Chapter One

Section 1.0

1. $m = \frac{y-9}{x-3}$. If x = 2.97, then $m = \frac{-0.1791}{-0.03} = 5.97$. If x = 3.001, then $m = \frac{0.006001}{0.001} = 6.001$.

   If x = 3 + h, then $m = \frac{(3+h)^2 - 9}{(3+h) - 3} = \frac{9 + 6h + h^2 - 9}{h} = 6 + h$. When h is very small (close to 0), 6 + h is very close to 6.

3. $m = \frac{y - \mathbf{4}}{x-2}$. If x = 1.99, then $m = \frac{-0.0499}{-0.01} = 4.99$. If x = 2.004, then $m = \frac{0.020016}{0.004} = 5.004$.

   If x = 2 + h, then $m = \frac{\{ (2+h)^2 + (2+h) - 2 \} - 4}{(2+h) - 2} = \frac{4+4h+h^2 + 2+h - 2 - 4}{h} = 5 + h$. When h is very small, 5 + h is very close to 5.

5. All of these answers are **approximate**. Your answers should be close to these numbers.

   (a) average rate of temperature change $\approx \frac{80^{o} - 64^{o}}{1 \text{ pm} - 9 \text{ am}} = \frac{16^{o}}{4 \text{ hours}} = 4^{o}$ per hour.

   (b) at 10 am, temperature was rising about 5^{o} per hour.
   at 7 pm, temperature was rising about -10^{o} per hour (**falling** about 10^{o} per hour).

7. All of these answers are **approximate**. Your answers should be close to these numbers.
   (a) average velocity $\approx \frac{300 \text{ ft} - 0 \text{ ft}}{20 \text{ sec} - 0 \text{ sec}} =$ 15 feet per second.
   (b) average velocity $\approx \frac{100 \text{ ft} - 200 \text{ ft}}{30 \text{ sec} - 10 \text{ sec}} = -5$ feet per second.
   (c) at t = 10 seconds, velocity $\approx$ 30 feet per second (between 20 and 35 ft/s).
   at t = 20 seconds, velocity ≈ -1 feet per second.
   at t = 30 seconds, velocity ≈ -40 feet per second.

9. (a) A(0) = 0, A(1) = 3, A(2) = 6, A(2.5) = 7.5, A(3) = 9.
   (b) the area of the rectangle bounded below by the x–axis, above by the line y = 3, on the left by the vertical line x = 1, and on the right by the vertical line x = 4.
   (c) Graph of y = A(x) = 3x.

Section 1.1

1. (a) 2 (b) 1 (c) DNE (does not exist) (d) 1

3. (a) 1 (b) –1 (c) –1 (d) 2

5. (a) –7 (b) (13/0) DNE

7. (a) 0.54 (remember, we are using radian mode) (b) –0.318 (c) –0.54

9. (a) 0 (b) 0 (c) 0   10. (a) –1 (b) +1 (c) DNE (does not exist)

11. (a) 0 (b) –1 (c) DNE

13. $\lim_{x \to 0^-} g(x) = 1$ $\lim_{x \to 0^+} g(x) = 1$ $\lim_{x \to 0} g(x) = 1$

    $\lim_{x \to 2^-} g(x) = 1$ $\lim_{x \to 2^+} g(x) = 4$ $\lim_{x \to 2} g(x)$ does not exist

$\lim_{x\to 4^-} g(x) = 2$ $\lim_{x\to 4^+} g(x) = 2$ $\lim_{x\to 4} g(x) = 2$

$\lim_{x\to 5^-} g(x) = 1$ $\lim_{x\to 5^+} g(x) = 1$ $\lim_{x\to 5} g(x) = 1$

15. (a) 1.0986 (b) 1 17. (a) 0.125 (b) 3.5

19. (a) $A(0) = 0$, $A(1) = 2.25$, $A(2) = 5$, $A(3) = 8.25$
 (b) $A(x) = 2x + x^2/4$
 (c) the area of the region bounded below by the x–axis, above by the line $y = x/2 + 2$, on the left by the vertical line $x = 1$, and on the right by the vertical line $x = 3$.

