

PROBLEM ANSWERS Chapter Four**Section 4.0**

1. (a) $(10)(12) + (8)(4) = 152$ (b) $(10)(20) - (3)(8) = 176$
3. $bh + \frac{1}{2} b(H - h) = bh + \frac{1}{2} bH - \frac{1}{2} bh = b \left(\frac{h + H}{2} \right)$
5. (a) $(1)(3) + (1)(2) = 5$ (b) {area of shaded region in Fig. 24b} < 5
7. $A(1) = 1$, $A(2) = 2.5$, $A(3) = 4.5$, $A(4) = 6$, $A(5) = 7$
9. $C(1) = 1.5$, $C(2) = 4$, $C(3) = 7.5$, and $C(x) = \text{rect.} + \text{triangle areas} = x + \frac{1}{2} x \cdot x = x + \frac{1}{2} x^2$
11. Distance = "area" $= (20)(30) + \frac{1}{2} (10)(30) = 600 + 150 = 750$ feet.
13. (a) A: 20 seconds to stop. B: 40 seconds to stop.
 (b) A: $\frac{1}{2} (20)(80) = 800$ feet to stop. B: $\frac{1}{2} (40)(40) = 800$ feet to stop.
15. miles, dollars, cubic feet, kilowatt•hours, people, square meals

Section 4.1

1. $2^2 + 3^2 + 4^2 = 29$ 3. $(1+1)^2 + (1+2)^2 + (1+3)^2 = 29$
5. $\cos(0) + \cos(\pi) + \cos(2\pi) + \cos(3\pi) + \cos(4\pi) + \cos(5\pi) = 1 + (-1) + 1 + (-1) + 1 + (-1) = 0$
7. $\sum_{k=3}^{94} k$ 9. $\sum_{k=3}^{12} k^2$ 11. $\sum_{k=1}^7 k \cdot 2^k$
13. (a) $(1+2) + (2+2) + (3+2) = 3 + 4 + 5 = 12$ (b) $(1+2+3) + (2+2+2) = 12$
15. (a) $5 \cdot 1 + 5 \cdot 2 + 5 \cdot 3 = 5 + 10 + 15 = 30$ (b) $5 \cdot (1 + 2 + 3) = 5 \cdot 6 = 30$
17. (a) $1^2 + 2^2 + 3^2 = 1 + 4 + 9 = 14$ (b) $(1 + 2 + 3)^2 = 6^2 = 36$
19. $f(0) + f(1) + f(2) + f(3) = 0^2 + 1^2 + 2^2 + 3^2 = 14$
21. $2 \cdot f(0) + 2 \cdot f(1) + 2 \cdot f(2) + 2 \cdot f(3) = 2 \cdot 0 + 2 \cdot 1 + 2 \cdot 4 + 2 \cdot 9 = 28$
23. $g(1) + g(2) + g(3) = 3 + 6 + 9 = 18$
25. $g^2(1) + g^2(2) + g^2(3) = 3^2 + 6^2 + 9^2 = 126$
27. $h(2) + h(3) + h(4) = \frac{2}{2} + \frac{2}{3} + \frac{2}{4} = \frac{13}{6}$

$$29. f(1)h(1) + f(2)h(2) + f(3)h(3) = (1)(2) + (4)(1) + (9)(2/3) = 12$$

$$31. (1^2 - 0^2) + (2^2 - 1^2) + (3^2 - 2^2) + (4^2 - 3^2) + \dots + (7^2 - 6^2) = 7^2 - 0^2 = 49$$

$$33. \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{6}\right) = 1 - \frac{1}{6} = \frac{5}{6}$$

$$35. (\sqrt{1} - \sqrt{0}) + (\sqrt{2} - \sqrt{1}) + (\sqrt{3} - \sqrt{2}) + \dots + (\sqrt{9} - \sqrt{8}) = \sqrt{9} - \sqrt{0} = 3$$

$$37. (i) [2, 3], [3, 4.5], [4.5, 6], [6, 7] (ii) 1, 1.5, 1.5, 1$$

$$(iii) \text{ mesh} = 1.5$$

$$(iv) 1 + 1.5 + 1.5 + 1 = 5$$

$$39. (i) [-3, -1], [-1, 0], [0, 1.5], [1.5, 2]$$

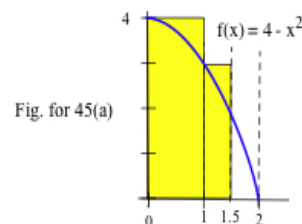
$$(ii) 2, 1, 1.5, 0.5$$

$$(iii) \text{ mesh} = 2$$

$$(iv) 2 + 1 + 1.5 + 0.5 = 5$$

$$41. (i) [3, 3.8], [3.8, 4.5], [4.5, 5.2], [5.2, 7] (ii) 0.8, 0.7, 0.7, 1.8 (iii) \text{ mesh} = 1.8 (iv) 0.8 + 0.7 + 0.7 + 1.8 = 4$$

$$43. \Delta x_1 + \Delta x_2 + \Delta x_3 + \dots + \Delta x_n = (x_1 - x_0) + (x_2 - x_1) + (x_3 - x_2) + \dots + (x_n - x_{n-1}) = x_n - x_0$$



