

A Message

Chapter 14 and the previous sections of Chapter 15 have focused on integrals of scalar and vector-valued functions in various ways, and they were mainly computational.

The remaining sections of Chapter 15 are also computational. But the main content introduces and examines important theorems relating line integrals, the gradient, the curl and the divergence to each other.

Part of the beauty of mathematics is that it shows and verifies connections between apparently very different ideas such as derivatives and integrals. Part of the power of mathematics is that once we have these connections we can trade one problem for an equivalent problem that might be easier to solve. For example, the beauty of the Fundamental Theorem of Calculus is that it shows the deep connection between derivatives and integrals. The power of the FTC is that it allows us to trade the difficult problem of finding the area under a curve, the integral of $f(x)$ from $x=a$ to $x=b$, for the much easier subtraction problem, $F(b)-F(a)$, if we can find a function F so that $F'(x)=f(x)$.

Each of these important new theorems (Fundamental Theorem of Line Integrals, Green's Theorem, Stoke's Theorem and the Divergence Theorem) shows deep connections between apparently different ideas and each allows us trade one kind of calculation for another one that might be easier. Each of our new theorems has beauty and power like the Fundamental Theorem of Calculus, and each generalizes the Fundamental Theorem of Calculus in some way. You still need to know how to do calculations, but it is vital to also focus on what each of these theorems says and the relationships among the different operations. Concepts are very important in the coming sections so think about what each of these new operations does and how it is related to the others. Think about each trade.