}{2(A-2)^2} + \frac{1}{2} \} = \frac{1}{2}$ .

11. $\int = \lim_{A\to\infty} \{ \frac{-1}{x+2} \Big|_{3}^{A} \} = \lim_{A\to\infty} \{ \frac{-1}{A+2} - \frac{-1}{3+2} \} = \lim_{A\to\infty} \{ \frac{-1}{A+2} + \frac{1}{5} \} = \frac{1}{5}$ .

13. $\int = \lim_{A\to 0^+} \{ 2\sqrt{x} \Big|_{A}^{4} \} = \lim_{A\to 0^+} \{ 2\sqrt{4} - 2\sqrt{A} \} = 4$ .

15. $\int = \lim_{A\to 0^+} \{ \frac{4}{3} x^{3/4} \Big|_{A}^{16} \} = \lim_{A\to 0^+} \{ \frac{4}{3}(16)^{3/4} - \frac{4}{3} A^{3/4} \} = \lim_{A\to 0^+} \{ \frac{32}{3} - \frac{4}{3} A^{3/4} \} = \frac{32}{3}$ .

17. $\int = \lim_{A\to 2^-} \{ \arcsin( \frac{x}{2} ) \Big|_{0}^{A} \} = \lim_{A\to 2^-} \{ \arcsin( \frac{A}{2} ) - \arcsin( \frac{0}{2} ) \} = \frac{\pi}{2} - 0 = \frac{\pi}{2}$ .

19. $\int = \lim_{A\to\infty} \{ -\cos(x) \Big|_{-2}^{A} \} = \lim_{A\to\infty} \{ -\cos(A) - -\cos(-2) \} = \lim_{A\to\infty} -0.416 - \cos(A)$ DNE. $\int$ DIVERGES.

21. $\int = \lim_{A\to\pi/2} \{ -\ln| \cos(x) | \Big|_{0}^{A} \} = \lim_{A\to\pi/2} \{ -\ln| \cos(A) | - -\ln| \cos(0) | \} = \lim_{A\to\pi/2} 0 - \ln| \cos(A) |$ DNE.

$\int$ DIVERGES.

23. (a) A = R: $\int_R^{2R} \frac{R^2P}{x^2}\, dx = R^2P(\frac{-1}{x})\Big|_R^{2R} = R^2P(\frac{-1}{2R} - \frac{-1}{R}) = \frac{1}{2}RP$.

A = 2R: $\int_R^{3R} \frac{R^2P}{x^2}\, dx = R^2P(\frac{-1}{x})\Big|_R^{3R} = R^2P(\frac{-1}{3R} - \frac{-1}{R}) = \frac{2}{3}RP$.

(b) $\lim_{A\to\infty} \int_R^{R+A} \frac{R^2P}{x^2}\, dx = \lim_{A\to\infty} R^2P(\frac{-1}{x})\Big|_R^{R+A} = \lim_{A\to\infty} R^2P(\frac{-1}{R+A} - \frac{-1}{R}) = R^2P(\frac{1}{R}) = RP.$

25. $\int_3^{\infty} \frac{1}{x(x^2+1)}\, dx = \int_3^{\infty} \frac{1}{x^3+x}\, dx < \int_3^{\infty} \frac{1}{x^3}\, dx$ which converges by the p–test.

Therefore, $\int_3^{\infty} \frac{1}{x(x^2+1)}\, dx$ converges.

27. Fof $x > 0$, $\ln(x) < x$ so $x + \ln(x) < 2x$ and $\frac{7}{x+\ln(x)} > \frac{7}{2x}$.

Then $\int_3^{\infty} \frac{7}{2x}\, dx = \frac{7}{2}\int_3^{\infty} \frac{1}{x}\, dx$ which diverges by the p–test. Therefore, $\int_3^{\infty} \frac{7}{x+\ln(x)}\, dx$ diverges.

29. $-1 \le \cos(x) \le 1$ so $0 \le 1 + \cos(x) \le 2$ and $0 \le \frac{1+\cos(x)}{x^2} \le \frac{2}{x^2}$. Then

$\int_7^{\infty} \frac{1+\cos(x)}{x^2}\, dx \le \int_7^{\infty} \frac{2}{x^2}\, dx$ which converges by the p–test. Therefore, $\int_7^{\infty} \frac{1+\cos(x)}{x^2}\, dx$ converges.

31. $V = \int_0^{\infty} \pi(\frac{1}{x^2+1})^2\, dx < \pi\int_0^{\infty} \frac{1}{x^4}\, dx$ which converges by the p–test.

Therefore, $V = \int_0^{\infty} \pi(\frac{1}{x^2+1})^2\, dx$ converges.

33. (a) $\int_1^{A} \frac{1}{x}\, dx < \sum_{k=1}^{A-1} \frac{1}{k}$ (b) $\int_1^{A} \frac{1}{x}\, dx > \sum_{k=2}^{A} \frac{1}{k}$

**Section 8.2**

1. $u = x^2 + 7$, $du = 2x\, dx$, $3\, du = 6x\, dx$: $\int = \int u^2\, 3\, du = u^3 + C = (x^2+7)^3 + C$.

3. $u = x^2 - 3$, $du = 2x\, dx$, $3\, du = 6x\, dx$: $\int = \int \frac{1}{\sqrt{u}}\, 3\, du = 3\cdot 2\sqrt{u} = 6\sqrt{x^2-3}\,\Big|_2^4 = 6\sqrt{13} - 6\sqrt{1} \approx 15.6$

5. $u = x^2 + 3$: $\int = 6 \ln(x^2 + 3) + C$.  7. $u = 3x + 2$: $\int = -\frac{1}{3}\cos(3x + 2) + C$.