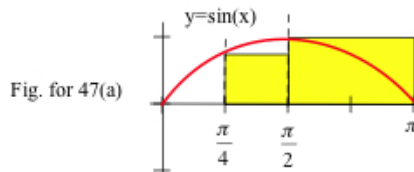
$$45. (a) f(0)(1) + f(1)(0.5) + f(2)(0.5) = (4)(1) + (3)(0.5) + (0)(0.5) = 5.5$$

$$(b) f(1)(1) + f(1.5)(0.5) + f(2)(0.5) = (3)(1) + (1.75)(0.5) + (0)(0.5) = 4.75$$

$$47a. (i) \text{ and } (ii) \text{ See the graph.}$$

$$(iii) f(0) = 0, f(\pi/4) \approx 0.707, f(\pi/2) = 1$$

$$(iv) (\pi/4)(0) + (\pi/4)(0.707) + (\pi/2)(1) \approx 2.13$$



$$49. (a) (2)(1) + (5)(2) + (17)(1) \leq RS \leq (5)(1) + (17)(2) + (26)(1) \text{ so } 29 \leq RS \leq 65.$$

$$(b) (2)(1) + (5)(1) + (10)(1) + (17)(1) \leq RS \leq (5)(1) + (10)(1) + (17)(1) + (26)(1) \text{ so } 34 \leq RS \leq 58.$$

$$(c) (2)(0.5) + (3.25)(0.5) + (5)(1) + (10)(1) + (17)(1) \leq RS \leq (3.25)(0.5) + (5)(0.5) + (10)(1) + (17)(1) + (26)(1) \text{ so } 34.625 \leq RS \leq 57.125.$$

$$51. (a) (0)(\pi/2) + (0)(\pi/2) \leq RS \leq (1)(\pi/2) + (1)(\pi/2) \text{ so } 0 \leq RS \leq \pi$$

$$(b) (0)(\pi/4) + (0.707)(\pi/4) + (0)(\pi/2) \leq RS \leq (0.707)(\pi/4) + (1)(\pi/4) + (1)(\pi/2) \text{ so } 0.56 \leq RS \leq 2.91$$

$$(c) (0)(\pi/4) + (0.707)(\pi/4) + (0.707)(\pi/4) + (0)(\pi/4) \leq RS \leq (0.707)(\pi/4) + (1)(\pi/4) + (1)(\pi/4) + (0.707)(\pi/4) \text{ so } 1.11 \leq RS \leq 2.68$$

$$53. (a) |7.402 - 7.362| = 0.04$$

$$(b) |7.390 - 7.372| = 0.018$$

$$55. |\text{error}| \leq (\text{base})(\text{height}) = \frac{4-2}{50} (65-9) = \frac{2}{50} (56) = \frac{112}{50} = 2.24$$

$$57. (a) \frac{100(101)}{2} = 5050 \quad (b) 1 + 2 + 3 + \dots + 100$$

$$\frac{100 + 99 + 98 + \dots + 1}{2}$$

$$101 + 101 + 101 + \dots + 101 = 100(101) = 10100. \quad \frac{1}{2}(10100) = 5050$$

$$59. 10 + 11 + 12 + \dots + 20 = (1+2+3+\dots+20) - (1+2+3+\dots+9) = \frac{20(21)}{2} - \frac{9(10)}{2} = 210 - 45 = 165$$

$$61. \sum_{k=1}^{10} (k^3 + k) = \sum_{k=1}^{10} k^3 + \sum_{k=1}^{10} k = \left(\frac{10(11)}{2} \right)^2 + \frac{10(11)}{2} = (55)^2 + 55 = 3080$$

Section 4.2

$$1. \int_0^4 2 + 3x \, dx$$

$$3. \int_2^5 x^3 \, dx$$

$$5. \int_1^5 x^3 \, dx$$

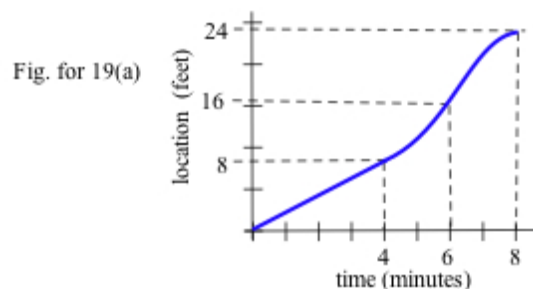
$$7. \int_{0.5}^2 x \sin(x) \, dx$$

$$9. \int_1^3 \ln(x) \, dx$$

$$11. \int_1^3 2x \, dx = 8$$

$$13. \int_{-1}^0 |x| \, dx = 1/2$$

$$15. \int_0^4 3 - \frac{x}{2} \, dx = 8$$



$$17. (a) 3 \quad (b) -1 \quad (c) 6 \quad (d) 8 \quad (e) 7$$

$$19. (a) \text{ see the graph} \\ (b) 24 \text{ feet.} \\ (c) 24 \text{ feet from the starting point.}$$

$$21. \text{ meters}$$

$$23. \text{ feet}^3 = \text{cubic feet}$$

$$25. \text{ gram} \cdot \text{meters}$$

$$27. \text{ feet/second} = \text{feet per second}$$

$$29. \Delta x = \frac{2-0}{n} = \frac{2}{n} \quad m_i = \frac{2}{n}(i-1) \quad \text{and} \quad M_i = \frac{2}{n}i \quad \text{so} \quad f(m_i) = \left\{ \frac{2}{n}(i-1) \right\}^3 \quad \text{and} \quad f(M_i) = \left\{ \frac{2}{n}i \right\}^3.$$

