13.0 INTRODUCTION TO FUNCTIONS OF SEVERAL VARIABLES

Any situation where the result or measurement (the output) depends on several circumstances (the inputs) is a function of those circumstances, a function of several variables. Such functions of several variables are extremely common in the world around us, and the main ideas and techniques of calculus extend in natural ways to describe and analyze such functions. The world around us is three (or more) dimensional and very few outcomes in experiments or our daily life depend on only a single input. Functions of several variables can be used to describe and illustrate these functions, and the calculus of functions of several variables can help us analyze them.

The temperature at each location (x,y) on the surface of a stove depends on the location (x,y) and is a function of the two variables x and y. Fig. 1 is a table of temperature values at locations on the top of the stove. Fig. 2 is a graphical represention of temperatures at many locations on the top of the stove. The temperature at each location (x,y,z) in a room also depends on the location (x,y,z) and is a function of the three variables x, y, and z. This situation requires a much larger table of values and is more difficult to represent graphically.

side \ front	0	2	4	6	8	10	12	14
0	80	80	80	80	80	80	80	80
2	80	100	110	100	80	80	80	80
4	80	110	280	110	80	80	80	80
6	80	100	110	100	80	80	80	80
8	80	80	80	80	100	110	110	100
10	80	80	80	80	110	480	480	110
12	80	80	80	80	110	480	480	110
14	80	80	80	80	100	110	110	100

Fig. 1: Table of Stove Top Temperatures

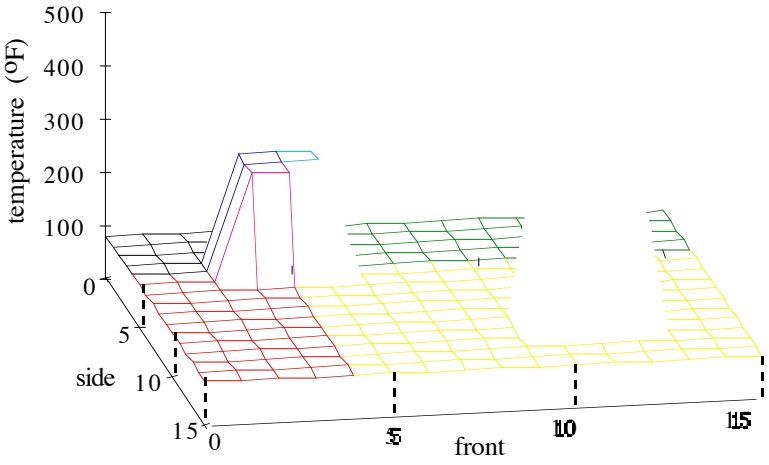

Fig. 2: Graph of Stove Top Temperatures

Almost every decision we make depends on several pieces of information. The amount of money you are willing to pay for a cool drink is a function of the temperature, your thirst (which may partially depend on the temperature), how much money you have with you, and probably several other variables. A politician's vote on a particular issue is typically a function of the politician's own view of the issue, the views of the voters in the politician's state, the closeness of the next election, and many other things. Even your calculus grade is a function of many variables:

calculusgrade(you) = f(effort, brains, time, testing skill, ... , mood of teacher, a little luck) .

It is not smart to rely on the impact of these last two variables for this function.

Chapter 13 extends the ideas and techniques and applications of **differential** calculus to functions of more than one variable.

- Section 13.1 introduces the basic definitions and tools needed to examine and visualize functions of several variables. Just as your earlier work in calculus used the interplay among functions given by data, the graphs of functions, and functions defined by formulas, this section has a similar emphasis. The main difference from earlier work is that there are several ways to graphically represent a function of 2 (or more variables), and you need to become familiar with these different graphical representations.

- Section 13.2 extends the ideas of limits and continuity to functions of several variables and examines the graphic meaning of these ideas for functions of two variables.

- Sections 13.3, 13.4, and 13.5 extend the ideas and definitions of derivatives, rates of change, and tangent lines of $y = f(x)$ to derivatives, rates of change in different directions, and tangent planes of $z = f(x,y)$ and functions of more than two variables.

- Sections 13.6 and 13.7 use the ideas of derivatives to introduce and show methods for solving maximum and minimum problems of functions of several variables.

- Section 13.8 discusses the Chain Rule (actually, Chain Rule**s**) for functions of several variables.

Chapters 14 and 15 extend the ideas, techniques, and applications of **integral** calculus to functions of several variables.

In these chapters we will focus primarily on functions of two variables because it is possible to visualize such functions and because most of the concepts and techniques and applications (and difficulties) of working with functions of several variables can be observed with functions of only two variables.

PROBLEMS

List 4 or 5 of the most important variables that determine

where you decided to attend college	what you are charged for automobile insurance
what you will eat tonight	whether you think a calculus teacher is good
what you will do next Saturday evening	what career you want to have
which personal computer you would buy	
how rapidly a disease spreads through a town	how much weight a bridge can support