Math 151

Show Your Work! March 11, 2018 Name ______________________________

Good Luck! Test #3A *(please print)*

1. **Show your work**. No work = no points.

   (a) $x^3y^2 + 2e^x = 2y^3 - 14$. $\dfrac{dy}{dx}$ = ______________________________

   At (0,2) slope = ______

(7)

   (b) $f(x) = Ae^{4x} + \dfrac{B}{x} + \cos(Cx) + 7$. (A, B and C are constants.)

   Calculate $f''(x) =$

(5)

2. True or False (circle the correct answer)

(2) True False If g(x) is continuous and decreasing for $1 \le x \le 6$ then $g'(4) < 0$.

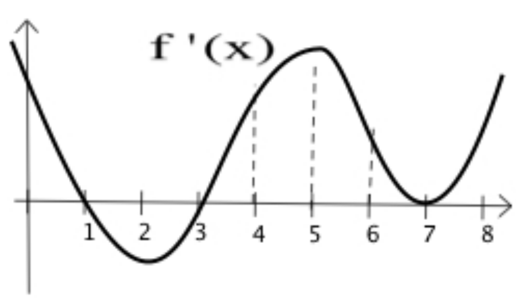
(2) True False If f(3) is a local minimum for f, then $f'(3) = 0$.

(2) True False If $g'(2)=0$ then g(2) is a local max or local min.

(2) True False If $\lim_{x\to 1} g(x) = 0$, then $\lim_{x\to 1}\{f(x)/g(x)\}$ does not exist.

(2) True False If $f'(2)=0$ and $f''(2)=3$ then f has a local minimum at x=2.

3. The graph of **y = f '(x)** is shown.



(2) (a) At x=1 f has a local MAX MIN NEITHER (circle one).

(2) (b) At x=7 f has a local MAX MIN NEITHER (circle one).

(2) (c) At x = 6 f is INCREASING DECREASING (circle one)

(2) (d) At x=4 f is concave UP DOWN NEITHER (circle one)

(2) (e) f has an Inflection Point at x = ____

4. $f(x) = 2x^3 - 9x^2 + 2$ on the interval $-1 \le x \le 5$. **Use CALCULUS** to answer these.

(4) (a) f has critical numbers at x = ____________________

(2) (b) f has local maximum(s) at x = ____________________

(2) (c) The global minimum value of f is _____________

(2) at x = _____________

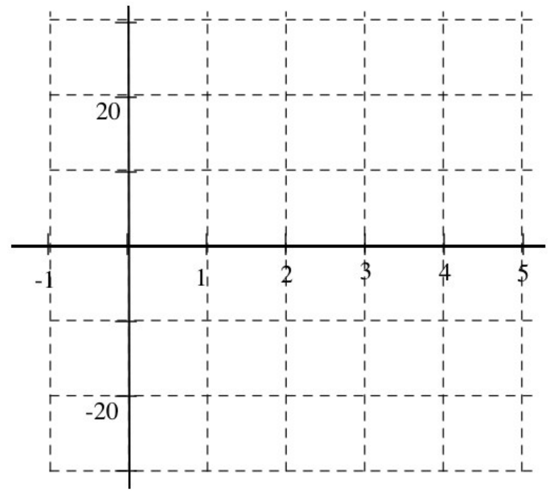
(2) (d) at x=3 the graph of f(x) is concave UP DOWN NEITHER

(2) (e) f has Inflection Point(s) at x = ______________

(4) (f) Sketch a **good** graph of f

(4) (g) According to the Mean Value Theorem there is a value x=c between -1 and 5 so that f '(c) = ____________

(Show your **calculus** work. No work = no points.)



5. **Use L'Hopital's Rule** to calculate these limits.

(a) $\lim_{x \to 0} \dfrac{5x + \sin(3x)}{4x} =$

(5)

(b) $\lim_{x \to \infty} \dfrac{x^2 + 5}{x \cdot \ln(x)} =$

(5)

6. $f(x) = Cx^4 - 12Tx^2$ (C and T are positive constants).

Find all x so that $f\,'(x) = 0$. x = ____________

(4)

7. Write the equation for each asymptote of $f(x) = \dfrac{4x^2 - 16}{x^2 - x - 2}$

(3) (a) f has horizontal asymptote(s) at ___________________

(3) (b) f has vertical asymptote(s) at ___________________

8. Write the **equation** for the horizontal asymptote(s) of $f(x) = \dfrac{3x^2 - x \cdot \sin(x)}{2x^2 + 77}$. ___________________

(3)

9. Some quickies

(a) $D(\ln(7x + \cos(x)))$ = __________________

(3)

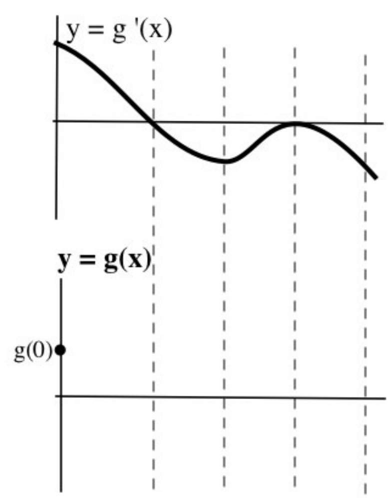
(b) The units of x are dollars and the units of y are miles.

Then the units of $\dfrac{d^2y}{dx^2}$ = y '' are _____________

(3)

10. The graph of y = g '(x) is shown on the top graph and g(0) is given on the bottom graph. On the lower axis sketch a good graph of y = g(x) .

(4)



11. Do **TWO** of these max/min problems. (If you do all 3 I will only grade A ad B. (7 points each)

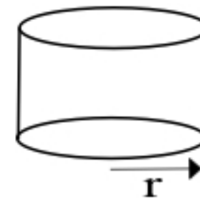
A. You have 72 square inches of tin to make into a cylindrical can (see figure).

Use calculus to find the dimensions of the can will maximize the volume of the can.

(Data: $V = \pi r^2 h$, surface area $= 2\pi r^2 + 2\pi rh$)

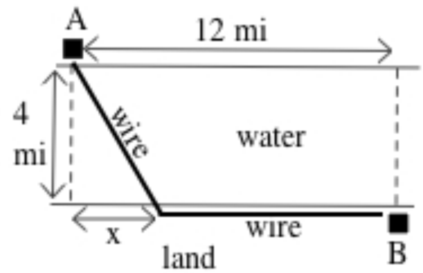
r= ________________ h = ______________ (2 decimal places)

(Show your work. Organize your work so I can understand it.



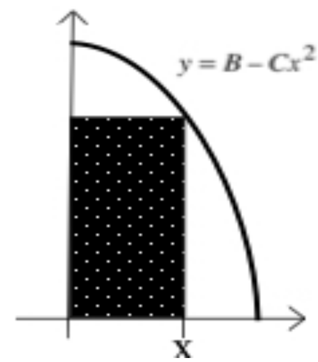
B. You want to connect towns A and B with a data wire (see figure). It costs $10 per mile for wire in the water and $2 per mile for wire on land. What value of x will minimize the total cost? x = ______________ (2 decimal places)

(Use calculus and show your work)



C. A rectangle has base on the x-axis and one corner on the curve $y = B - Cx^2$.

Find the value of x that maximizes the area of the rectangle. x = _________



BONUS (+2 if correct)

Find a function f so that $f'(x) = 6x^2 + 2\cos(x) + 3e^x$ and f(0) = 7 f(x) = ____________________________

The end!! (Total = 102 + 2 bonus.) Tomorrow: test back