$$(a) \quad LS = \sum_{i=1}^n f(m_i) \Delta x = \sum_{i=1}^n \left\{ \frac{2}{n}(i-1) \right\}^3 \frac{2}{n} = \frac{2}{n} \frac{8}{n^3} \left\{ \sum_{i=1}^n i^3 - 3 \sum_{i=1}^n i^2 + 3 \sum_{i=1}^n i - \sum_{i=1}^n 1 \right\} \\ = \frac{16}{n^4} \left\{ \left(\frac{1}{4} n^4 + \frac{1}{2} n^3 + \frac{3}{12} n^2 \right) - 3 \left(\frac{1}{3} n^3 + \frac{1}{2} n^2 + \frac{2}{12} n \right) + 3 \left(\frac{1}{2} n^2 + \frac{1}{2} n \right) - n \right\} \\ = \frac{16}{n^4} \left\{ \frac{1}{4} n^4 - \frac{1}{2} n^3 + \frac{1}{4} n^2 \right\} = 4 - \frac{8}{n} + \frac{4}{n^2} \longrightarrow 4.$$

$$(b) \quad US = \sum_{i=1}^n f(M_i) \Delta x = \sum_{i=1}^n \left\{ \frac{2}{n}i \right\}^3 \frac{2}{n} = \frac{2}{n} \frac{8}{n^3} \left\{ \sum_{i=1}^n i^3 \right\} = \frac{16}{n^4} \left\{ \frac{1}{4} n^4 + \frac{1}{2} n^3 + \frac{3}{12} n^2 \right\} \\ = 4 + \frac{8}{n} + \frac{4}{n^2} \longrightarrow 4.$$

Section 4.3

$$1. 5$$

$$3. 0$$

$$5. 3$$

$$7. 0$$

$$9. -5$$

$$11. -5$$

$$13. 0$$

$$15. 4.5 + 5 = 9.5$$

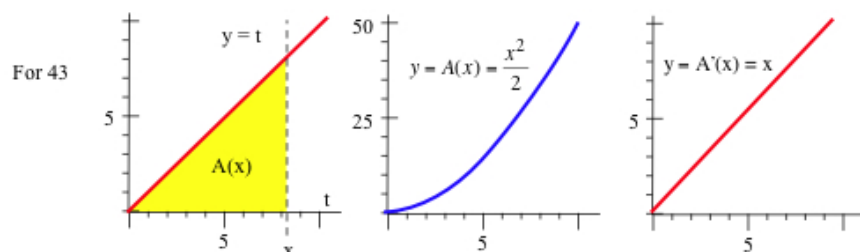
$$17. 10 + 3 = 13$$

$$19. 5 + 2 = 7$$

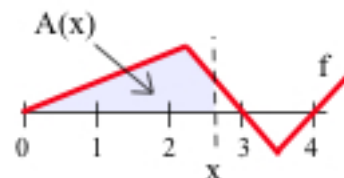
$$21. 1$$

$$23. -1$$

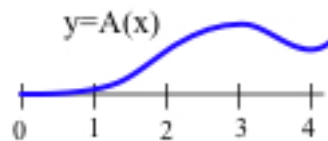
25. 2 27. 1 29. 1 31. (a) $8 \cdot 6 = 48$ (b) 24
 33. (a) 32 (b) $8^2 = 64$ 35. 8 37. 2.5
 39. 3 41. 7
 43. (a) graph $y = A(x) = x^2/2$ (b) graph $y = A'(x) = x$



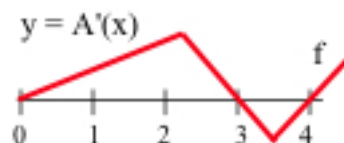
45. (a) graph of $y = A(x)$ in Figure 45a
 (b) graph of $y = A'(x)$ in Figure 45b
 47. (a) f is continuous on $[1,4]$
 (b) f is not differentiable on $[1,4]$ (not diff. at $x \approx 2.3$ and 3.3)
 (c) f is integrable on $[1,4]$



49. (a) f is not continuous on $[1,4]$ (not cont. at $x = 2$ and 3)
 (b) f is not differentiable on $[1,4]$ (not diff. at $x = 2$ and 3)
 (c) f is integrable on $[1,4]$.



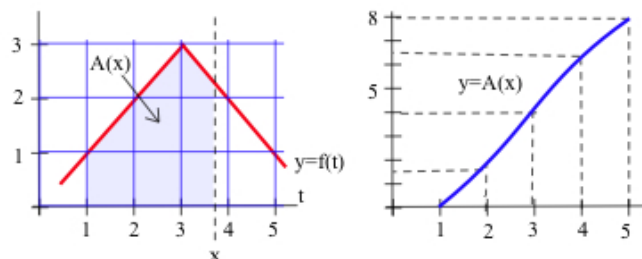
51. $\int_1^4 \text{velocity } dt = \int_1^2 \text{velocity } dt + \int_2^4 \text{velocity } dt = 35 + 50 = 85$
 miles