Section 1.2

1. (a) 2 (b) 0 (c) DNE (does not exist)
 (d) 1.5

3. (a) 1 (b) 3 (c) 1 (d) ≈ 0.8

5. See Fig. 1.2P5 .

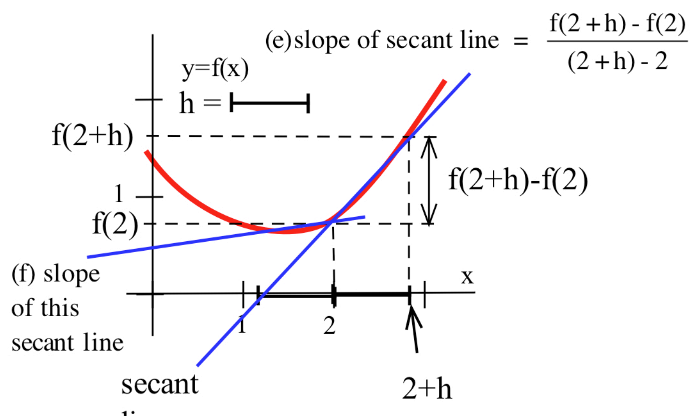

Fig. 1.2P5

7. (a) 2 (b) –1 (c) DNE (d) 2
 (e) 2 (f) 2 (g) 1 (h) 2 (i) DNE

9. (a) When $v = 0$, $L = A$.

 (b) $\lim_{v\to c^-} A\sqrt{1 - \frac{v^2}{c^2}} = 0$

11. (a) 4 (b) 1 (c) 2 (d) 0 (e) 1 (f) 1

13. (a) Slope of the line tangent to the graph of $y = \cos(x)$ at the point $(0,1)$. (b) Slope = 0.

15. (a) ≈ 1 (b) ≈ 3.43 (c) ≈ 4

17. at $x = -1$: a at $x = 0$: b at $x = 1$: c at $x = 2$: d
 at $x = 3$: c at $x = 4$: b at $x = 5$: a

19. Verify each step.

21. Several different lists will work. Here is one example.

 Put $a_n = 1/(n\pi)$ for $n = 1, 2, 3, \ldots$ so a_n approaches 0 and $\sin(a_n) = \sin\left(\frac{1}{1/(n\pi)}\right) = \sin(n\pi) = 0$ for all n.

 Put $bn = \frac{1}{2n\pi + \pi/2}$ for $n = 1, 2, 3, \ldots$ so bn approaches 0 and
 $\sin(bn) = \sin(2n\pi + \pi/2) = \sin(\pi/2) = 1$ for all n.

 Therefore, $\lim_{h\to 0} \sin(1/x)$ does not exist.

Section 1.3

1. Discontinuous at 1, 3, and 4.
3. (a) Discontinuous at $x = 3$. Fails condition (i) there.
   (b) Discontinuous at $x = 2$. Fails condition (i) there.
   (c) Discontinuous where $\cos(x)$ is negative, (e.g., at $x = \pi$). Fails condition (i) there.
   (d) Discontinuous where x^2 is an integer (e.g., at $x = 1$ or $\sqrt{2}$). Fails condition (ii) there.
   (e) Discontinuous where $\sin(x) = 0$ (e.g., at $x = 0, \pm\pi, \pm 2\pi, \ldots$). Fails condition (i) there.
   (f) Discontinuous at $x = 0$. Fails condition (i) there.
   (g) Discontinuous at $x = 0$. Fails condition (i) there.
   (h) Discontinuous at $x = 3$. Fails condition (i) there.
   (i) Discontinuous at $x = \pi/2$. Fails condition (i) there.

5. (a) $f(x) = 0$ for at least 3 values of x , $0 \le x \le 5$.
   (b) 1 (c) 3 (d) 2 (e) Yes. It does not have to happen, but it is possible.

