Math 152

Show Your Work! March 14, 2011 (= pi day) Name ______________________________

Good Luck! TEST **3D** (please print =1 point)

1. **(a)** Find $\bar{x}$ for the shaded region R in Fig. 1.

(Timesaving hint: area of region R is 1/4)
Do this using antiderivatives -- SHOW WORK.

$\bar{x}$ = __________

(4)(4)

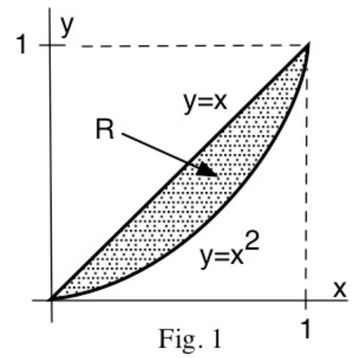


Fig. 1

(b) Find the **volume** (using antiderivatives) when region R is rotated around the line x=1:

$V = \int$ = _________ (fraction)

(4)(3)

2. "The initial population P of the town was 1700 people and the rate of change of the population was proportional to the square root of the population."

(a) Translate this statement into an **initial value differential equation.**

(4) (Note: This is NOT the exponential growth model we did in class.)

(3) (b) Separate the variables from part (a) **(Do Not solve the D.E.)**:

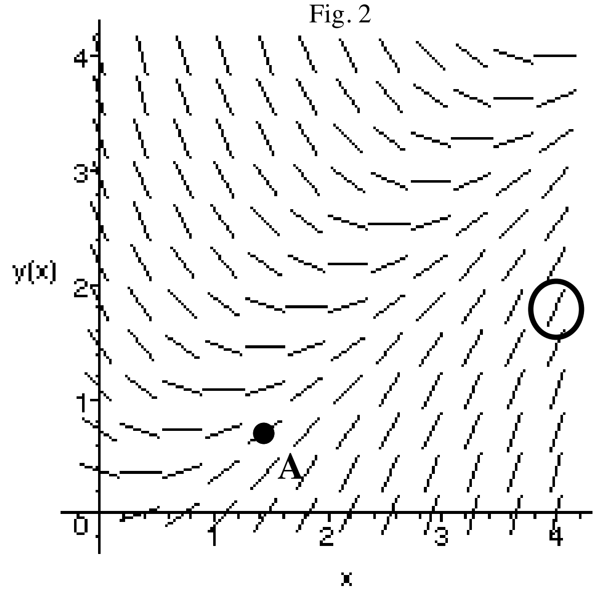
3. $\dfrac{dy}{dx} = x - y$ (see Fig. 2)

(2) (a) Is $y = x - 1$ a solution of this differential equation? **Yes** **No**

(b) Is $y = x - 1 + e^{-x}$ a solution of this differential equation? **Yes** **No**

(2)

Fig. 2



(c) What is the slope of the circled "tangent segment" in Fig. 2?

(2) Slope = __________

(d) On the direction field in Fig. 2, sketch the

(3) solution that goes through point **A.**

(e) On the direction field in Fig. 2, sketch the solution that satisfies the initial

(3) condition $y(0) = 3$

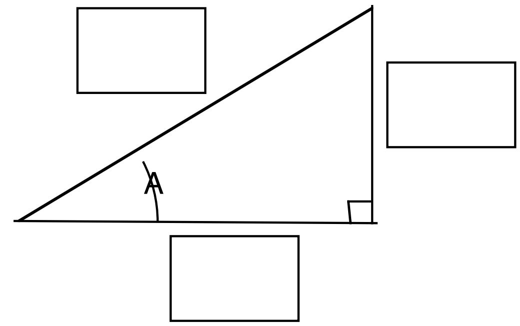
4. Solve $\dfrac{dy}{dx} = 10y^2x$, $y(0) = 11.$ y = ______________________________

(7)

5. $\dfrac{dP}{dt} = (10 - P)\left(\dfrac{P}{20} - 1\right)$. If P(0) = 15, then for "large" vales of t P(t) will approach the value ________

(4)

6. (a) Label the edges of the triangle so that $A = \arcsin(x)$



(3)

(b) Then $\tan(\arcsin(x)) =$ ____________

(3)

7. $D\left(\arcsin(3x+1)\right) =$ ________________ $D\left(\arctan(2+e^{x})\right) =$ ________________

(5)(5)

8. Some integrals. SHOW YOUR WORK.

(a) $\displaystyle\int \frac{7}{\sqrt{9-x^2}}\,dx =$ ________________ (b) $\displaystyle\int \frac{3}{25+4x^2}\,dx =$ ________________

(5)(5)

(c) $\displaystyle\int \frac{6x+5}{4+x^2}\,dx =$ ________________

(5)

(d) $\displaystyle\int \frac{3\cos(x)}{1+\sin^2(x)}\,dx =$ ________________

(5)

9. If the integral converges, give the value. If the integral diverges, write DIV. **Show your work.**

(a) $\displaystyle\int_3^{\infty} \frac{4x}{1+x^2}\,dx$ = ____________ (b) $\displaystyle\int_3^{7} \frac{6}{\sqrt{x-3}}\,dx$ = ____________

(5)(5)

10. **Quickies** (sort of):

(a) If $w(t)$ is a solution of the differential equation $w' = 43 - w^3$, then when w = 2 the graph of w is (circle one) increasing decreasing horizontal

(2)

(b) If $\dfrac{dy}{dx} = Ay$ and y(0) = B then y = ________________

(2)

(b) If the "half life" of radioactive material $R(t) = R_0 e^{kt}$ is 157 years, then k = _________

(2)

(c) $\displaystyle\int_3^{\infty} \frac{2+\sin(x)}{5+x^3}\,dx$ is (circle one) **Finite** **Infinite**

(2)

(1) (c) What is the other area (besides math) Perci Diaconis works in? ________________

(1) (d) How old was Evariste Galois when he died? ________________

the end (total = 102 points)