Graphs for problem 45

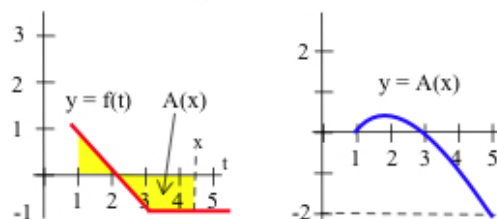
Section 4.4

1. (a) see Figure
 (b) $A(1) = 0$, $A(2) = 1.5$, $A(3) = 4$, $A(4) = 6.5$
 (c) $A'(1) = 1$, $A'(2) = 2$, $A'(3) = 3$, $A'(4) = 2$



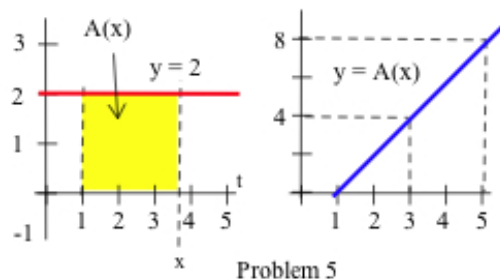
Problem 1

3. (a) see Figure
 (b) $A(1) = 0$, $A(2) = 0.5$, $A(3) = 0$, $A(4) = -1$
 (c) $A'(1) = 1$, $A'(2) = 0$, $A'(3) = -1$, $A'(4) = -1$

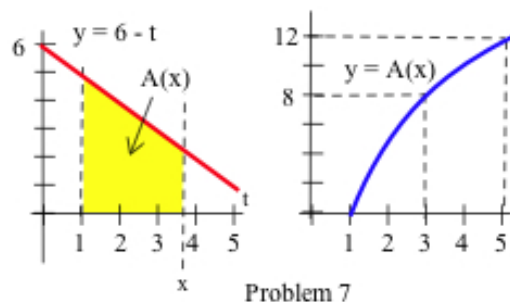


Problem 3

5. (a) see Figure
 (b) $A(1) = 0$, $A(2) = 2$, $A(3) = 4$, $A(4) = 6$
 (c) $A'(1) = 2$, $A'(2) = 2$, $A'(3) = 2$, $A'(4) = 2$



7. (a) see Figure
 (b) $A(1) = 0$, $A(2) = 4.5$, $A(3) = 8$, $A(4) = 10.5$
 (c) $A'(1) = 5$, $A'(2) = 4$, $A'(3) = 3$, $A'(4) = 2$



9. $x^2 \Big|_0^3 = 9$, $x^2 \Big|_1^3 = 8$, $x^2 \Big|_0^1 = 1$

11. $2x^3 \Big|_1^3 = 52$, $2x^3 \Big|_1^2 = 14$, $2x^3 \Big|_0^3 = 54$

13. $x^4 \Big|_0^3 = 81$, $x^4 \Big|_1^3 = 80$, $x^4 \Big|_0^1 = 1$

15. $x^3 \Big|_{-3}^3 = 54$, $x^3 \Big|_{-3}^0 = 27$, $x^3 \Big|_0^3 = 27$

17. $x^3 \Big|_0^2 = 8$, $x^3 \Big|_1^3 = 26$

19. (a) distance = $\int_0^{10} 2t \, dt = t^2 \Big|_0^{10} = 100$ feet.

(b) Find T so $50 = \int_0^T 2t \, dt = t^2 \Big|_0^T = T^2$. $T = \sqrt{50} \approx 7.07$ seconds.

21. (a) distance = $\int_0^{10} 4t^3 \, dt = t^4 \Big|_0^{10} = 10,000$ feet.

(b) $5000 = \int_0^T 4t^3 \, dt = T^4$. $T = \sqrt[4]{5000} \approx 8.41$ seconds.

23. (a) velocity = $75 - 3t^2 = 0$ when $t = 5$ seconds. (b) distance = $\int_0^5 75 - 3t^2 \, dt = 75t - t^3 \Big|_0^5 = 250$ feet.

(c) $125 = \int_0^T 75 - 3t^2 \, dt = 75t - t^3 \Big|_0^T = 75T - T^3$ so $T^3 - 75T + 125 = 0$ (solve using

Newton's method or by examining the graph of $y = x^3 - 75x + 125$) and $T \approx 1.74$ seconds.

25. The total area is $\int_0^3 x^2 dx = \frac{1}{3} x^3 \Big|_0^3 = 9$.

(a) Find T so $\frac{1}{2} \cdot 9 = \frac{9}{2} = \int_0^T x^2 dx = \frac{1}{3} x^3 \Big|_0^T = \frac{1}{3} T^3$. $T = \sqrt[3]{27/2} \approx 2.38$.

(b) Find T so $\frac{1}{3} \cdot 9 = 3 = \int_0^T x^2 dx = \frac{1}{3} x^3 \Big|_0^T = \frac{1}{3} T^3$. $T = \sqrt[3]{9} \approx 2.08$.

Then find T so $\frac{2}{3} \cdot 9 = 6 = \int_0^T x^2 dx = \frac{1}{3} x^3 \Big|_0^T = \frac{1}{3} T^3$. $T = \sqrt[3]{18} \approx 2.62$.

Section 4.5

1. (a) $A(x) = x^3$. Then $A'(x) = 3x^2$, and $A'(1) = 3$, $A'(2) = 12$, and $A'(3) = 27$.

(b) $A'(x) = \mathbf{D}\left(\int_0^x 3t^2 dt\right) = 3x^2$. $A'(1) = 3$, $A'(2) = 12$, and $A'(3) = 27$.