9. $u = e^x + 3$, $du = e^x\,dx$: $\int = \int \sec^2(u)\,du = \tan(u) = \tan(e^x + 3)\Big|_0^1 = \tan(e + 3) - \tan(1+3) \approx -1.79$

11. $u = \ln(x)$, $du = \frac{1}{x}\,dx$: $\int = \int u\,du = \frac{1}{2}u^2 + C = \frac{1}{2}(\ln(x))^2 + C$.

13. $u = \sin(x)$, $du = \cos(x)\,dx$: $\int = \int e^u\,du = e^u + C = e^{\sin(x)} + C$.

15. $u = 3x$, $du = 3\,dx$: $\int = \int \frac{5}{1 + u^2}\,du = \frac{5}{3}\arctan(u) = \frac{5}{3}\arctan(3x)\Big|_1^3 = \frac{5}{3}\arctan(9) - \frac{5}{3}\arctan(3) \approx 0.35$

17. $u = \frac{1}{x}$, $du = \frac{-1}{x^2}\,dx$: $\int = \int -\cos(u)\,du = -\sin(u) = -\sin(\frac{1}{x})\Big|_1^2 = (-\sin(\frac{1}{2}) ) - (-\sin(1)) \approx 0.36$

19. $u = 5 + \sin^2(x)$, $du = 2\cdot\sin(x)\cdot\cos(x)\,dx$: $\int = \int \frac{1}{u}\,3\,du = 3\ln|u| + C = 3\ln|5 + \sin^2(x)| + C$.

21. $\int = 5\ln|2x + 5| + C$.

23. $\int = 2\ln|5x^2 + 3|\Big|_1^3 = 2\ln|48| - 2\ln|8| = 2\ln|\frac{48}{8}| = \ln(36) \approx 3.58$

25. $\int = \frac{7}{2}\arctan(\frac{x+3}{2})\Big|_0^1 = \frac{7}{2}\arctan(2) - \frac{7}{2}\arctan(1.5) \approx 0.44$

27. $u = e^x$, $du = e^x\,dx$: $\int = \int \frac{1}{1 + u^2}\,du = \arctan(u) + C = \arctan(e^x) + C$.

29. $u = 1 + \ln(x)$, $du = \frac{1}{x}\,dx$: $\int = \int \frac{3}{u}\,du = 3\ln|u| = 3\ln|1 + \ln(x)|\Big|_1^e = 3\ln|2| \approx 2.08$

31. $u = 1 - x^2$, $du = -2x\,dx$: $\int = \int -\sqrt{u}\,du = \frac{-2}{3}u^{3/2} = \frac{-2}{3}(1 - x^2)^{3/2}\Big|_0^1 = \frac{-2}{3}(0)^{3/2} - \frac{-2}{3}(1)^{3/2} = \frac{2}{3}$

33. $u = 1 + \sin(x)$, $du = \cos(x)\,dx$: $\int = \int u^3\,du = \frac{1}{4}u^4 + C = \frac{1}{4}(1 + \sin(x))^4 + C$.

35. $u = \ln(x)$, $du = \frac{1}{x}\,dx$: $\int = \int \sqrt{u}\,du = \frac{2}{3}u^{3/2} = \frac{2}{3}(\ln(x))^{3/2}\Big|_1^e = \frac{2}{3}(\ln(e))^{3/2} - \frac{2}{3}(\ln(1))^{3/2} = \frac{2}{3}$.

37. $u = 5 + \tan(x)$, $du = \sec^2(x)\,dx$: $\int = \int \frac{1}{u}\,du = \ln| u | + C = \ln| 5 + \tan(x) | + C$ .

39. $u = x - 5$, $du = dx$: $\int = \int \tan(u)\,du = \ln| \sec(u) | + C = \ln| \sec(x-5) | + C$ .

41. $u = 5x$, $du = 5\,dx$: $\int = \int \frac{1}{5} e^u\,du = \frac{1}{5} e^u = \frac{1}{5} e^{5x} \Big|_0^1 = \frac{1}{5} e^5 - \frac{1}{5} e^0 \approx 29.48$

43. $\int = \int \frac{7}{(x+2)^2 + 1}\,dx = 7\text{•}\arctan(x+2) + C$ .

45. $\int = \int \frac{2}{(x-3)^2 + 49}\,dx = \frac{2}{7}\text{•}\arctan\left(\frac{x-3}{7}\right) + C$ .

47. $\int = \int \frac{3}{(x+5)^2 + 4}\,dx = \frac{3}{2}\text{•}\arctan\left(\frac{x+5}{2}\right) + C$ .

49. $\int = \ln| x^2 + 4x + 5 | + 7\text{•}\arctan(x+2) + C$ .

51. $\int = 2\text{•}\ln| x^2 - 6x + 10 | + 19\text{•}\arctan(x-3) + C$ .

53. $\int = \int \frac{6x - 12}{x^2 - 4x + 13}\,dx + \int \frac{17}{(x-2)^2 + 9}\,dx = 3\text{•}\ln| x^2 - 4x + 13 | + \frac{17}{3}\text{•}\arctan\left(\frac{x-2}{3}\right) + C$ .

**Section 8.3**

1. $\int 12x\text{•}\ln(x)\,dx$ $u = \ln(x)$. Then $dv = 12x\,dx$, $du = \frac{1}{x}\,dx$, and $v = 6x^2$ .

$= uv - \int v\,du = \ln(x)\text{•}6x^2 - \int 6x^2\text{•}\frac{1}{x}\,dx = 6x^2\ln(x) - \int 6x\,dx =$ $\mathbf{6x^2 \ln(x) - 3x^2 + C}$ **.**

3. $\int x^4 \ln(x)\,dx$ $dv = x^4\,dx$. Then $u = \ln(x)$ , $du = \frac{1}{x}\,dx$, and $v = \frac{1}{5} x^5$ .