7. (a) $f(0) = 0, f(3) = 9$ and $0 \le 2 \le 9$. $c = \sqrt{2} \approx 1.414$
   (b) $f(-1) = 1, f(2) = 4$ and $1 \le 3 \le 4$. $c = \sqrt{3} \approx 1.732$
   (c) $f(0) = 0, f(\pi/2) = 1$ and $0 \le 1/2 \le 1$. $c =$ (inverse sine of $1/2$) ≈ 0.524
   (d) $f(0) = 0, f(1) = 1$ and $0 \le 1/3 \le 1$. $c = 1/3$
   (e) $f(2) = 2, f(5) = 20$ and $2 \le 4 \le 20$. $c = (1 + \sqrt{17})/2 \approx 2.561$.
   (f) $f(1) = 0, f(10) \approx 2.30$ and $0 \le 2 \le 2.30$. $c =$ (inverse of $\ln(2)$) $= e^2 \approx 7.389$.

9. Neither student is correct. The bisection algorithm converges to the root labeled C.

11. (a) D
    (b) D
    (c) hits B

13. $[-0.9375, -0.875], \approx -0.879$
    $[1.3125, 1.375], \approx 1.347$
    $[2.5, 2.5625], \approx 2.532$

15. $[2.3125, 2.375], \approx 2.32$.

17. $[-0.375, -0.3125], \approx -0.32$.

19. See the three graphs in Fig. 1.3P19.

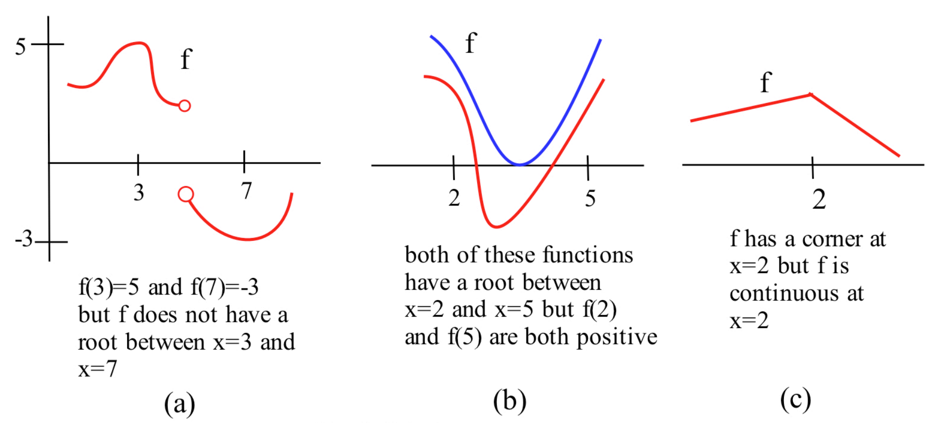

Fig. 1.3P19

21. (a) $A(2.1) - A(2)$ is the area of the region bounded below by the x–axis, above by the graph of f, on the left by the vertical line $x = 2$, and on the right by the vertical line $x = 2.1$.

$$\frac{A(2.1) - A(2)}{0.1} \approx f(2) \text{ or } f(2.1) \text{ so } \frac{A(2.1) - A(2)}{0.1} \approx 1.$$

   (b) $A(4.1) - A(4)$ is the area of the region bounded below by the x–axis, above by the graph of f, on the left by the vertical line $x = 4$, and on the right by the vertical line $x = 4.1$. $\dfrac{A(4.1) - A(4)}{0.1} \approx f(4) \approx 2$.

23. (a) Yes. You supply the justification. (b) Yes (c) Try it.

Section 1.4

1. (a) If x is within **0.5** unit of 3. (b) If x is within **0.3** unit of 3.
   (c) If x is within **0.02** unit of 3. (d) If x is within $\varepsilon/2$ unit of 3.