3. $A'(x) = 2x$ so $A'(1) = 2$, $A'(2) = 4$, $A'(3) = 6$. 4. $A'(x) = 2x$ so $A'(1) = 2$, $A'(2) = 4$, $A'(3) = 6$.

5. $A'(x) = 2x$ so $A'(1) = 2$, $A'(2) = 4$, $A'(3) = 6$. 6. $A'(x) = 3 - x^2$ so $A'(1) = 2$, $A'(2) = -1$, $A'(3) = -6$.

7. $A'(x) = \sin(x)$ so $A'(1) \approx 0.84$, $A'(2) \approx 0.91$, $A'(3) \approx 0.14$.

8. $A'(x) = |x - 2|$ so $A'(1) = 1$, $A'(2) = 0$, $A'(3) = 1$.

9. $A'(x) = f(x)$ so $A'(1) = 2$, $A'(2) = 1$, $A'(3) = 2$. 10. $A'(x) = f(x)$ so $A'(1) = 0$, $A'(2) = -1$, $A'(3) = 0$.

11. $A'(x) = f(x)$ so $A'(1) = 1$, $A'(2) = 2$, $A'(3) = 2$. 12. $A'(x) = f(x)$ so $A'(1) = -1$, $A'(2) = -1$, $A'(3) = 0$.

13. $F(1) - F(0) = 6 - 5 = 1$

15. $F(3) - F(1) = 9 - \frac{1}{3} = \frac{26}{3}$ 17. $F(5) - F(1) \approx 1.61 - 0 = 1.61$ 19. $F(3) - F(1/2) \approx 1.10 - (-0.69) = 1.79$

21. $F(\pi/2) - F(0) = 1 - 0 = 1$ 23. $F(1) - F(0) \approx 0.67 - 0 = 0.67$ 25. $F(7) - F(1) = \frac{2}{3}(7)^{3/2} - \frac{2}{3} \approx 11.68$

27. $F(9) - F(1) = 3 - 1 = 2$ 29. $F(3) - F(-2) \approx 20.09 - 0.14 = 19.95$

31. $F(\pi/4) - F(0) = 1 - 0 = 1$ 33. $F(3) - F(0) = \frac{2}{3}(10)^{3/2} - \frac{2}{3} \approx 20.42$

35. $F(x) = \frac{1}{3} x^3$. $F(2) - F(-1) = \frac{8}{3} - (-\frac{1}{3}) = 3$. 37. $F(x) = \ln(x)$. $F(e) - F(1) = 1 - 0 = 1$.

39. $F(x) = \frac{2}{3} x^{3/2}$. $F(100) - F(25) = \frac{2000}{3} - \frac{250}{3} = \frac{1750}{3} \approx 583.33$

41. $F(x) = -1/x$. $F(10) - F(1) = -0.1 - (-1) = 0.9$ 43. $F(x) = e^x$. $F(1) - F(0) = e - 1 \approx 1.718$

45. $F(x) = \tan(x)$. $F(\pi/4) - F(\pi/6) \approx 1 - 0.577 = 0.423$

47. The integral goes from -3 to 3 so even without knowing an antiderivative, $\int_{-3}^3 \sin(x) \cdot \ln(x) \, dx = 0$.

49. $\text{area} = \int_0^{\pi} \sin(x) \, dx = -\cos(x) \Big|_0^{\pi} = -(-1) - (-1) = 2$ 51. $\text{area} = \int_0^{3.5} \text{INT}(x) \, dx = 0 + 1 + 2 + \frac{1}{2}(3) = 4.5$.

53. $\text{area} = \int_0^3 (x-2)^2 \, dx = \int_0^3 x^2 - 4x + 4 \, dx = \frac{1}{3}x^3 - 2x^2 + 4x \Big|_0^3 = 3 - 0 = 3$.

55. $\mathbf{D}(A(3x)) = \mathbf{3} \cdot \tan(3x)$, $\mathbf{D}(A(x^2)) = \mathbf{2x} \cdot \tan(x^2)$, $\mathbf{D}(A(\sin(x))) = \mathbf{\cos(x)} \cdot \tan(\sin(x))$

57. $\sqrt{1+5x} \cdot (5)$

59. $\sqrt{1+\sin(x)} \cdot \cos(x)$

61. $\{3(1-2x)^2 + 2\} \cdot (-2)$

63. $-\cos(3x)$

65. $\tan(x^2) \cdot 2x - \tan(x)$

67. $5 \cdot \ln(x) \cdot \cos(3 \cdot \ln(x)) \cdot \frac{1}{x}$

Section 4.6

$$1. \text{ Left side} = \frac{1}{4} x^4 \Big|_1^2 = \frac{15}{4} . \quad \text{Right side} = \left\{ \frac{1}{3} x^3 \Big|_1^2 = \frac{7}{3} \right\} \cdot \left\{ \frac{1}{2} x^2 \Big|_1^2 = \frac{3}{2} \right\} = \frac{7}{2} \neq \text{left side}.$$

$$3. \text{ Left side} = \frac{1}{4} . \quad \text{Right side} = \left(\frac{1}{3} \right) \cdot \left(\frac{1}{2} \right) = \frac{1}{6} \neq \text{left side}.$$

$$5. \frac{1}{3} \sin(3x) + C \quad 7. -\cos(2 + e^x) + C \quad 9. \tan(\sin(x)) + C \quad 11. \frac{5}{2} \ln|3 + 2x| + C$$