$= uv - \int v\,du = \ln(x)\text{•}\frac{1}{5} x^5 - \int \frac{1}{5} x^5\text{•}\frac{1}{x}\,dx = \frac{1}{5} x^5\text{•}\ln(x) - \int \frac{1}{5} x^4\,dx =$ $\mathbf{\frac{1}{5} x^5 \text{•}\ln(x) - \frac{1}{25} x^5 + C}$ .

5. $\int x\text{•}\arctan(x)\,dx$ $dv = x\,dx$. Then $u = \arctan(x)$, $du = \frac{1}{1+x^2}\,dx$ , and $v = \frac{1}{2} x^2$ .

$= uv - \int v\,du = \arctan(x)\text{•}\frac{1}{2} x^2 - \int \frac{1}{2} x^2\text{•}\frac{1}{1+x^2}\,dx = \frac{1}{2} x^2\text{•}\arctan(x) - \frac{1}{2}\int 1 - \frac{1}{1+x^2}\,dx$

$= \frac{1}{2} x^2\text{•}\arctan(x) - \frac{1}{2}\{ x - \arctan(x) \} + C =$ $\mathbf{\frac{1}{2} x^2 \text{•}\arctan(x) - \frac{1}{2}\text{•}x + \frac{1}{2}\text{•}\arctan(x) \} + C}$ **.**

7. $\int_0^1 \frac{x}{e^{3x}}\,dx = \int_0^1 x\text{•}e^{-3x}\,dx$. Put $u = x$. Then $dv = e^{-3x}\,dx$, $du = dx$, and $v = \frac{-1}{3} e^{-3x}$ .

$= uv - \int v\,du = x\text{•}\frac{-1}{3} e^{-3x} - \int \frac{-1}{3} e^{-3x}\,dx = x\text{•}\frac{-1}{3} e^{-3x} - \frac{1}{9} e^{-3x} \Big|_0^1$

$= \{ 1\text{•}\frac{-1}{3} e^{-3(1)} - \frac{1}{9} e^{-3(1)} \} - \{ 0\text{•}\frac{-1}{3} e^{-3(0)} - \frac{1}{9} e^{-3(0)} \} = \mathbf{\frac{1}{9} - \frac{4}{9} e^{-3}}$ .

9. $\int x\cdot\sec(x)\cdot\tan(x)\,dx$. Put $u = x$. Then $dv = \sec(x)\cdot\tan(x)\,dx$, $du = dx$, and $v = \sec(x)$.

$= uv - \int v\,du = x\cdot\sec(x) - \int \sec(x)\,dx = \mathbf{x\cdot\sec(x) - \ln|\sec(x) + \tan(x)| + C}$ .

11. $\int_{\pi/3}^{\pi/2} 7x\cdot\cos(3x)\,dx$ Put $u = 7x$. Then $dv = \cos(3x)\,dx$, $du = 7\,dx$, and $v = \frac{1}{3}\sin(3x)$ .

$= uv - \int v\,du = 7x\cdot\frac{1}{3}\sin(3x) - \int \frac{1}{3}\sin(3x)\cdot 7\,dx = \frac{7}{3}x\cdot\sin(3x) + \frac{7}{9}\cos(3x)\Big|_{\pi/3}^{\pi/2}$

$= \{ \frac{7}{3}\cdot\frac{\pi}{2}\cdot\sin(3\cdot\frac{\pi}{2}) + \frac{7}{9}\cos(3\cdot\frac{\pi}{2}) \} - \{ \frac{7}{3}\cdot\frac{\pi}{3}\cdot\sin(3\cdot\frac{\pi}{3}) + \frac{7}{9}\cos(3\cdot\frac{\pi}{3}) \} \approx -2.887$ .

13. $\int 12x\cdot\cos(3x^2)\,dx$ . **Use u–substitution!** Put $u = 3x^2$ . Then $du = 6x\,dx$ and $2\,du = 12x\,dx$.

$\int = \int \cos(u)\cdot 2\,du = 2\cdot\sin(u) + C = \mathbf{2\cdot\sin(3x^2) + C}$ .

15. $\int_1^3 \ln(2x + 5)\,dx$ . Put $u = \ln(2x + 5)$. Then $dv = dx$, $du = \frac{2}{2x+5}\,dx$, and $v = x$ .

$= uv - \int v\,du = \ln(2x + 5)\cdot x - \int x\cdot\frac{2}{2x+5}\,dx = x\cdot\ln(2x + 5) - \int 1 - \frac{5}{2x+5}\,dx$

$= x\cdot\ln(2x + 5) - \{ x - \frac{5}{2}\cdot\ln|2x + 5| \}\Big|_1^3 = \{ 3\cdot\ln(11) - 3 + \frac{5}{2}\cdot\ln|11| \} - \{ 1\cdot\ln(7) - 1 + \frac{5}{2}\cdot\ln|7| \}$

$= \frac{11}{2}\cdot\ln(11) - \frac{7}{2}\cdot\ln(7) - 2 \approx \mathbf{4.38}$ .

17. $\int_1^e (\ln(x))^2\,dx$ . Put $u = (\ln(x))^2$ . Then $dv = dx$, $du = 2\cdot\ln(x)\cdot\frac{1}{x}\,dx$ , and $v = x$.

$= uv - \int v\,du = (\ln(x))^2 x - \int x\cdot 2\cdot\ln(x)\cdot\frac{1}{x}\,dx$

$= x\cdot(\ln(x))^2 - \int 2\cdot\ln(x)\,dx = x\cdot(\ln(x))^2 - 2\{ x\cdot\ln(x) - x \}\Big|_1^e$

$= \{ e(\ln(e))^2 - 2e\cdot\ln(e) + 2e \} - \{ 1(\ln(1))^2 - 2\cdot\ln(1) + 2 \} = e - 2 \approx \mathbf{0.718}$ .

19. $\int \arcsin(x)\,dx$ . Put $u = \arcsin(x)$. Then $dv = dx$, $du = \frac{1}{\sqrt{1-x^2}}\,dx$ , and $v = x$ .