3. (a) If x is within **0.25** unit of 2. (b) If x is within **0.1** unit of 2.
   (c) If x is within **0.02** unit of 2. (d) If x is within $\varepsilon/4$ unit of 2.

5. Problem 1: slope = 2, $\delta = \varepsilon/2$. Problem 3: slope = 4, $\delta = \varepsilon/4$.
   General pattern: $\delta = \varepsilon/slope$ for linear functions

7. Each board must be within 0.06/3 = 0.02 inches of 10 inches in length.

9. (a) $1.957433821 < x < 2.040827551$ (b) $1.995824623 < x < 2.004158016$

11. (a) $0 < x < 8$ (b) $2.99920004 < x < 3.00080004$

13. Each piece of wire must be within 0.005996404 inches of 5 inches.

15. & 17. See Figures

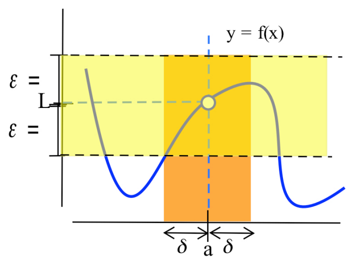

Fig. for Problem 15

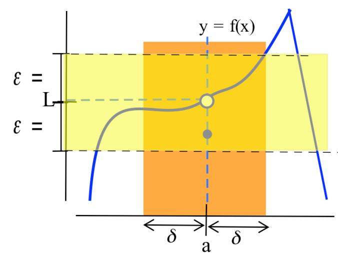

Fig. for Problem 17

19. Take $\varepsilon = 1/2$ (or smaller).

    If $x > 2$ and $|f(x) - L| < \varepsilon = 1/2$ then $|2 - L| < 1/2$ so $3/2 < L < 5/2$.

    If $x < 2$ and $|f(x) - L| < \varepsilon = 1/2$ then $|3 - L| < 1/2$ so $5/2 < L < 7/2$.

    There is no value of L that is both larger than 5/2 and smaller than 5/2 so the limit does not exist.

21. Take $\varepsilon = 1/2$ (or smaller) and suppose x is within 1 of 2 ($1 < x < 3$).

    If $1 < x < 2$ and $|f(x) - L| = |x - L| = |L - x| < \varepsilon = 1/2$ then $-1/2 < L - x < 1/2$

    so $x - 1/2 < L < x + 1/2$ and $L < 2.5$.

    If $2 < x < 3$ and $|f(x) - L| = |f(x) - L| = |L - 6 + x| < \varepsilon = 1/2$ then $-1/2 < L - 6 + x < 1/2$

    so $5.5 < L + x < 7.5$ and $2.5 < L$.

    There is no value of L that is both larger than 2.5 and smaller than 2.5 so the limit does not exist. .

23. This proof is very similar to the proof of the second theorem on page 9.

Assume that $\lim\limits_{x \to a} f(x) = L$ and $\lim\limits_{x \to a} g(x) = M$. Then, given any $\varepsilon > 0$, we know $\varepsilon/2 > 0$ and that there are deltas for f and g , δ_f and δ_g , so that

if $|x - a| < \delta_f$, then $|f(x) - L| < \varepsilon/2$ ("if x is within δ_f of a, then f(x) is within $\varepsilon/2$ of L", and

if $|x - a| < \delta_g$, then $|g(x) - M| < \varepsilon/2$ ("if x is within δ_g of a, then g(x) is within $\varepsilon/2$ of M").

Let δ be the smaller of δ_f and δ_g. If $|x - a| < \delta$, then $|f(x) - L| < \varepsilon/2$ and $|g(x) - M| < \varepsilon/2$

so $|(f(x) - g(x)) - (L - M))| = |(f(x) - L) + (M - g(x))|$ (rearranging the terms)

$\leq |f(x) - L| + |M - g(x)|$ (by the Triangle Inequality for absolute values)

$< \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon.$ (by the definition of the limits for f and g).