Math 152

Show Your Work!   March 14, 2011 (= pi day)   Name _____
  Good Luck!   TEST **3A**   (please print =1 point)

1.   **(a)** Find $\bar{x}$ for the shaded region R in Fig. 1.

(Timesaving hint: area of region R is 1/4)
Do this using antiderivatives -- SHOW WORK.

$\bar{x}$ = _____

(4)(4)

Fig. 1
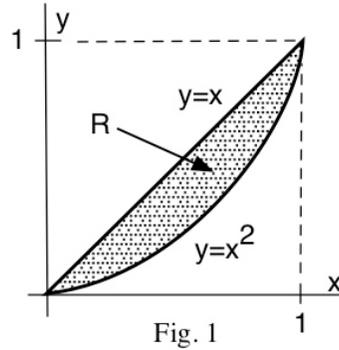
(b) Find the **volume** (using antiderivatives) when region R is rotated around the line x=1:

V = $\displaystyle\int$ = _____ (fraction)

(4)(3)

2.   "The initial population P of the town was 1700 people and the rate of change

of the population was proportional to the square root of the population."

(a)   Translate this statement into an **initial value differential equation.**

(4)   (Note: This is NOT the exponential growth model we did in class.)

(3)   (b)   Separate the variables from part (a)   (**Do Not solve the D.E.**):

3. $\dfrac{dy}{dx} = x - y$ (see Fig. 2)

(2)   (a)   Is $y = x - 1$ a solution of this differential equation?   **Yes**   **No**

(b)   Is $y = x - 1 + e^{-x}$ a solution of this differential equation?   **Yes**   **No**

(2)

Fig. 2

(c)   What is the slope of the circled "tangent segment" in Fig. 2?

(2)   Slope = _____

(d)   On the direction field in Fig. 2, sketch the

(3)   solution that goes through point **A.**

(e)   On the direction field in Fig. 2, sketch the solution that satisfies the initial

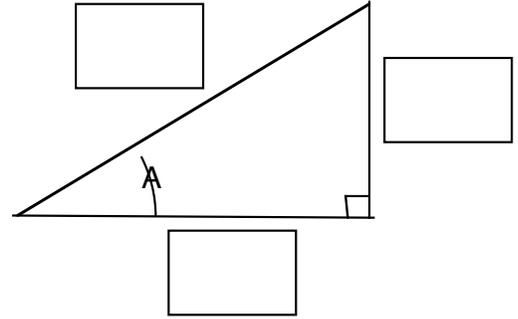(3)   condition $y(0) = 3$
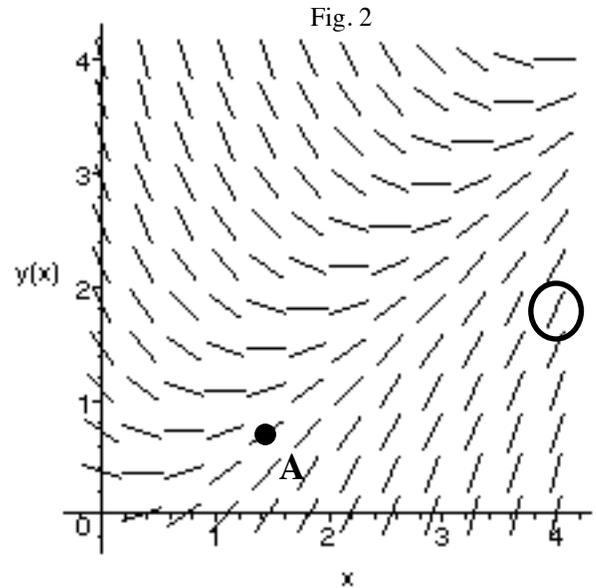
4.   Solve $\dfrac{dy}{dx} = 12y^2 x$, $y(0) = 7$. $y = $ _____

(7)

5. $\dfrac{dP}{dt} = (10 - P)\left(\dfrac{P}{20} - 1\right)$. If $P(0) = 15$, then for "large" vales of t $P(t)$ will approach the value _____

(4)

6. (a) Label the edges of the triangle so that $A = \arcsin(x)$

(3)

(b) Then $\tan(\arcsin(x)) =$ _____

(3)

7. $D\left(\arcsin(3x+1)\right) =$ _____ $D\left(\arctan(2 + e^x)\right) =$ _____

(5)(5)

8. Some integrals. SHOW YOUR WORK.

(a) $\displaystyle\int \frac{7}{\sqrt{9 - x^2}}\ dx\ =$ _____   (b) $\displaystyle\int \frac{3}{25 + 4x^2}\ dx\ =$ _____

(5)(5)

(c) $\displaystyle\int \frac{6x + 5}{4 + x^2}\ dx\ =$ _____

(5)

(d) $\displaystyle\int \frac{3\cos(x)}{1 + \sin^2(x)}\ dx\ =$ _____

(5)

9. If the integral converges, give the value. If the integral diverges, write DIV. **Show your work.**

(a) $\displaystyle\int_{3}^{\infty}\frac{4x}{1+x^2}\,dx$ = _____

(b) $\displaystyle\int_{3}^{7}\frac{6}{\sqrt{x-3}}\,dx$ = _____

(5)(5)

10. **Quickies** (sort of):

(a) If $w(t)$ is a solution of the differential equation $w' = 43 - w^3$, then when $w = 2$

the graph of w is (circle one)   increasing decreasing   horizontal

(2)

(b) If $\dfrac{dy}{dx} = Ay$ and $y(0) = B$ then $y =$ _____

(2)

(b) If the "half life" of radioactive material $R(t) = R_0 e^{kt}$ is 127 years, then $k =$ _____

(2)

(c) $\displaystyle\int_{3}^{\infty}\frac{2+\sin(x)}{5+x^3}\,dx$ is (circle one) **Finite   Infinite**

(2)

(1) (c) What is the other area (besides math) Perci Diaconis works in? _____

(1) (d) How old was Evariste Galois when he died? _____

the end (total = 102 points)