$= uv - \int v\,du = \arcsin(x)\cdot x - \int x\cdot\frac{1}{\sqrt{1-x^2}}\,dx = x\cdot\arcsin(x) - \int \frac{x}{\sqrt{1-x^2}}\,dx$ (use u–sub with $u = 1 - x^2$)

$= x\cdot\arcsin(x) - \int \frac{1}{\sqrt{u}}\left(\frac{-1}{2}\right)du = x\cdot\arcsin(x) + \sqrt{u} + C = \mathbf{x\cdot\arcsin(x) + \sqrt{1-x^2} + C}$ .

21. $\int x\cdot\arctan(3x)\,dx$ . Put $u = \arctan(3x)$. Then $dv = x\,dx$, $du = \frac{3}{1+9x^2}\,dx$, and $v = \frac{1}{2}x^2$ .

$= uv - \int v\,du = \arctan(x)\cdot\frac{1}{2}x^2 - \int \frac{1}{2}x^2\cdot\frac{3}{1+9x^2}\,dx = \frac{1}{2}x^2\cdot\arctan(3x) - \frac{1}{2}\int \frac{1}{3} - \frac{1/3}{1+9x^2}\,dx$

$= \frac{1}{2}x^2\cdot\arctan(3x) - \frac{1}{2}\{\frac{x}{3} - \frac{1}{9}\cdot\arctan(3x)\} + C = \mathbf{\frac{1}{2}x^2\cdot\arctan(3x) - \frac{1}{6}x + \frac{1}{18}\arctan(3x)\} + C}$.

23. $\int_1^2 \frac{\ln(x)}{x}\,dx$. **Use u–substitution!** Put $u = \ln(x)$. Then $du = \frac{1}{x}\,dx$.

$\int = \int u\,du = \frac{1}{2}u^2 = \frac{1}{2}(\ln(x))^2\Big|_1^2 = \frac{1}{2}(\ln(2))^2 - \frac{1}{2}(\ln(1))^2 = \frac{1}{2}(\ln(2))^2 \approx \mathbf{0.240}$ .

25. (a) $\int \sin^3(x)\,dx = \frac{1}{3}\{-S^2\cdot C + 2\int S\,dx\} = \frac{1}{3}\{-S^2\cdot C - 2C\} + K$

$= \mathbf{\frac{1}{3}\{-\sin^2(x)\cdot\cos(x) - 2\cdot\cos(x)\} + K}$ .

(b) $\int \sin^4(x)\,dx = \frac{1}{4}\{-S^3\cdot C + 3\int S^2\,dx\} = \frac{1}{4}\{-S^4\cdot C + 3[\frac{1}{2}(-SC + x)]\} + K$

$= \frac{1}{4}\{-S^3\cdot C - \frac{3}{2}SC + \frac{3}{2}x\} + K$

$= \mathbf{\frac{1}{4}\{-\sin^3(x)\cdot\cos(x) - \frac{3}{2}\sin(x)\cdot\cos(x) + \frac{3}{2}x\} + K}$ (c) on your own.

27. (a) $\int \sec^3(x)\,dx = \frac{1}{2}\{\sec(x)\tan(x) + \int \sec(x)\,dx\} = \mathbf{\frac{1}{2}\{\sec(x)\cdot\tan(x) + \ln|\sec(x) + \tan(x)|\} + K}$

(b) and (c) on your own.

29. $\int \cos^3(2x+3)\,dx$ . First do a substitution: $u = 2x + 3$. Then $du = 2\,dx$ and $dx = \frac{1}{2}\,du$ .

$= \int \frac{1}{2}\cos^3(u)\,du = \frac{1}{2}\{\frac{1}{3}(C^2\cdot S + 2\int C\,du)\} = \frac{1}{6}\{C^2\cdot S + 2S\} + K$

$= \frac{1}{6}\{\cos^2(u)\cdot\sin(u) + 2\sin(u)\} + K = \mathbf{\frac{1}{6}\{\cos^2(2x+3)\cdot\sin(2x+3) + 2\sin(2x+3)\} + K}$

31. $\int x\cdot(2x+5)^{19}\,dx$ .

(a) By parts: put $u = x$. Then $dv = (2x+5)^{19}\,dx$ , $du = dx$ , and $v = \frac{1}{40}(2x+5)^{20}$ .

$\int = uv - \int v\,du = x\cdot\frac{1}{40}(2x+5)^{20} - \int \frac{1}{40}(2x+5)^{20}\,dx$

$= x\cdot\frac{1}{40}(2x+5)^{20} - \frac{1}{40}\frac{1}{42}(2x+5)^{21} + C.$

(b) Substitution: put $u = 2x + 5$. Then $du = 2\,dx$ and $dx = \frac{1}{2}\,du$. Also, $x = \frac{1}{2}(u-5)$ .

$\int = \int \frac{1}{2}(u-5)\cdot u^{19}\,\frac{1}{2}\,du = \frac{1}{4}\int u^{20} - 5u^{19}\,du = \frac{1}{4}\{\frac{1}{21}u^{21} - \frac{5}{20}u^{20}\} + C$

$= \frac{1}{84}(2x+5)^{21} - \frac{5}{80}(2x+5)^{20} + C.$

The answers (antiderivatives) in parts (a) and (b) look different, but you can check that the derivative of each answer is $x\cdot(2x+5)^{19}$ .

33. (a) Make an informed prediction.

(b) $\int_0^1 \sin(x)\,dx = -\cos(x)\Big|_0^1 = (-\cos(1)) - (-\cos(0)) = \cos(0) - \cos(1) \approx 1 - 0.54 = \mathbf{0.46}$ .

$\int_0^1 x\text{•}\sin(x)\,dx = -x\text{•}\cos(x) + \sin(x)\Big|_0^1 = \{-1\text{•}\cos(1) + \sin(1)\} - \{-0\text{•}\cos(0) + \sin(0)\} = \sin(1) - \cos(1) \approx \mathbf{0.30}$ .