$$13. -\frac{1}{3} \cos(1 + x^3) + C \quad 15. \frac{1}{4} \sin(4x) + C \quad 17. \frac{1}{48} (5 + x^4)^{12} + C \quad 19. \ln|2 + x^3| + C$$

$$21. \frac{1}{2} (\ln(x))^2 + C \quad 23. \frac{1}{24} (1 + 3x)^8 + C \quad 25. \sec(e^x) + C \quad 27. \frac{1}{3} \sin(3x) \Big|_0^{\pi/2} = -\frac{1}{3}$$

$$29. -\cos(2 + e^x) \Big|_0^1 = \cos(3) - \cos(2 + e) \approx -0.996$$

$$31. \frac{1}{18} (1 + x^3)^6 \Big|_{-1}^1 = \frac{32}{9} \quad 33. \frac{5}{2} \ln|3 + 2x| \Big|_0^2 = \frac{5}{2} \ln\left(\frac{7}{3}\right) \quad 35. -\frac{1}{3} (1 - x^2)^{3/2} \Big|_0^1 = \frac{1}{3}$$

$$37. \frac{2}{9} (1 + 3x)^{3/2} \Big|_0^1 = \frac{16}{9} - \frac{2}{9} = \frac{14}{9} \quad 39. \frac{1}{2} x - \frac{1}{20} \sin(10x) + C$$

$$41. \frac{1}{4} \sin(2x) + C \quad 43. \frac{1}{2} x - \frac{1}{4} \sin(2x) \Big|_0^{\pi} = \frac{\pi}{2} \quad 45. \frac{1}{7} x^7 + \frac{3}{5} x^5 + x^3 + x + C$$

$$47. \frac{1}{2} e^{2x} + 2e^x + x + C \quad 49. \frac{1}{6} x^6 + \frac{1}{4} x^4 + \frac{5}{3} x^3 + 5x + C \quad 51. \frac{1}{2} e^{2x} + \frac{1}{4} e^{4x} + C$$

$$53. \frac{2}{7} x^{7/2} + \frac{6}{5} x^{5/2} - \frac{4}{3} x^{3/2} + C \quad 55. 3x - 3 \cdot \ln|x + 1| + C \quad 57. \frac{1}{2} x^2 - x + C$$

$$59. (\text{divide first}) x^2 - 11x + 7 \cdot \ln|x - 1| + C \quad 61. (\text{divide first}) x + 3 \cdot \ln|x - 1| + C$$

$$63. \frac{2}{3} x^{3/2} + 8 x^{1/2} + C \quad 65. (\text{area of semicircle with radius 1}) = \frac{1}{2} \pi(1)^2 = \frac{\pi}{2}$$

$$67. (\text{area of semicircle with radius 3}) = \frac{1}{2} \pi(3)^2 = \frac{9}{2} \pi$$

$$69. (\text{area of rectangle}) + (\text{area of semicircle of radius 1}) = (2)(2) + \frac{1}{2} (\pi(1)^2) = 4 + \frac{\pi}{2}$$

Section 4.7

1. between 11 (using left endpoints of intervals) and 6 (using right endpoints)
3. between 4 (using left endpoints of intervals) and 6 (using right endpoints)
5. Using left endpoint widths: $(0)(40)+(70)(40)+(55)(40)+(90)(40)+(130)(40)+(115)(40) = 18,400 \text{ ft}^2$.
Right endpoint widths (70, 55, ...) and average widths $(70/2, 125/2, \dots)$ give the same result, $18,400 \text{ ft}^2$.
All of these are reasonable methods for estimating the area of the island.

$$7. 9 \qquad 9. 1 \qquad 11. \frac{1}{2} \cdot e^2 - \frac{3}{2} \qquad 13. \frac{1}{32} \pi^2 + \frac{1}{4} \pi - \frac{\sqrt{2}}{2}$$

$$15. e^2 - 3 \qquad 17. 3 - \frac{\pi}{4}$$

$$19. \text{ Estimate using midpoints of unit intervals: } \frac{1}{4} \{f(1)(1)+f(2)(1)+f(3)(1)+f(4)(1)\} = \frac{19}{4} . \text{ About } \frac{19}{4} .$$

$$21. \text{ Estimate using midpoints of unit intervals: } \frac{1}{2} \{f(2)(1)+f(3)(1)\} = 5 . \text{ About } 5 .$$

$$23. \text{ average} \approx 1 \qquad 25. \text{ average} \approx \frac{11}{5} \qquad 27. \text{ average} = 5 \qquad 29. \text{ average} = \frac{13}{3} \qquad 31. \text{ average} = \frac{2}{\pi}$$

$$33. (a) C = 1: \text{ average} = \frac{2}{3} \qquad (b) C = 9: \text{ average} = 2 \qquad (c) C = 81: \text{ average} = 6 \qquad (d) C = 100: \text{ average} = \frac{20}{3}$$

In general, $\text{average} = \frac{2}{3} \sqrt{C}$.