35. (a) Make an informed prediction.

(b) See problem 17: $V_{x-axis} = \int_{x=1}^{e} \pi(\ln(x))^2\,dx = \pi(e-2) \approx \mathbf{2.257}$ .

$V_{y-axis} = \int_{y=0}^{1} 2\pi\text{•}y\text{•}e^y\,dy = 2\pi \int_{y=0}^{1} y\text{•}e^y\,dy$ (use integration by parts with $u = y$, $dv = e^y\,dy$ : see Example 2)

$= 2\pi(y\text{•}e^y - e^y)\Big|_0^1 = 2\pi(1\text{•}e^1 - e^1) - 2\pi(0\text{•}e^0 - e^0) = 2\pi(0) - 2\pi(-1) = 2\pi \approx \mathbf{6.283}$ .

37. On your own.

39. $\int x^2\text{•}\arctan(x)\,dx$. Put $u = \arctan(x)$. Then $dv = x^2\,dx$, $du = \frac{1}{x^2+1}\,dx$ , and $v = \frac{1}{3}x^3$ .

$= uv - \int v\,du = \arctan(x)\text{•}\frac{1}{3}x^3 - \int \frac{1}{3}x^3 \frac{1}{x^2+1}\,dx$

$= \frac{1}{3}x^3\text{•}\arctan(x) - \frac{1}{3}\int x - \frac{x}{x^2+1}\,dx$ (dividing $x^3$ by $x^2+1$)

$= \frac{1}{3}x^3\text{•}\arctan(x) - \frac{1}{3}\{\frac{1}{2}x^2 - \frac{1}{2}\ln(x^2+1)\} + C$

$= \mathbf{\frac{1}{3}x^3\text{•}\arctan(x) - \frac{1}{6}x^2 + \frac{1}{6}\ln(x^2+1)\,)\} + C}$ .

40 – 50. On your own.

**Section 8.4**

1. $= \frac{A}{x} + \frac{B}{x+1} = \frac{2}{x} + \frac{5}{x+1}$

3. $= \frac{A}{x+1} + \frac{B}{x+8} = \frac{2}{x+1} + \frac{9}{x+8}$

5. Divide first: $\frac{2x^2+15x+25}{x^2+5x} = 2 + \frac{5x+25}{x^2+5x} = 2 + \frac{5(x+5)}{x(x+5)} = 2 + \frac{5}{x}$ .

7. $\frac{6x^2+9x-15}{x(x+5)(x-1)} = \frac{A}{x} + \frac{B}{x+5} + \frac{C}{x-1} = \frac{A(x^2+4x-5) + B(x^2-x) + C(x^2+5x)}{x(x+5)(x-1)}$ .

Solving
$x^2$: A + B + C = 6
x: 4A – B + 5C = 9
k: –5A = – 15 we get A = 3, B = 3, and C = 0 so

$$\frac{6x^2 + 9x - 15}{x(x+5)(x-1)} = \frac{3}{x} + \frac{3}{x+5} + \frac{0}{x-1} = \frac{3}{x} + \frac{3}{x+5} \ .$$

9. $$\frac{8x^2 - x + 3}{x(x^2+1)} = \frac{A}{x} + \frac{Bx + C}{x^2+1} = \frac{A(x^2+1) + x(Bx + C)}{x(x^2+1)} \ .$$

Solving
$$\begin{array}{llllll} x^2: & A + & B & & = 8 & \\ x: & & & C & = -1 & \\ k: & A & & & = 3 & \text{we get } A = 3, B = 5, \text{ and } C = -1 \text{ so} \end{array}$$

$$\frac{8x^2 - x + 3}{x(x^2+1)} = \frac{3}{x} + \frac{5x - 1}{x^2+1} \ .$$

11. $$\frac{11x^2 + 23x + 6}{x^2(x+2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+2} = \frac{A(x)(x+2) + B(x+2) + C(x^2)}{x^2(x+2)} \ .$$

Solving
$$\begin{array}{llllll} x^2: & A + & & C & = 11 & \\ x: & 2A + & B & & = 23 & \\ k: & & 2B & & = 6 & \text{we get } A = 10, B = 3, \text{ and } C = 1 \text{ so} \end{array}$$

$$\frac{11x^2 + 23x + 6}{x^2(x+2)} = \frac{10}{x} + \frac{3}{x^2} + \frac{1}{x+2} \ .$$

13. $$\int \frac{3x + 13}{(x+2)(x-5)} \, dx = \int \frac{-1}{x+2} + \frac{4}{x-5} \, dx = \mathbf{-\ln|\, x + 2 \,| + 4\cdot\ln|\, x - 5 \,| + C} \ .$$

15. $$\int_2^5 \frac{2}{x^2 - 1} \, dx = \int_2^5 \frac{-1}{x+1} + \frac{1}{x-1} \, dx = -\ln|\, x+1 \,| + \ln|\, x - 1 \,|$$

$$= \ln\left|\frac{x-1}{x+1}\right| \Bigg|_2^5 = \ln(\tfrac{4}{6}) - \ln(\tfrac{1}{3}) \approx \mathbf{0.693} \ .$$

17. (1) $\displaystyle\int \frac{2}{x} + \frac{5}{x+1} \, dx = \mathbf{2\cdot\ln|\, x \,| + 5\cdot\ln|\, x + 1 \,| + C}$

(2) $\displaystyle\int \frac{3}{x+3} + \frac{4}{x-1} \, dx = \mathbf{3\cdot\ln|\, x + 3 \,| + 4\cdot\ln|\, x - 1 \,| + C}$

19. $$\int \frac{2x^2 + 5x + 3}{x^2 - 1} \, dx = \int 2 + \frac{5}{x-1} \, dx = \mathbf{2x + 5\cdot\ln|\, x - 1 \,| + C} \ .$$

21. $$\int \frac{3x^2 + 19x + 24}{x^2 + 6x + 5} \, dx = \int 3 + \frac{2}{x+1} + \frac{-1}{x+5} \, dx = \mathbf{3x + 2\cdot\ln|\, x + 1 \,| - \ln|\, x + 5 \,| + C} \ .$$

23. $\displaystyle\int \frac{3x^2 - 1}{x^3 - x} \, dx$ . Use u–substitution with $u = x^3 - x$. Then $du = 3x^2 - 1$ so

$\displaystyle\int \; = \int \frac{1}{u} \, du = \ln|\, u \,| + C = \ln|\, x^3 - x| + C$. A partial fraction decomposition also works but takes longer.

25. $\int \frac{x^3 + 3x^2 - 4x + 30}{x^2 + 3x - 10}\, dx = \int x + \frac{6}{x-2}\, dx = \mathbf{\frac{1}{2} x^2 + 6 ln| x - 2 | + C}$ .

27. $\int \frac{12x^2 + 19x - 6}{x^3 + 3x^2}\, dx = \int \frac{7}{x} + \frac{-2}{x^2} + \frac{5}{x+3}\, dx = \mathbf{7 \cdot ln| x | + \frac{2}{x} + 5 \cdot ln| x+3 | + C}$ .

29. $\int \frac{7x^2 + 3x + 7}{x^3 + x}\, dx = \int \frac{7}{x} + \frac{3}{x^2 + 1}\, dx = \mathbf{7 \cdot ln| x | + 3 \cdot arctan( x ) + C}$ .

31. (a) $\int \frac{1}{x^2 + 2x + \mathbf{2}}\, dx = \int \frac{1}{(x+1)^2 + 1}\, dx = \mathbf{arctan( x+1 ) + C}$ .

(b) $\int \frac{1}{x^2 + 2x + \mathbf{1}}\, dx = \int \frac{1}{(x+1)^2}\, dx = -( x + 1 )^{-1} + C = \mathbf{\frac{-1}{x+1} + C}$ .

(c) $\int \frac{1}{x^2 + 2x + \mathbf{0}}\, dx = \int \frac{1}{x(x+2)}\, dx = \int \frac{1/2}{x} + \frac{-1/2}{x+2}\, dx = \mathbf{\frac{1}{2} ln| x | - \frac{1}{2} ln| x+2 | + C}$ .

33. Prob. 1: $f(x) = \frac{2}{x} + \frac{5}{x+1} = 2 \cdot (x)^{-1} + 5 \cdot (x+1)^{-1}$ . Then

$f'(x) = -2 \cdot (x)^{-2} - 5 \cdot (x+1)^{-2}$ and $f''(x) = 4 \cdot (x)^{-3} + 10 \cdot (x+1)^{-3}$ .

Prob. 3: $g(x) = \frac{3}{x+3} + \frac{4}{x-1} = 3 \cdot (x+3)^{-1} + 4 \cdot (x-1)^{-1}$ . Then

$g'(x) = -3 \cdot (x+3)^{-2} - 4 \cdot (x-1)^{-2}$ and $g''(x) = 6 \cdot (x+3)^{-3} + 8 \cdot (x-1)^{-3}$ .

35. Prob. 5: $f(x) = \frac{2x^2 + 15x + 15}{x^2 + 5x} = 2 + \frac{5}{x}$ so $f'(x) = \frac{-5}{x^2}$ and $f''(x) = \frac{10}{x^3}$ .

Prob. 6: On your own.

37. (a) Solve $\frac{dx}{dt} = x(100 - x)$ .

Separate the variables: $\frac{1}{x(100-x)}\, dx = dt$ .

Use partial fractions: $\frac{1}{x(100-x)} = \frac{A}{x} + \frac{B}{100-x} = \frac{0.01}{x} + \frac{0.01}{100-x}$ (solving $-A+B = 0$ and $100A = 1$)

so $\left\{ \frac{0.01}{x} + \frac{0.01}{100-x} \right\} dx = dt$ and, integrating, $\int \frac{0.01}{x} + \frac{0.01}{100-x}\, dx = \int 1\, dt$ . Then

$0.01 \cdot \ln| x | - 0.01 \cdot \ln| 100-x | = t + C$ so $\ln\left| \frac{x}{100-x} \right| = 100t + K$. ( $K = 100C$ is a constant )

Using the initial condition $x(0) = 150$: $\ln\left| \frac{150}{100-150} \right| = \ln( 3 ) = 100 \cdot (0) + K$ so $K = \ln( 3 )$.

Finally, $\ln\left| \frac{x}{100-x} \right| = 100 \cdot t + \ln(3)$ so $\left| \frac{x}{100-x} \right| = e^{100t} \cdot e^{\ln(3)} = 3 \cdot e^{100t}$ and

$$\mathbf{x = \frac{-300 \cdot e^{100t}}{1 - 3 \cdot e^{100t}}}$$

. Graph this on your own.

(b) In this part it is easier to use the form $\left| \frac{x}{100-x} \right| = 3\cdot e^{100t}$ .

Put $x = 120$ and solve $\left| \frac{120}{100-120} \right| = 3\cdot e^{100t}$ : $6 = 3\cdot e^{100t}$ so $t = \frac{1}{100}\ln(2) \approx$ **0.0069** .

Put $x = 110$ and solve $\left| \frac{110}{100-110} \right| = 3\cdot e^{100t}$ : $11 = 3\cdot e^{100t}$ so $t = \frac{1}{100}\ln(\frac{11}{3}) \approx$ **0.013** .

Put $x = 100$ . Then $\left| \frac{x}{100-x} \right|$ is undefined (division by 0) so $x(t)$ is never equal to 100 .

(c) limit (as t becomes arbitrarily large) of $x = \dfrac{-300\cdot e^{100t}}{1 - 3\cdot e^{100t}}$ is $\dfrac{-300}{-3} = 100$.