$$35. (a) \text{ Graphically, average} \approx 3000 \cdot 1000 \frac{\text{calls}}{\text{hour}} = \frac{3000000}{60} \frac{\text{calls}}{\text{min}} \approx 50,000 \frac{\text{calls}}{\text{min}} . (b) \text{ About } 58,333 \frac{\text{calls}}{\text{min}} .$$

$$37. (a) \text{ Similar to Example 5: work} = 1,950 \text{ foot-pounds} \quad (b) \text{ work} = 1,312.5 \text{ foot-pounds}$$

$$39. (a) \text{ work} = 1,200 \text{ foot-pounds} \qquad (b) \text{ work} = 600 \text{ foot-pounds} \qquad (c) \text{ work} = 400 \text{ foot-pounds}$$

$$41. \text{ work} = 1,275 \text{ foot-pounds}$$

Section 4.8

$$1. \text{ Table \#35, } a = 2: \frac{1}{2} \arctan\left(\frac{x}{2}\right) + C \qquad 3. \text{ Table \#35, } a = 5: x^2 + \frac{2}{5} \arctan\left(\frac{x}{5}\right) + C$$

$$5. \text{ Table \#37, } a = 3: \left(\frac{1}{3}\right) \ln \left| \frac{x+3}{x-3} \right| + C \qquad 7. \text{ Table \#35, } a = \sqrt{3}: \frac{1}{\sqrt{3}} \arctan\left(\frac{x}{\sqrt{3}}\right) + C$$

$$9. \text{ Table \#35, } a = \sqrt{2}: e^x + \frac{7}{\sqrt{2}} \arctan\left(\frac{x}{\sqrt{2}}\right) + C$$

11. Table #34, $a = \sqrt{5}$: $3 \cdot \arcsin\left(\frac{x}{\sqrt{5}}\right) + C$ 13. Table #35, $a = \frac{2}{5}$: $\frac{1}{10} \arctan\left(\frac{5}{2}x\right) + C$

15. First substitute $u = 2x$, $du = 2 dx$. Then use Table #34 with $a = 1$: $\frac{5}{2} \cdot \arcsin(u) + C = \frac{5}{2} \cdot \arcsin(2x) + C$

17. Table #43 and substitution $u = 3x$: $\frac{2}{3} \ln|3x + \sqrt{1+9x^2}| + C$

19. Table #38 and substitution $u = x+1$: $(x+1) \cdot \ln|x+1| - (x+1) + C$ or $(x+1) \cdot \ln|x+1| - x + C_2$

21. Table #38 and substitution $u = 5x^2+7$: $\frac{3}{10} \{ (5x^2+7) \cdot \ln|5x^2+7| - (5x^2+7) \} + C$

23. Table #38 and substitution $u = \sin(x)$: $\sin(x) \cdot \ln|\sin(x)| - \sin(x) + C$

25. Table #44, $a = 2$: $\frac{x}{2} \sqrt{x^2+4} + \frac{1}{2} (4) \cdot \ln|x + \sqrt{x^2+4}| + C$

27. Table #44, $a = 4$: $\frac{x}{2} \sqrt{x^2+16} + \frac{1}{2} (16) \cdot \ln|x + \sqrt{x^2+16}| + C$

29. Table #35, $a = 5$: $x^2 + \frac{2}{5} \arctan\left(\frac{x}{5}\right) \Big|_1^3 = 8 + \frac{2}{5} \left\{ \arctan\left(\frac{3}{5}\right) - \arctan\left(\frac{1}{5}\right) \right\}$

31. Table #35, $a = \sqrt{3}$: $\frac{1}{\sqrt{3}} \arctan\left(\frac{x}{\sqrt{3}}\right) \Big|_{-1}^1 = \frac{1}{\sqrt{3}} \left\{ \arctan\left(\frac{1}{\sqrt{3}}\right) - \arctan\left(\frac{-1}{\sqrt{3}}\right) \right\}$

33. Table #34, $a = \sqrt{5}$: $3 \arcsin\left(\frac{x}{\sqrt{5}}\right) \Big|_1^2 = 3 \cdot \left\{ \arcsin\left(\frac{2}{\sqrt{5}}\right) - \arcsin\left(\frac{1}{\sqrt{5}}\right) \right\}$

35. Table #34, $a = 1/2$: $\frac{5}{2} \arcsin\left(\frac{x}{1/2}\right) \Big|_0^{0.1} = \frac{5}{2} \arcsin(0.2)$

37. $7 \cdot \ln(7) - 6$ 39. $3 \cdot \ln(3) - 2 \cdot \ln(2) - 1$ 41. $3\sqrt{18} + \frac{9}{2} \cdot \ln\left(\frac{3+\sqrt{18}}{-3+\sqrt{18}}\right)$

43. Table #19a: $\frac{-\sin^2(x) \cdot \cos(x)}{3} - \frac{2}{3} \cos(x) + C$ 45. Table #20: $\frac{\cos^4(x) \cdot \sin(x)}{5} + \frac{4}{5} \{ \text{answer to number 44} \}$

47. Table #29: $x^2 \cdot \sin(x) + 2x \cdot \cos(x) - 2 \cdot \sin(x) + C$

49. Average of $\sin(x)$ on $[0, \pi]$ is $\frac{2}{\pi}$. Average of $\sin^2(x)$ on $[0, \pi]$ is $\frac{1}{2}$. $\frac{2}{\pi} > \frac{1}{2}$.

51. Using results from #50: (a) $\frac{1}{e-1}$ (b) $\frac{1}{9} \{ 10 \cdot \ln(10) - 9 \} \approx \frac{14.03}{9} \approx 1.56$
 (c) $\frac{1}{99} \{ 100 \cdot \ln(100) - 99 \} \approx 361.52/99 \approx 3.65$ (d) $\frac{1}{199} \{ 200 \cdot \ln(200) - 199 \} \approx 860.66/199 = 4.32$

53. (c) is largest.