(d) The population $x(t)$ is decling to the "carrying capacity" $M = 100$ of the enviroment.

39. (a) Solve $\frac{dx}{dt} = (7-x)(5-x)$ by separating the variables and using partial fractions to rewrite the fraction.

$\dfrac{7-x}{5-x} = \dfrac{7}{5}e^{2kt}$ and $x(t) = \dfrac{7\cdot e^{2kt} - 7}{\frac{7}{5}\cdot e^{2kt} - 1}$ . (As t gets big, x approaches 5.)

(b) Solve $\frac{dx}{dt} = (6-x)(6-x) = (6-x)2$ by separating the variables: $\dfrac{1}{(6-x)^2}\,dx = dt$ and integrating.

$\dfrac{1}{6-x} = t + C$ $(C = \frac{1}{6})$ so $x = \dfrac{6t+1}{t+\frac{1}{6}} = \dfrac{36t+6}{6t+1}$ . (As t gets big, x approaches 6.)

**Section 8.5**

1. $x = 3\cdot\sin(\theta)$ (a) $9 - x^2 = 9 - 9\sin^2(\theta) = 9(1 - \sin^2(\theta)) = 9\cos^2(\theta)$ so $\dfrac{1}{\sqrt{9-x^2}} = \dfrac{1}{3\cos(\theta)}$ .

(b) $dx = 3\cos(\theta)\,d\theta$

3. $x = 3\cdot\sec(\theta)$ (a) $x^2 - 9 = 9\sec^2(\theta) - 9 = 9(\sec^2(\theta) - 1) = 9\tan^2(\theta)$ so $\dfrac{1}{\sqrt{x^2-9}} = \dfrac{1}{3\tan(\theta)}$ .

(b) $dx = 3\sec(\theta)\tan(\theta)\,d\theta$

5. $x = \sqrt{2}\,\tan(\theta)$ (a) $2 + x^2 = 2 + 2\tan^2(\theta) = 2(1 + \tan^2(\theta)) = 2\sec^2(\theta)$ so $\dfrac{1}{\sqrt{2+x^2}} = \dfrac{1}{\sqrt{2}\,\sec(\theta)}$ .

(b) $dx = \sqrt{2}\,\sec^2(\theta)\,d\theta$

7. $x = 3\cdot\sin(\theta)$ (a) $\theta = \arcsin(x/3)$

(b) & (c) $f(\theta) = \cos(\theta)\cdot\tan(\theta) = \cos(\arcsin(x/3))\cdot\tan(\arcsin(x/3)) = \dfrac{\sqrt{9-x^2}}{3}\cdot\dfrac{x}{\sqrt{9-x^2}} = \dfrac{x}{3}$ .

9. $x = 3\cdot\sec(\theta)$ (a) $\theta = \text{arcsec}(x/3)$

(b) & (c) $f(\theta) = \sqrt{1+\sin^2(\theta)} = \sqrt{1 + \sin^2(\text{arcsec}(x/3))} = \sqrt{1 + \left(\dfrac{\sqrt{x^2-9}}{x}\right)^2}$

11. $x = 5\cdot\tan(\theta)$ (a) $\theta = \arctan(x/5)$

$$\text{(b) \& (c)} \quad f(\theta) = \frac{\cos^2(\theta)}{1 + \cot(\theta)} = \frac{\left(\frac{5}{\sqrt{25 + x^2}}\right)^2}{1 + \left(\frac{5}{x}\right)} = \frac{\frac{25}{25 + x^2}}{1 + \frac{5}{x}}$$

13. Same as Practice 3. Sorry.

15. $x = 7\cdot\tan(\theta)$. $dx = 7\sec^2(\theta)\, d\theta$ . $x^2 + 49 = 49\tan^2(\theta) + 49 = 49(\tan^2(\theta) + 1) = 49\sec^2(\theta)$ .

$$\int \frac{1}{\sqrt{x^2 + 49}}\, dx = \int \frac{1}{\sqrt{49\sec^2(\theta)}}\, 7\sec^2(\theta)\, d\theta$$

$$= \int \sec(\theta)\, d\theta = \ln|\sec(\theta) + \tan(\theta)| + C = \ln|\sec(\arctan(x/7)) + \tan(\arctan(x/7))| + C$$

$$= \ln\left|\frac{\sqrt{x^2 + 49}}{7} + \frac{x}{7}\right| + C .$$

17. $x = 6\cdot\sin(\theta)$. $dx = 6\cos(\theta)\, d\theta$ . $36 - x^2 = 36\cos^2(\theta)$ .

$$\int \sqrt{36 - x^2}\, dx = \int \sqrt{36 - \cos^2(\theta)}\; 6\cos(\theta)\, d\theta = 36 \int \cos^2(\theta)\, d\theta = \text{(use Table \#14)}$$

$$= 36\left\{\frac{1}{2}\theta + \frac{1}{2}\sin(\theta)\cdot\cos(\theta)\right\} + C = 36\left\{\frac{1}{2}\arcsin(x/6) + \frac{1}{2}\left(\frac{x}{6}\right)\cdot\left(\frac{\sqrt{36 - x^2}}{6}\right)\right\} + C .$$

19. $x = 6\cdot\tan(\theta)$. $dx = 6\cdot\sec^2(\theta)\, d\theta$ . $36 + x^2 = 36\sec^2(\theta)$ .

$$\int \sqrt{36 + x^2}\, dx = \int \frac{1}{\sqrt{36 + x^2}}\; 6\cdot\sec^2(\theta)\, d\theta = \int \sec(\theta)\, d\theta = \text{(use Table \#11)}$$

$$= \ln|\sec(\theta) + \tan(\theta)| + C = \ln\left|\frac{\sqrt{36 + x^2}}{6} + \frac{x}{6}\right| + C \text{ or } \ln\left|\sqrt{36 + x^2} + x\right| + K.$$