55. approximately (a) 1.57 (b) 2.94 (c) 3.04 (d) 3.07 (e) 3.09

Section 4.9

1. $h = \frac{(6-2)}{4} = 1$. $T_4 = \frac{1}{2} \{ 2.1 + 2(3.8) + 2(0.3) + 2(-0.9) + 2.2 \} = 5.35$
 $S_4 = \frac{1}{3} \{ 2.1 + 4(3.8) + 2(0.3) + 4(-0.9) + 2.2 \} = 5.5$
3. $h = \frac{1 - (-3)}{8} = 0.5$.
 $T_8 = \frac{0.5}{2} \{ 4.2 + 2(1.8) + 2(0.7) + 2(1.5) + 2(3.4) + 2(4.3) + 2(3.5) + 2(-0.3) + -4.6 \} = 7.35$
 $S_8 = \frac{0.5}{3} \{ 4.2 + 4(1.8) + 2(0.7) + 4(1.5) + 2(3.4) + 4(4.3) + 2(3.5) + 4(-0.3) + -4.6 \} = 22/3 = 7.3333$
5. $T_4 = 4$ 7. $T_4 = 0.75$ 9. $T_4 = 1.896118898$
 $S_4 = 4$ $S_4 = 2/3 = 0.666$ $S_4 = 2.004559755$
 exact = 4 exact = 2/3 exact = 2
11. $T_6 = 1.088534906$ 13. $T_6 = 3.815780054$
 $S_6 = 1.090560447$ $S_6 = 3.826350295$
15. $T_6 = 0.8159928163$
 $S_6 = 0.8120491229$
17. (a) $M_2 = 0$ so the error bound = 0 (the trapezoidal approximation is exact)
 (b) $M_4 = 0$ so the error bound = 0 (the Simpson's rule approximation is exact)
 (c) $n = 1$ (d) $n = 2$ (must be an even integer)
19. $M_2 = \{ \max. \text{ of } 6x \text{ on } [-1, 1] \} = 6$. $f'''(x) = 0$ so $M_4 = 0$.
 (a) $| \text{error} | \leq \frac{2^3}{(12)(4^2)} (6) = 0.25$
 (b) $M_4 = 0$ so the error bound = 0 (the Simpson's rule approximation is exact)
 (c) Set $0.001 = \frac{2^3}{(12)(n^2)} (6)$ and solve for $n^2 = \frac{48}{(12)(0.001)} = 4,000$ so $n = 63.25$.
 Take $n = 64$ (to be certain that we have enough subintervals, always round UP).
 (d) $n = 2$ (must be an even integer)

21. $M_2 = \{ \text{max. of } |-\sin(x)| \text{ on } [0, \pi] \} = 1$. $M_4 = \{ \text{max. of } |\sin(x)| \text{ on } [0, \pi] \} = 1$.

(a) $|\text{error}| \leq \frac{\pi^3}{(12)(4^2)}(1) = 0.1614910244$ (b) $|\text{error}| \leq \frac{\pi^5}{(180)(4^4)}(1) = 0.0066410522$

(c) Set $0.001 = \frac{\pi^3}{(12)(n^2)}(1)$ so $n^2 = \frac{\pi^3}{(12)(0.001)} = 2,583.9$ and $n = 50.83$. **$n = 51$** .

(d) Set $0.001 = \frac{\pi^5}{(180)(n^4)}(1)$ so $n^4 = \frac{\pi^5}{(180)(0.001)} = 1,700.1$ and $n = 6.42$. **$n = 8$** .

23. $h = 30$. $S_6 = \frac{30}{3} \{ 50 + 4(62) + 2(92) + 4(86) + 2(74) + 4(50) + 40 \} = 12,140$ square feet.
($T_6 = 12,270$ square feet.)

25. $h = 50$.

$S_6 = \frac{50}{3} \{ 100 + 4(150) + 2(170) + 4(160) + 2(120) + 4(70) + 30 \} = 37,166.7$ square feet area.

volume = (area)(depth) = $(37,166.7 \text{ ft}^2)(22 \text{ ft}) = 817,667 \text{ ft}^3$.

$T_6 = \frac{50}{2} \{ 100 + 2(150) + 2(170) + 2(160) + 2(120) + 2(70) + 30 \} = 36,750$ square feet area.

volume = (area)(depth) = $(36,750 \text{ ft}^2)(22 \text{ ft}) = 808,500 \text{ ft}^3$.

27. Distance traveled by jogger is area under the graph of velocity ($f(x)$) vs. time (x).

area $\approx T_{10} = \frac{1}{2} \{ 0+840+1080+600+1000+1160+1040+880+720+520+180 \} = 4,010$ ft.

29. – 32. on your own

33. (a) $L_4 = 3.5$

(b) $R_4 = 4.5$

(c) $M_4 = 4$

(d) exact = 4

35. (a) $L_4 = 0.75$

(b) $R_4 = 0.75$

(c) $M_4 = 0.625$

(d) exact ≈ 0.6667

37. (a) $L_4 = 1.8961$

(b) $R_4 = 1.8961$

(c) $M_4 = 2.0523$

(d) exact = 2