21. Similar to 19: $x = 7\cdot\tan(\theta)$. $\displaystyle\int \frac{1}{\sqrt{49 + x^2}}\, dx = \ln\left|\sqrt{49 + x^2} + x\right| + C.$

23. $x = 5\cdot\sin(\theta)$. $\displaystyle\int = -5\cos(\arcsin(x/5)) + C = -5\,\frac{\sqrt{25 - x^2}}{5} + C = -\sqrt{25 - x^2} + C.$

25. $x = 7\cdot\tan(\theta)$. $\displaystyle\int = -\ln|\cos(\arctan(x/7))| + C = -\ln\left|\frac{7}{\sqrt{49 + x^2}}\right| + C$ (now some algebra)

$$= -\ln|7| + \ln\left|\sqrt{49 + x^2}\right| + C = \frac{1}{2}\ln|49 + x^2| + K .$$

(A u–substitution with $u = 49 + x^2$ is **much** easier.)

27. $x = 3\cdot\sec(\theta)$. $\int = \frac{1}{9}\int \frac{\cos(\theta)}{\sin^2(\theta)}\,d\theta$ = (put $u = \sin(\theta)$ ) $\frac{-1}{9}\frac{1}{\sin(\theta)} + C = -\csc(\theta) + C = \frac{-1}{9}\frac{x}{\sqrt{x^2+9}} + C$ .

29. $x = 5\cdot\sec(\theta)$. $\int = \frac{1}{2}\,\theta + C = \frac{1}{2}\,\mathrm{arcsec}\left(\frac{x}{5}\right) + C$ .

31. $x = 5\cdot\sin(\theta)$. $\int = \frac{1}{5}\ln|\sec(\theta) + \tan(\theta)| + C = \frac{1}{5}\ln\left|\frac{5}{\sqrt{25-x^2}} + \frac{x}{\sqrt{25-x^2}}\right| + C$

= (after lots of algebra) $\frac{1}{10}\ln\left|\frac{5+x}{5-x}\right| + C$.

33. Similar to 19. $\int = \ln\left|\frac{\sqrt{a^2+x^2}}{a} + \frac{x}{a}\right| + C = \ln\left|\sqrt{a^2+x^2} + x\right| + K$.

35. $x = a\cdot\tan(\theta)$. $\int = \frac{-1}{a^2}\frac{\sqrt{a^2+x^2}}{x} + C$.

37. $x + 1 = u$. Then $u = 3\cdot\tan(\theta)$ . $\int = \ln|\sec(\theta) + \tan(\theta)| + C = \ln\left|\frac{\sqrt{u^2+9}}{3} + \frac{u}{3}\right| + C$

$= \ln\left|\sqrt{u^2+9} + u\right| \mathbf{- \ln(3) + C} = \ln\left|\sqrt{u^2+9} + u\right| + \mathbf{K} = \ln\left|\sqrt{(x+1)^2+9} + (x+1)\right| + K$.

39. $\frac{1}{2}\arctan\left(\frac{x+5}{2}\right) + C$

41. $\ln\left|(x+2) + \sqrt{x^2+4x+3}\right| + C$.

**Section 8.6**

1. $\int \sin^2(3x)\,dx = \frac{x}{2} - \frac{\sin(6x)}{12} + C = \frac{x}{2} - \frac{\sin(3x)\cdot\cos(3x)}{6} + C$ .

3. Put $u = \sin(e^x)$ . $\int = \frac{1}{2}\sin^2(e^x) + C$ . ( If you put $w = \cos(e^x)$ then $\int = -\frac{1}{2}\cos^2(e^x) + C$ .)

5. $\frac{3}{8}\pi$

7. $\frac{4}{21}$

9. $\frac{1}{14}\sin^2(7x) + C$ or $-\frac{1}{14}\cos^2(7x) + C$

11. Put $u = \cos(7x)$. $\int = -\frac{1}{28}\cos^4(7x) + C$ .

13. $\int \sin^2(3x)\cos^2(3x)\,dx = \int \frac{1}{2}(1-\cos(6x))\frac{1}{2}(1+\cos(6x))\,dx$

$$= \frac{1}{4}\int 1-\cos^2(6x)\,dx = \frac{x}{4} - \frac{1}{4}\int \cos^2(6x)\,dx$$

$$= \frac{x}{4} - \frac{1}{4}\left\{ \frac{x}{2} + \frac{\sin(12x)}{24} \right\} + C\,.$$

15. $\int = \frac{1}{10}\tan^2(5x) + C.$

17. $\int = \frac{1}{9}\sec^3(3x) + C\,.$

19. $m \neq n$. $\displaystyle\int_0^{2\pi} \sin(mx)\sin(nx)\,dx = \frac{1}{2}\left\{ \frac{\sin((m-n)x)}{m-n} - \frac{\sin((m+n)x)}{m+n} \right\}\Bigg|_0^{2\pi}$

$$= \frac{1}{2}\left\{ \frac{\sin((m-n)2\pi)}{m-n} - \frac{\sin((m+n)2\pi)}{m+n} \right\} - \frac{1}{2}\left\{ \frac{\sin((m-n)0)}{m-n} - \frac{\sin((m+n)0)}{m+n} \right\}$$

$$= \frac{1}{2}\{0-0\} - \frac{1}{2}\{0-0\} = 0\,.$$

21. $\displaystyle\int_0^{2\pi} \sin(mx)\sin(mx)\,dx = \int_0^{2\pi} \sin^2(mx)\,dx = \frac{x}{2} - \frac{\sin(mx)\cos(mx)}{2m}\Bigg|_0^{2\pi}$

$$= \left\{ \frac{2\pi}{2} - \frac{\sin(m2\pi)\cos(m2\pi)}{2m} \right\} - \left\{ \frac{0}{2} - \frac{\sin(0)\cos(0)}{2m} \right\} = \pi\,.$$

23. On your own.